Comparison Between Various Methods of Parametric System Identification Using Lab View

Parth S. Thaker, Prof. Chirag S. Dalal

Department of Instrumentation and Control

Dharamsinh Desai University, Nadiad.

ABSTRACT

This paper discusses and represents the comparison between results of various methods for parametric and non parametric system identification using Lab-VIEW. Here system identification is concerned as control point of view. Controller design for a linear system can be estimated by knowing the process dynamics. The results of system identification by various well known methods such as Box-Jerkins (BJ), Recursive BJ, Auto-Regressive (ARX), Auto-Regressive Moving Average X (ARMAX), and output-error(OE) method are shown.

Keywords - System Identification, Transfer function estimation, linear system modeling.

1. INTRODUCTION.

System identification and modeling of linear process models has remained always a key problem for control system design. To achieve robust and optimized control of any linear system without knowing its dynamics is near to impossible task. Sometimes it is also required for the testing purpose of the new operating process conditions and/or new control strategies. Also most effective means of efficiency enhancement of any process is improvement of its control system. An essential tool for such an improvement is a valid system model. Methods for obtaining such a model are found in various open literatures however some methods are specific to particular system only. The objective of this paper is to represent the most commonly used methods for system identification and model design such as Auto Regressive method(ARX), Auto Regressive Moving Average method (ARMAX), Output-Error method(OE), Box-Jerkins method(BJ) and Recursive ARMAX method. The basic requirement for the identification of any system is the data. For estimation of any process with block diagram as shown below in fig 1. following data is required in adequate amount.

1.) Process Variable Data(PV)
2.) Manipulated Variable Data(MV)
3.) Set Value(SV)
4.) Controller parameters.

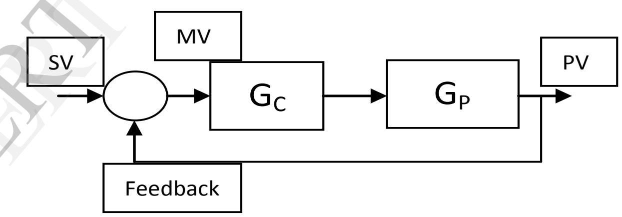

Fig 1. A closed loop system.

Another important task linked with the system identification and modeling is the validation of the derived model. The designed model of any system should represent the actual system in every possible mean and tries to follow the input output characteristics of actual process. Lab VIEW is very powerful tool for completion of such task. Model estimation and validation is done in the 2nd part and comparison responses of the models derived by the above mentioned methods are compared in 3rd part. The conclusion and comments are given in the 4 part.

2. LAB VIEW SYSTEM IDENTIFICATION TOOL KIT.

System identification, the first step in the model-based control design process, involves building mathematical models of a dynamic system based on a set of measured stimulus and response data samples. We can use system identification in a wide range of applications, including mechanical engineering, biology, physiology, meteorology, economics, and model-based control design. For example, engineers use a system model of the relationship between the fuel flow and the shaft speed of a turbojet engine to optimize the efficiency and operational stability of the engine. Biologists and physiologists use system identification techniques in areas such as eye pupil response and heart rate control. Meteorologists and economists build mathematical models based on historical data for use in forecasting.

From the various function pallets available with the LAB View we will use the following for the case study of system identification:

1.) ARX MODEL
2.) ARMAX MODEL
3.) Box Jenkins MODEL and
4.) Output Error MODEL

2.1 ARX MODEL:

This section includes the algorithms and references used by the si_Est ARX Model (SISO).vi, which is the core VI to estimate the coefficients of an ARX (SISO) model using the input-output data of a system. The si_Est ARX Model (SISO).vi is located in the labview, vi.lib, add-ons, System Identification, Parametric Estimation Subs.llb directory.

The ARX (SISO) model is described as

$$A(q)y(n) = B(q)u(n-k)+e(n)$$

Where, $A(q) = 1 + \sum_{i=1}^{N_a} a_i\, q^{-i}$,

$$B(q) = [B_1(q), B_2(q), B_3(q), \ldots .B_N(q)]$$

$$B_i(q) = 1 + \sum_{j=0}^{N_{ib-1}} b_{ij}\, q^{-j}$$

$$u(n-k) = [u_1(n-k_1), u_2(n-k_2), u_3(n-k_3) \ldots u_N(n-k_N)]^T$$

k is the delay of the system, and $u(n)$, $y(n)$, and $e(n)$ are the input, output, and disturbance of a system, respectively. q is the backward shift operator, which means

$$q^{-i}y(n) = y(n-i) \ldots\ldots\ldots(1)$$

The purpose of this VI is to estimate the coefficients $[[a_1, a_2, a_3 \ldots\ldots\ldots]$ and $[b_1, b_2, b_3 \ldots\ldots\ldots]$ using the input-output data of a system.

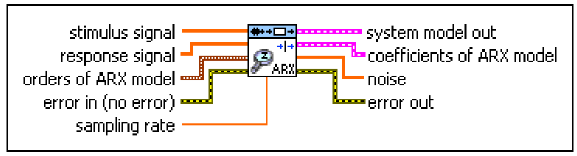

Fig 2. Block of SI Est. ARX Model (SISO).vi.**[5]**

Fig 2 shows the block element for the identification of the ARX model with the required connections.

2.1.1 Least Mean Square Algorithm[1]

The least square solution for the ARX model estimation is

$$\hat{\theta}_N^{LS} = \left[\frac{1}{N}\sum_{t=0}^{N} \varphi(t)\varphi^T(t)\right]^{-1} \frac{1}{N}\sum_{t=0}^{N} \varphi(t)y(t)$$

..........(2)

where

$$\varphi(t) = [-y(t-1) - y(t-2) \ldots - y(N_a - 1)u(t)u(t-1) \ldots u(t - N_b + 1)]^T$$

You can rewrite Equation 2 as the solution of the linear equations:

$$AX = Y$$

where

$$A = \begin{bmatrix} \varphi^T(p) \\ \varphi^T(p+1) \\ \vdots \\ \varphi^T(N) \end{bmatrix}, X = \begin{bmatrix} a_1 \\ \vdots \\ a_{N_a} \\ b_0 \\ \vdots \\ b_{N_b-1} \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} y(p) \\ y(p+1) \\ \vdots \\ y(N) \end{bmatrix}$$

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2.2 ARMAX MODEL:

This section includes the algorithms and references used by the si_Est ARMAX Model (SISO).vi, which is the core VI to estimate the coefficients of an ARMAX (SISO) model based on the input output data of a system. The si_Est ARMAX Model (SISO).vi is located in the labview, vi.lib, add-ons, System identification, parametric estimation subs.llb directory.

The ARMAX (SISO) model is described as

$$A(q)y(n) = B(q)u(n-k)+c(q)e(n)\ldots\ldots(3)$$

where

$$A(q) = 1 + \sum_{i=1}^{N_a} a_i\, q^{-i}$$

$$B(q) = 1 + \sum_{i=0}^{N_{b-1}} b_i\, q^{-i} \; and$$

$$C(q) = 1 + \sum_{i=0}^{N_c} c_i\, q^{-i}$$

u(n), *y(n)* and *e(n)* are the input, output, and disturbance of a system, respectively. *q* is the backward shift operator, which means

$$q^{-i}y(n) = y(n-i)$$

k is the delay of the system. The purpose of this VI is to estimate the coefficients

$[a_1, a_2, a_3 \ldots\ldots a_N]$, $[b_1, b_2, b_3 \ldots\ldots b_{N_b-1}]$ and $[c_1, c_2, c_3 \ldots\ldots c_{N_c}]$ based on the input-output data of a system.

The multi-stage method is applied to have a coarse estimation for *A(q)*, *B(q)*, and *(q)*, and then the Gauss-Newton**[3]** minimization method is applied to refine the results of *A(q)*, *B(q)*, and *C(q)*.

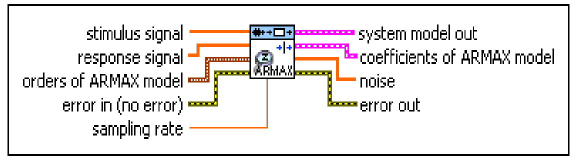


Fig 3 Block for SI Est. ARMAX Model (SISO).vi.**[5]**

2.2.1 Multi-Stage coarse estimation[2]

This method is generally described on page 337 of Ref [2]. Here is the deduction specific for an ARMAX model.

Let

$$v(t) = c(q)e(t)$$

Then Equation 3 becomes

$$A(q)y(t)=B(q)u(t-k)+ v(t)$$

Since *v(t)* here is not white Gaussian noise, the System Identification Toolkit applies the instrumental variable (IV) method to estimate *A(q)* and *B(q)*. You then can have

$$\hat{v}(t) = \hat{A}(q)y(t)-\hat{B}(q)u(t-k) = C(q)e(t) \quad \ldots\ldots(4)$$

Here, as elsewhere, $\hat{f}$ denotes the estimation of f. Thus, Equation 4 can be rewritten as

$$\frac{1}{C(q)}\hat{v}(t) = e(t)$$

Equation 4 can be treated as a high order AR model, though theoretically, it represents an AR model with infinite orders. However, in practice, this kind of AR model is selected as the dimension of the system $N_a+N_b+N_c$. With the AR model estimation, we have an estimation for *e(t)*. Since the $\hat{v}(t)$ and $\hat{e}(t)$ are known now, Equation 4 can be rewritten as

$$\hat{v}(t) = (C(q)-1)\hat{e}(t)+e(t)$$

Equation represents one form of ARX model. *C(q)* can be estimated by using ARX model estimation methods with $\hat{v}(t)$ and $\hat{e}(t)$ as the output and input, respectively.

2.3 BOX JENKINS MODEL:

This section includes the algorithms and references used by the si_Est BJ Model (SISO).vi, which is the core VI to estimate the coefficients of a Box-Jenkins (BJ) (SISO) model based on the input-output data of a system. The si_Est BJ Model (SISO).vi is located in the labview, vi.lib, add-ons, System Identification, Parametric Estimation Subs.llb directory.

A BJ (SISO) model is described as

$$y(n) = \frac{B(q)}{F(q)}u(n-k) + \frac{C(q)}{D(q)}e(n)$$

….(5)

where

$$B(q) = \sum_{i=0}^{N_b-1} b_i q^{-i}$$

$$F(q) = 1 + \sum_{i=1}^{N_f} f_i q^{-i}$$

$$C(q) = 1 + \sum_{i=1}^{N_c} C_i q^{-i}$$

$$D(q) = 1 + \sum_{i=1}^{N_d} D_i q^{-i}$$

and q is the backward shift operator, which means:

$$q^{-i}y(n) = y(n-i) \qquad \ldots(6)$$

k is the delay of the system. The purpose of this function is to estimate the coefficients $[b_1, b_2, b_3 \ldots\ldots b_{N_b-1}]$ $[f_1, f_2, f_3 \ldots\ldots f_{N_f}]$, $[c, c_2, c_3 \ldots\ldots c_{N_c}]$ and $[d_1, d_2, d_3 \ldots\ldots d_{N_d}]$ based on the input-output data of a system.

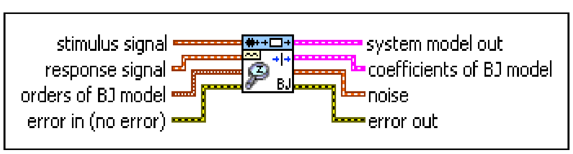


Fig. 4 Block of SI Est. BJ Model (SISO).vi. **[5]**

2.3.1 Multi-Stage Coarse Estimation[2]

This method is generally described on page 337 of Ref [3]. Here is the deduction specific for BJ model.

$$\varepsilon(n) = F(q)\frac{C(q)}{D(q)}e(n)$$

Then Equation 6 becomes

$$F(q)y(n) = B(q)u(n-k) + \varepsilon\ (n)$$

We then apply the instrumental variable (IV) method to estimate *B(q)* and *F(q)* and have the following equation:

$$v\hat{}(n) = y(n) - \frac{B\hat{}(q)}{F\hat{}(q)}u(n-k) = \frac{C(q)}{D(q)}e(n)$$

Here, as elsewhere, $f\hat{}$ denotes the estimation of f. If N_c=0 above Equation can be re-written as

$$D(q)v\hat{}(n) = e(n)$$

Above equation can be treated as an AR model. With the AR model estimation, you have an estimation for *D(q)*.

If $N_c \neq 0$ above Equation can be re-written as

$$\frac{D(q)}{C(q)}V\hat{}(n) = e(n)$$

Then a high order AR model is applied to estimate $e\hat{}(n)$. Since the $v\hat{}(n)$ and $e\hat{}(n)$ are known now, above Equation can be rewritten as

$$D(q)v\hat{}(n) = (C(q)-1)e\hat{}(n) + e(n)$$

Above Equation is one form of the ARX model. *C(q)* and *D(q)* can be estimated by using ARX model estimation methods with $v\hat{}(n)$ and $e\hat{}(n)$ as the output and input, respectively.

2.4 OE MODEL:

This section includes the algorithms and references used by the si_Est OE Model (SISO).vi, which is the core VI to estimate the coefficients of an output-error (OE) (SISO) model based on the input-output data of a system. The si_Est OE Model (SISO).vi is located in the labview, vi.lib, add-ons, System Identification, Parametric Estimation Subs.llb directory.

An OE (SISO) model is described as

$$y(n) = \frac{B(q)}{F(q)} u(n-k) + e(n) \quad \text{........(7)}$$

where,

$$B(q) = \sum_{i=0}^{N_b - 1} b_i q^{-i}$$

$$F(q) = 1 + \sum_{i=1}^{N_f} f_i q^{-i}$$

u(n), *y(n)* and *e(n)* are the input, output, and disturbance of a system respectively. *q* is the backward shift operator, which means

$$q^{-i} y(n) = y(n-i) \quad \text{........ (8)}$$

k is the delay of the system. The purpose of this VI is to estimate the coefficients $[b_1, b_2, b_3 \dots \dots b_{N_b - 1}]$ and $[f_1, f_2, f_3 \dots \dots f_{N_f}]$ based on the input-output data of a system.

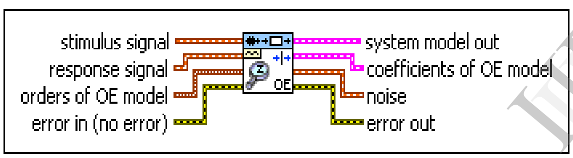


Fig. 5 Block of SI Est. OE Model (SISO).vi **[5]**

The multi-stage method is applied to have a coarse estimation for *B(q)* and *F(q)*, and then the Gauss-Newton minimization method is applied to refine the results of *B(q)* and *F(q)*.

2.4.1 Multi-Stage Coarse Estimation[2]

This method is generally described on page 337 of Ref [3]. Here is the deduction specific for OE model.

Let

$$v(t) = F(q)e(t)$$

Then Equation 7 becomes

$$F(q)y(t) = B(q)u(t-k) + v(t)$$

Above Equation is in the form of ARX model. Since *v(t)* here is not white Gaussian noise, the System Identification Toolkit applies the instrumental variable (IV) method to estimate *F(q)* and *B(q)*.

3. SIMULATED SYSTEM.

For comparison among various methods of system identification we need to identify the same system using the above mention methods and then compare the results. For this purpose we will consider a notch band reject filter with a very narrow rejection band. The Transfer function of the filter is:

$$G(s) = \frac{0.9907s^2 + 0.6123s + 0.9907}{s^2 + 0.6123s + 0.9813}$$

[4]

This filter is closed loop adaptive IIR notch filter for the suppression of narrowband interference in the Direct Sequence Spread Spectrum (DSSS) communication system. The concept is applicable to secure satcom systems especially UHF communications where the communication channels are affected by narrow band interferences.

The notch filter can also be perceived as a band stop filter with a high Q factor. For this filter the Q=100.

4. SYSTEM IDENTIFICATION AND COMPARISION.

With the above mentioned system in section 3 by providing the input sequence to the system we will generate the output sequence. By means of these two data sets of input and output we will identify the system in between and then validate the system model. We will then compare the results of various methods used for system identification.

Procedure for the system identification using LabVIEW is as mentioned below:

4.1 Importing the data into LabVIEW.

This task can be done by several different ways, one of the easiest method is to use the "read spreadsheet.vi" to directly read the data stored in the excel file. Other way is to read the data from the text file. In this experiment we will read our data from the text files.

4.2 Selecting the signal for identification

From the available data for identification of system we will use a portion of data for identification and other for the validation of the identified model. For this purpose we will use the "SI Split signal.vi" as shown in figure below.

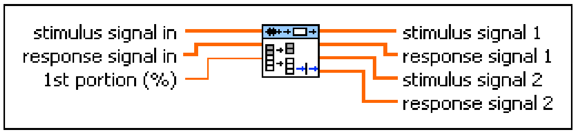


Fig. 6 SI Split signal.vi **[5]**

Here we will divide the available signal into 66% and 34% ratio the later will be used for the identification and the remaining is used for the validation of the identified model.

4.3 System Identification

For the identification of the model we will use (one by one) different VI's which are mentioned in the section 2.

For instance let us select the case of Box-Jenkins model, referring to Fig. 4 in section 2 and Fig.6 wire the stimulus signal-1 of SI Split Signal.vi to stimulus signal of SI Est. BJ Model.vi. Similarly wire the response signal-1 of SI Split Signal.vi to response signal of SI Est. BJ Model.vi. Now create a cluster control for the "orders of BJ model" terminal. This cluster contains 5 numeric controls viz.

i) Order of B

ii) Order of F

iii) Order of C

iv) Order of D and

v) Delay.

Now create a cluster indicator of 4 arrays to view the coefficients of the Box-Jenkins model. Connect this cluster to the "coefficients of BJ model" terminal of SI Est. BJ model.vi.

Similarly by connecting the appropriate terminals among the VI's we can estimate other models.

4.4 Validation of models

For the validation of the identified models we will compare their output with the actual system output. For this purpose we will use SI Model simulation.vi. This VI provides the response of the simulated model when feed with the stimulus signal.

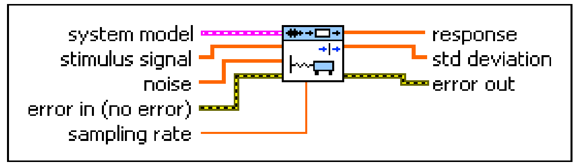


Fig. 7 SI Model Simulation.vi **[5]**

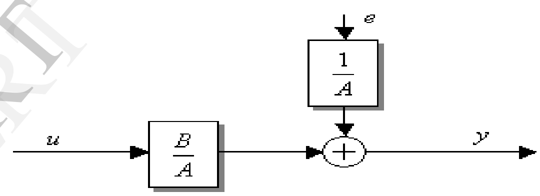
Response signal of the VI shown in the figure 7 is then compared with the actual system output.

5. RESULTS.

This section discusses the results obtained by performing the experiment mentioned in the previous sections. It discusses the obtained model and its response for the same stimulus signal. The comparison between the simulated response of each identified model and the actual model are also shown.

5.1 ARX MODEL:

The realization of ARX model is as shown in figure 8.



$$A(q)y(t) = B(q)u(t-k) + e(t)$$

Fig. 8 ARX Model

The coefficients of identified ARX model and simulated response of the identified model are shown in figure 9 and 10 respectively.

Equation

$$\frac{1.00009s^2 + 0.618086s + 1.00004}{1.00007s^2 + 0.618052s + 1}$$

Fig. 9 ARX model from its identified coefficients

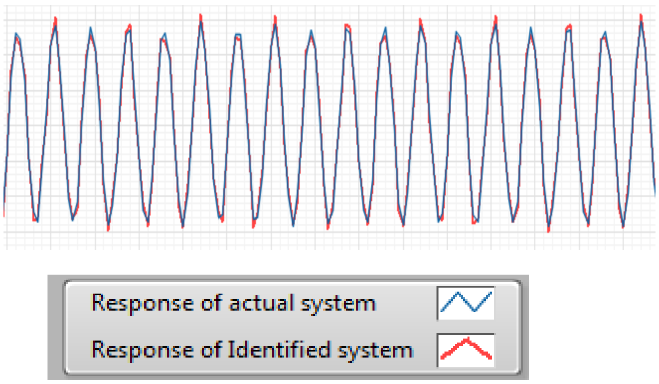
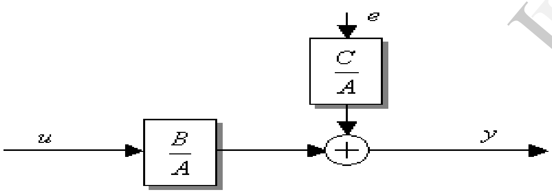


Fig. 10 Comparison of responses of identified ARX model and response of actual system

5.2 ARMAX MODEL:

The realization of ARMAX model is as shown in figure 11.



$$A(q)y(t) = B(q)u(t-k) + C(q)e(t)$$

Fig. 11 Realization of ARMAX model

The system coefficients and the error coefficients are shown in fig 12 and the comparison of responses of identified model and actual response of system are shown in fig 13.

Equation ARMAX

$$\frac{1.00007s^2 + 0.618074s + 1.00003}{1.00003s^2 + 0.618043s + 1}$$

Equation Disturbance

$$\frac{0.63024s^2 + 0.988002s + 1}{1.00003s^2 + 0.618043s + 1}$$

Fig. 12 Equations for ARMAX model

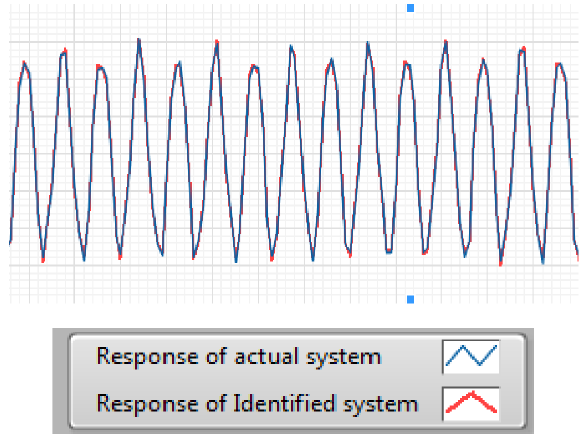
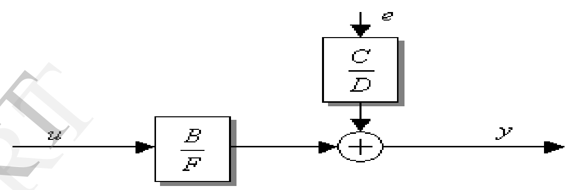


Fig. 13 Comparison of responses of identified ARMAX model and response of actual system

5.3 BOX-JENKINS MODEL:

The realization of ARMAX model is as shown in figure 14.



$$y(t) = \frac{B(q)}{F(q)}u(t-k) + \frac{C(q)}{D(q)}e(t)$$

Fig. 14 Realization of Box-Jenkins model

The system coefficients and the error coefficients of Box-Jenkins model are shown in fig 15 and the comparison of responses of identified model and actual response of system are shown in fig 16.

Equation BJ model

$$\frac{0.982474s^2 + 0.6168s + 1.01646}{0.999897s^2 + 0.617993s + 1}$$

Equation BJ Disturbance

$$\frac{0.992468s + 1}{0.347343s + 1}$$

Fig. 15 Equation of BJ model

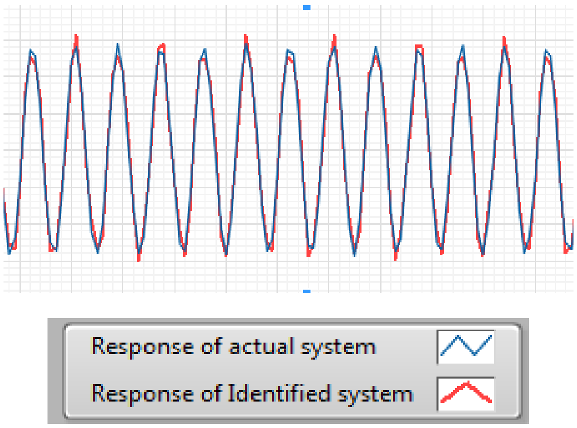
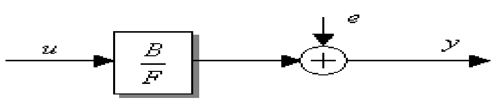


Fig. 16 Comparison of responses of identified BJ model and response of actual system

5.4 OUTPUT ERROR MODEL:

The realization of OE model is as shown in fig 17.



$$y(t) = \frac{B(q)}{F(q)} u(t-k) + e(t)$$

Fig. 17 Realization of OE model

The system coefficients and the error coefficients of Output-Error model are shown in fig 18 and the comparison of responses of identified model and actual response of system are shown in fig 19.

Equation OE model

$$\frac{1.00006s^2 + 0.618077s + 1.00004}{0.999963s^2 + 0.618021s + 1}$$

Fig. 18 Equation of OE model

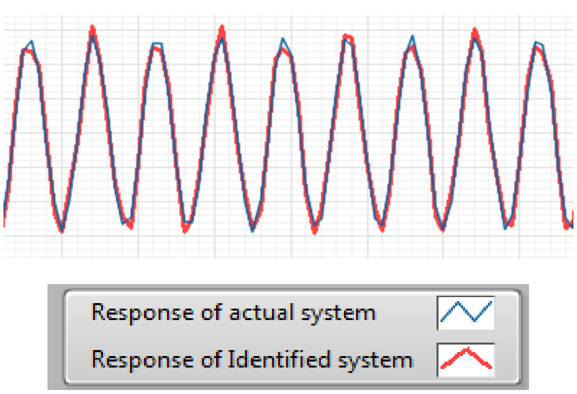


Fig. 19 Comparison of responses of identified OE model and response of actual system

6. CONCLUSION.

So we had identified the same system using the different models available in the LabVIEW system identification toolkit and compared their response with the actual system response. The Box-Jenkins model seems to be the best representation of the system. Other models responses are also consistent and are acceptable too.

7. REFERENCES

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