Comparative Study On Image Restoration Techniques Using The Partial Differential Equation And Filters

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Abstract

This paper explains how image restoration can be achieved by using Partial Differential Equations i.e., Heat Equation and Perona-Malik Equation and other conventional filters. In conventional filters like Mean, Median and Wiener filters the original image is lost as it is not done under a mathematical framework which is reversible. Whereas PDE based model uses a standard mathematical technique which is able to reverse the filtering process, provided some portion of original data is available. Based on the type of noise present in the image, the partial differential equation based approach gives better results. In this paper denoising is applied to digital images.

Keywords: Restoration, Partial Differential Equations, Linear Filters, Heat Equation, Perona-Malik Diffusion Equation.

1. Introduction

Image restoration is the operation of taking a corrupted/noisy image and estimating the clean original image. Image restoration is an essential pre-processing step for many image analysis applications. The proposed method focuses on comparison between denoising using partial differential equations and denoising using linear filters. This paper deals with image restoration using partial differential equations and evaluates performance of the proposed method and compares its performance with the Mean filter, Median filter and Wiener filter. The experiments are performed using various noisy images of Lena and Cameraman. The images are in JPEG format. The images are of size 256 x 256. Image noise is the random variation of brightness or colour information in images produced by the sensor and circuitry of a scanner or digital camera. Image noise is considered as an undesirable by-product of image capture. The types of Noise are following :- (i) *Speckle noise* - Speckle noise is a granular noise that inherently exists in and degrades the quality of the active radar and synthetic aperture radar (SAR) images. SAR is caused by unified processing of backscattered signals from multiple distributed targets. (ii) *Amplifier noise* (*Gaussian noise*) - The standard model of amplifier noise is additive. Gaussian noise is independent at each pixel and independent of the signal intensity. (iii) *Salt-and-pepper noise* - An image containing salt-and-pepper noise will have dark pixels in bright regions and bright pixels in dark regions. (iv) *Shot noise* (*Poisson noise*) -Poisson noise or shot noise is a type of electronic noise that occurs when the finite number of particles that carry energy i.e. electrons in an electronic circuit or photons in an optical device, is small enough to give rise to noticeable statistical fluctuations in a measurement. The Partial Differential Equations (PDEs) used for denoising are Heat Equation (HE) and Perona-Malik Equation (PME). Heat Equation implements isotropic diffusion process where as Perona-Malik Equation implements anisotropic diffusion process. Isotropic diffusion smoothens the entire image region including edges uniformly. Anisotropic diffuses the image region in all directions at the same time preserving the edges. Perona–Malik diffusion, also called anisotropic diffusion, is a method which aims at reducing image noise without removing significant parts of the image content, mainly edges, lines or other details that are important for the interpretation of the image. By using a constant diffusion coefficient the Perona-Malik equation can be reduced to heat equation which works like a Gaussian filter. This is good for removing noise but at the same this blurs edges also. When the diffusion coefficient is replaced by a seeking function, the resulting equations enables diffusion within the region and limits the diffusion across the edges. Hence the edges can be retained while removing noise from the image.

3. Literature Survey

The fundamental and the simplest of these algorithms is the Mean Filter. It is also known as Average filter and it is poor in preserving edges. The Median Filter is a non linear filtering method which scans each pixel by pixel of an image and replaces each pixel with the median of the neighboring pixels. It removes impulsive or salt and pepper noise in an image. The main goal of Wiener filter approach is to filter out the noise that has corrupted a signal. Wiener filter is generalized based on statistical approach [1]. The PDE based methods are used for denoising, edge detection, image enhancement, image segmentation, inpainting etc. In paper [2] PDE based approach is used for image restoration so that the damaged region is inpainted without blurring the edges. It diffuses the information equally in all directions but at the same time preserves the boundary region. The results show that they have good inpainting. Antoni Buades et al [3] have proposed a non local means algorithm, which computes the average of all pixels in the image by adding weights, which is based on the similarity between the pixels. The similarity between the pixels depends on the similarity of the intensity gray level vectors. A pixel in the nearby region of neighbourhood gives larger weight where as pixels away from the neighbourhood gives smaller weight. Non local means compares the grey level in a single point and also the geometrical configurations in the whole neighbourhood. The main difference of non-local means algorithm with respect to local filters or frequency domain filters is the systematic use of all possible self-predictions that the image noise reduction and edge protection capabilities can provide. The paper, A new multiscale filter based on a relatively new entropy manipulation method [4], uses a generalized neighbourhood operation where all pixels inside the neighbourhood window is updated in such a way that the spatial entropy inside the neighbourhood window is increased. The interesting similarity between the proposed filter and Perona-Malik filter is that the proposed filter can be considered as a generalization to the Perona-Malik equation. This filter when compared with the Perona-Malik filter, gives better and improved performance.

4. Mathematical Formulation

Heat Diffusion equation in 2D and 3D can be

generalized as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad \text{(2D)}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \quad \text{(3D)}$$

$\partial u/\partial t = \partial^2 u/\partial x^2$, describes the diffusion of temperature. u denotes the temperature, t denotes time and (x, y, z) denotes the 3 spatial coordinates.

Consider the bounded domain $\Omega \subset R^n$, with n = 1, and use the one-dimensional Perona-Malik Equation (PME).

$$u_t = \frac{\partial}{\partial x}(D(u)u_x) \text{ where } D(u) = \frac{1}{1+\epsilon|u_x|^2}$$

i.e. $u_t = \left(\frac{u_x}{1+\epsilon|u_x|^2}\right)_x$ where $0 < \varepsilon < 1$.

The above 1D PME is applied to each row of the image separately.

5. Results

Noisy images are given as input and Heat Equation and Perona-Malik Equation are applied on it. Now the output image obtained after applying these PDE's, are compared by the output image obtained by using the following conventional filters: Mean, Median and Wiener filters. The Peak signal-to-noise ratio (PSNR) is commonly used for measurement of the quality of reconstructed image. With the PSNR we will try to give the quality of the denoised image compared to the noisy image or compared to the original image. The PSNR is defined using the mean squared error (MSE) which is given by

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[I(i,j) - K(i,j)]$$

Where I and K are images and M, N are the size of the image respectively, in both directions. We will consider

K the noisy approximation of I. Let us define MAX_I as the maximum possible pixel value of the image so we have $MAX_I = 1$. With this the PSNR is defined as

$$PSNR = 10.log_{10}\left(\frac{MAX_I^2}{MSE}\right)$$

$$= 20.log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$$

$$= 20.log_{10}\left(\frac{1}{\sqrt{\frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[I(i,j)-K(i,j)]^2}}\right)$$

Figure: 1.a Speckle Noise Added Image, 1.b Applying Mean Filter, 1.c Applying Median Filter, 1.d Applying Wiener Filter, 1.e Applying HE,1.f Applying PME.

TableNo: 1

	Speckle Noise	
Methods	**MSE**	**PSNR**
Mean Filter	21.6188	69.5650
Median Filter	31.3011	66.3504
Wiener Filter	19.8040	70.3265
Heat Equation	21.4860	69.6185
Perona-Malik Equation	0.0678	119.6325

Figure: 2.a Gaussian Noise Added Image, 2.b Applying Mean Filter, 2.c Applying Median Filter, 2.d Applying Wiener Filter, 2.e Applying HE, 2.f Applying PME.

Table No: 2

	Gaussian Noise	
Methods	**MSE**	**PSNR**
Mean Filter	26.7276	67.7224
Median Filter	29.069	66.9927
Wiener Filter	24.876	68.3460
Heat Equation	27.3074	67.5360
Perona-Malik Equation	0.0915	117.0288

Table No: 3

	Salt and Pepper Noise	
Methods	**MSE**	**PSNR**
Mean Filter	24.0928	68.6239
Median Filter	25.4038	68.1636
Wiener Filter	21.5290	69.6011
Heat Equation	23.8686	68.7051
Perona-Malik Equation	0.0766	118.5743

Figure: 3.a Salt and Pepper Noise Added Image, 3.b Applying Mean Filter, 3.c Applying Median Filter, 3.d Applying Wiener Filter, 3.e Applying HE, 3.f Applying PME.

Figure: 4.a Poisson Noise Added Image, 4.b Applying Mean Filter, 4.c Applying Median Filter, 4.d Applying Wiener Filter, 4.e Applying HE, 4.f Applying PME.

Table No: 4

	Poisson Noise	
Methods	**MSE**	**PSNR**
Mean Filter	15.1958	72.6272
Median Filter	17.5840	71.3593
Wiener Filter	12.8261	74.0997
Heat Equation	15.7698	72.3051
Perona-Malik Equation	0.0416	123.8901

6. Conclusion

The performance of the PDE based model (Heat Equation and Perona-Malik Equation) are compared with some denoising filters like Mean, Median and Wiener filter. Depending on the noise involved, the PME has desirable results. The salt and pepper noise is the worst of all the four noises mentioned in this paper. This is because the percentage of changed pixels in the resultant image is more for salt and pepper noise. But when Gaussian noise is involved the denoised image with the Perona Malik Equation nicely resembles with the original image, meaning that edges are preserved.

7. References

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Topics: (a) Digital signal processing, (b) Anisotropic diffusion, (c) PSNR, (d) Von Neumann stability analysis, (e) Greyscale, (f) Image noise, (g) Gaussian noise, (h) Heat Equation, (i) Scale space, (j) Sarrus' rule

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