

# Comparative Study of Soil Slopes Using Static and Pseudo-Static Approaches

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**Abstract** – This paper presents a comparative study of three embankment slopes by both static and pseudo-static approaches. In pseudo-static approach, the effects of an earthquake are represented by constant vertical ( $k_v$ ) and horizontal ( $k_h$ ) seismic acceleration coefficients. The vertical force has less influence on the factor of safety (FOS) because it reduces both the driving force and resisting force and hence neglected here. The analysis is done by Fellenius method. Using GEO5 these embankment slopes are analyzed by different methods of slope stability and the results are compared. It is observed that the factor of safety is affected by type of soil, density, shear strength parameters and earthquake loadings.

**Keywords** - Factor of Safety. Static and pseudo static. Fellenius method. Horizontal seismic acceleration. GEO5

## INTRODUCTION

Slope stability is required in the design and construction of embankments, earth dams, railways, roadways, levees and other geotechnical structures. When an embankment is constructed, the load increases and hence the pore pressure also increases. With the passing of time, this excess pore water pressure dissipates to reach the equilibrium condition. The effective stress is minimum immediately

after the completion of construction and it increases gradually afterwards. Hence the most critical condition occurs at the end-of-construction or undrained condition. Again during earthquakes, the effect of induced ground shaking causes failure of slopes that were marginally stable before earthquakes.

In the present study, three embankment slopes have been analyzed in critical condition (undrained condition) by Fellenius and Jumikis method for  $c-\phi$  soil. The same slopes are also analyzed by different methods with the help of GEO 5 and a comparison is made amongst them.

## Methods for Slope Stability Analysis

Baker, 1980 [ 1] suggested a slip circle for determining the factor of safety using the Spencer method and used the dynamic programming. Celestino and Duncan, 1981 [3] also described a method for determining the critical noncircular slip surfaces using Spencer's method. Cala and Flisiak, 2001 [4] determined the factors of safety of slopes having different geometry with respect to the shear strength reduction technique, and the results were compared with the conventional limit equilibrium analysis. Shahgoli et al. 2001 [10] suggested the horizontal slice method for analysing the slope in seismic

condition at reinforced soil slopes. The slip surface was assumed and that surface was divided into a large no of horizontal slices.

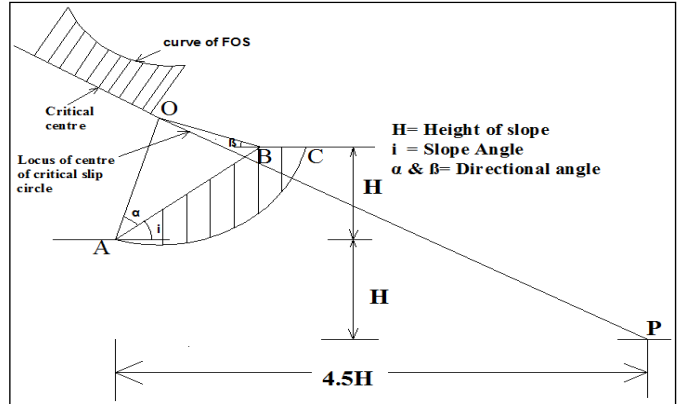
The slope-stability analysis is usually carried out by Fellenius (Fellenius, 1936) [16] method. It ignores inter slice forces and considers only moment equilibrium. It provides only moment factor of safety. It provides the factor of safety on lower side and thus results into a more conservative design of slope. The Janbu method (Janbu, 1956) [17] considers inter slice normal force only and ignores the inter slice shear force. It provides force equilibrium only and gives force factor of safety. The Bishop simplified method (Bishop, 1960) [15] considers inter slice normal force only, it ignores inter slice shear force. The slip surface is assumed to be circular and gives moment factor of safety only. The Morgenstern-Price method (Morgenstern and Price, 1965) [18] considers both inter slice normal force and shear force and provides both moment equilibrium and force equilibrium. It gives both moment and force factor of safety. Spencer method (Spencer, 1967) [19] also considers both inter slice normal and shear force and gives the moment and force factors of safety.

*Analysis by Fellenius Method*

Since the determination of the minimum factor of safety for a slope is extremely important for design, it is important to locate the most critical slip circle with as few trials as possible. In a random trial and error approach, the three geometric parameters such as the centre of rotation, the radius of slip circle and the distance of intercept in front of the toe are varied and the minimum factor of safety is obtained. This requires a very large number of trials. In order to reduce the number of trials and to find the centre of critical slip circle, Fellenius(1936) suggested an empirical procedure to find the centre of the most critical circle in a  $\phi_u=0$  soil. As shown in figure1, the centre O for the toe failure condition can be located at the intersection of the two lines drawn from the ends A and B of the slope at angle  $\alpha$  and  $\beta$  which are called directional angles. The angles  $\alpha$  and  $\beta$  vary with the slope angle  $i$  (Table 2). For a homogeneous  $c-\phi$  soil, the centre of slip circle lie on a line OP, in which the point P has its co-ordinates H (height of slope) downwards from toe and 4.5 H horizontally away as shown in Figure 1. The failure wedge is divided into a number of vertical slices of equal thickness 0.2 m. Trial centres are assumed on OP and factor of safety corresponding to each centre is calculated by method of slices. These various factors of safety so obtained are plotted as ordinates on the corresponding centres and a

smooth curve is drawn. The centre corresponding to the lowest factor of safety is the critical circle centre.

For the pseudo static analysis, a horizontal ( $k_h W$ ) and a vertical ( $k_v W$ ) force will act at the CG of each of the slices. The various parameters of a representative slice are detailed in Figure 2.



**Fig 1** Fellenius method for locating centre of critical slip circle.

The forces acting on the wedge slices have been calculated by the following parameters:

- $i$  = slope angle in degrees.
- $W$  = weight per unit length in kN/m.
- $K_h$  = horizontal seismic coefficient.
- $K_v$  = vertical seismic coefficient.
- $S$  = Shear strength along the slip surface.
- $\Phi$  = angle of internal friction in degrees.

**Table 1** Recommended Horizontal Seismic Coefficient [11]

Horizontal seismic coefficient, $K_h$	Description	Recommended FOS
0.1	“severe” earthquakes	> 1.0
0.2	“violent, destructive” earthquakes	
.5	“catastrophic” earthquakes	

The vertical seismic coefficient has less influence on the factor of safety because it reduces both the driving force and the resisting force and hence it is neglected here.

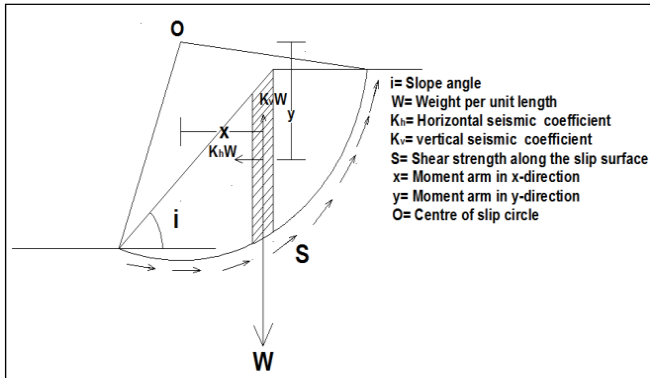


Fig 2 Force diagram of soil above failure surface in pseudo static slope stability analysis

Considering total length of the slip surface =  $\hat{L}$

Driving forces at static condition =  $\Sigma T$

Driving forces at pseudo static condition =  $\Sigma T + K_h W$

Driving moment at static condition =  $\Sigma T \times x$

Driving moment at pseudo static condition =  $(\Sigma T \times x) + (K_h W \times y)$

Normal stress,  $N = W \times \gamma$

Resisting forces =  $C\hat{L} + \Sigma N \tan \phi$

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Resisting forces =  $C\hat{L} + \Sigma N \tan \phi$

Resisting moment =  $(C\hat{L} + \Sigma N \tan \phi) R$

The factors of safety against sliding are-

$$\text{Static factor of safety} = \frac{C\hat{L} + \sum N \tan \phi}{\sum T \times x}$$

$$\text{Pseudo static factor of safety} = \frac{(C\hat{L} + \sum N \tan \phi) R}{(\sum T \times x) + (k_h \times y)}$$

The physical parameters of the slopes and types of soil are Shown in Table 2. The  $K_h$  values are chosen as 0.1, 0.2 and 0.5 as recommended by Terzaghi (Table 1) for different earthquake conditions.

**Table 2** Fellenius criteria for locating the centre circle of a slope in a  $\phi = 0$  soil [13]

Slope angle ( $i^\circ$ )	Slope ratio	Directional angle	
		( $\alpha^\circ$ )	( $\beta^\circ$ )
60	0.58:1	29	40
45	1:1	28	37
33.8	1.5:1	26	35
26.6	2:1	25	35
18.4	3:1	25	35
11.3	5:1	25	35

#### Result and discussion:

The results have been represented by graphs to illustrate the effects of the type of soil, density and shear strength parameters on the factor of safety of soil slope. Fig. 3 shows the change in FOS with  $k_h$  value. It is observed that FOS decreases with increase in earthquake loading. But in every case FOS is higher at MDD than at field density. Fig. 4 shows the effect on density on FOS. In all the cases FOS increases with the increases of density of soil. The cohesion of soil ( $C$ ) also influences the FOS. In Fig. 5 it is clear that FOS increases with the increases of  $C$  value. The factor of safety increases with the decrease of the angle of internal friction. Since the field soil is at the drier side of OMC, the deviator stress is more in field condition than in MDD condition. Hence, the angle of shearing resistance, which is the slope of the failure envelope, is higher in field condition than in MDD condition. But the overall shear strength of soil is more in MDD than field. That is why the FOS is increasing in spite of decrease of the  $\phi$  value. With the increases of angle of internal friction ( $\phi$ ), the FOS of slop decreases. While comparing the different types of soils it was seen that clayey soils possess higher FOS than silty sands at different earthquake loadings Fig 7.

Table 3 Details of slopes and types of soil

Slope	Slope length (m)	Slope angle	Soil type	Specific gravity	MDD (kN/m <sup>3</sup> )	Optimum Moisture content (%)	Liquid Limit (%)	Plastic Limit (%)	Plasticity Index
A	14.20	40°	Clayey sand	2.67	19.50	11.50	29.00	21.74	7.26
B	10.70	42°	Silty sand	2.39	19.76	11.76	20.20	17.69	2.51
C	17.00	39°	Silty sand	2.63	19.60	12.00	18.15	15.37	2.78

Table 4 Properties at field density

Slope	C kN/m <sup>2</sup>	$\phi$	W (%)	$\gamma$ kN/m <sup>3</sup>	Static FOS	pseudo-static FOS		
					$K_h=0.0$	$K_h=0.1$	$K_h=0.2$	$K_h=0.5$
A	16.70	31°	10.75	18.98	1.73	1.38	1.23	0.77
B	11.63	21°	11.52	19.68	1.19	1.02	0.88	0.59
C	13.00	24°	11.52	19.48	1.21	1.02	0.88	0.58

Table 5 Properties at MDD

Slope	C kN/m <sup>2</sup>	$\phi$	W (%)	$\gamma$ kN/m <sup>3</sup>	Static FOS	pseudo-static FOS		
					$K_h=0.0$	$K_h=0.1$	$K_h=0.2$	$K_h=0.5$
A	120.70	17°	11.50	19.50	6.411	5.135	4.204	2.708
B	124.60	19°	11.76	19.76	5.522	4.288	3.884	2.592
C	109.82	22°	12.00	19.60	5.943	4.050	3.543	2.494

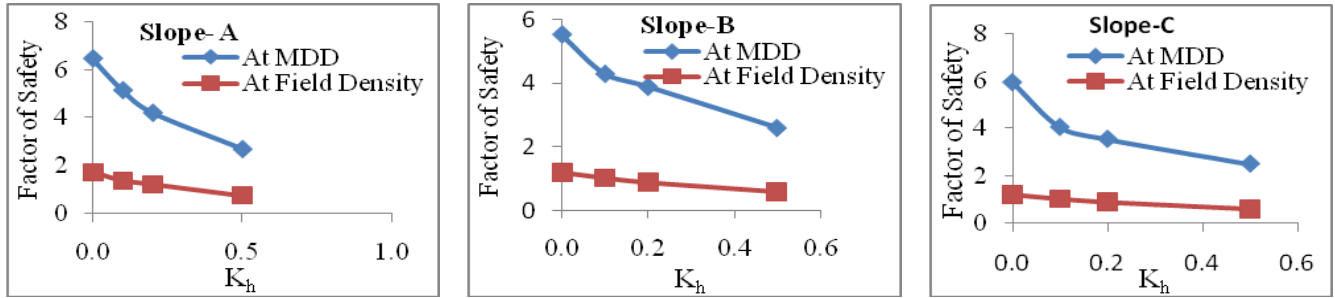


Fig. 3 Variation of factor of safety with earthquake loading ( $k_h$ )

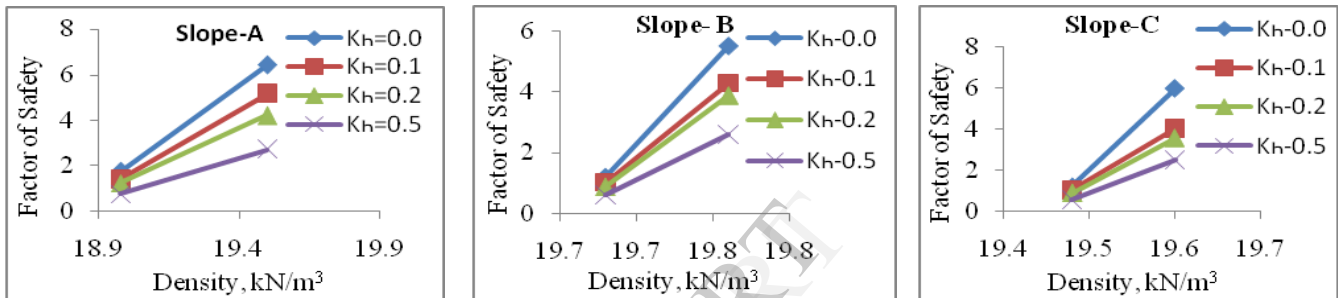


Fig. 4 Variation of factor of safety at different densities of soli

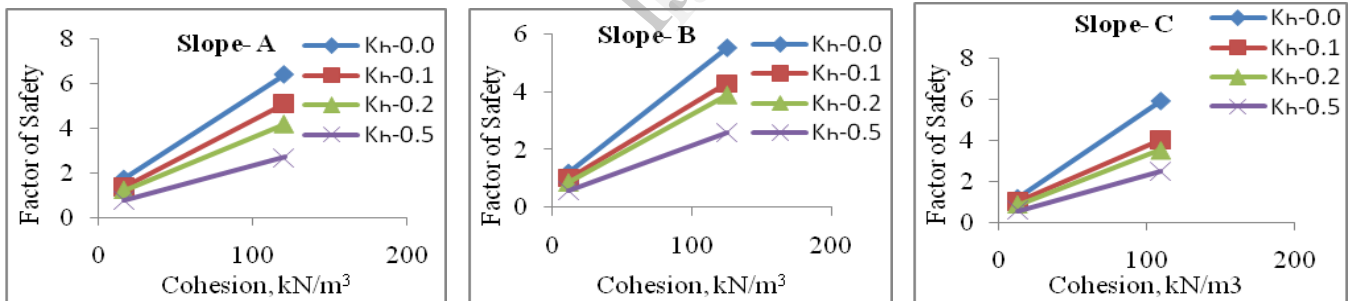


Fig. 5 Variation of factor of safety at different cohesion values (C)

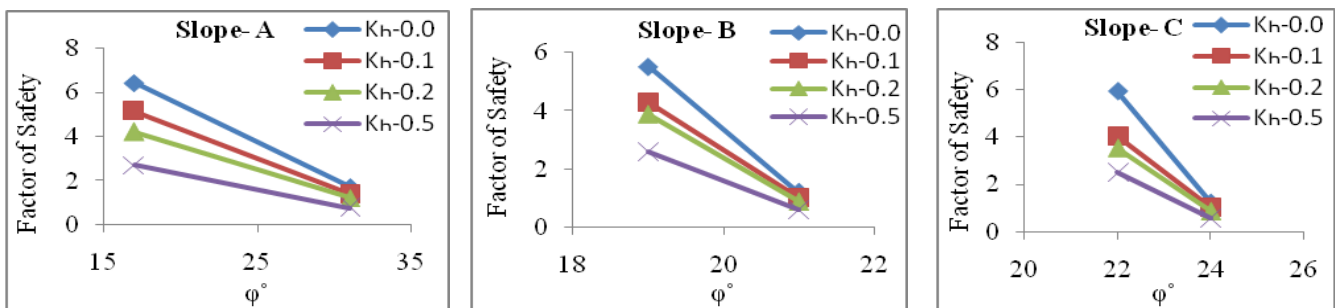


Fig. 6 Variation of factor of safety at different angle of shearing Resistance ( $\phi$ )

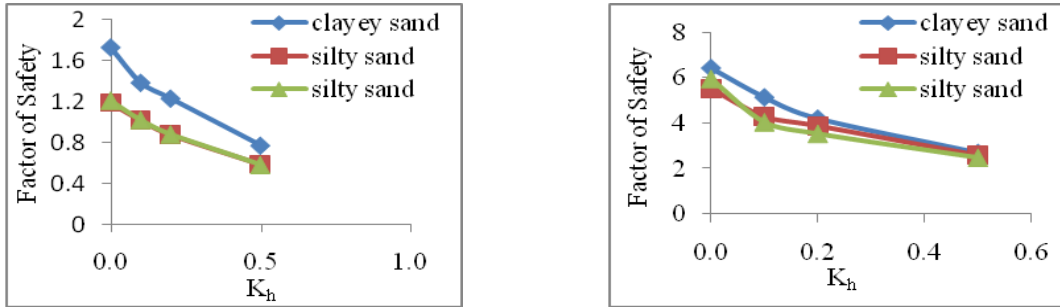
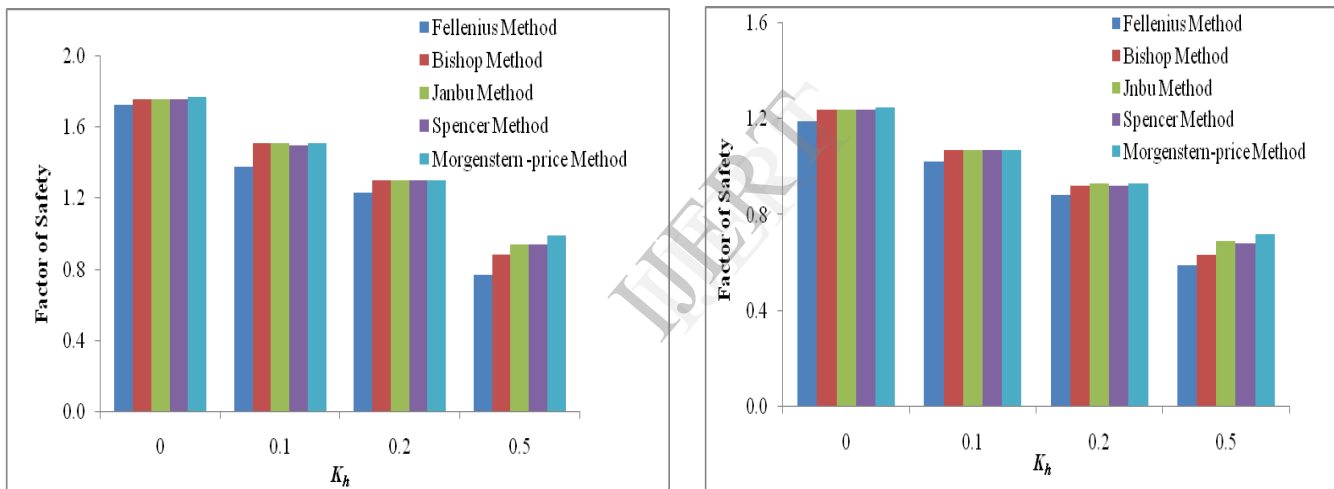
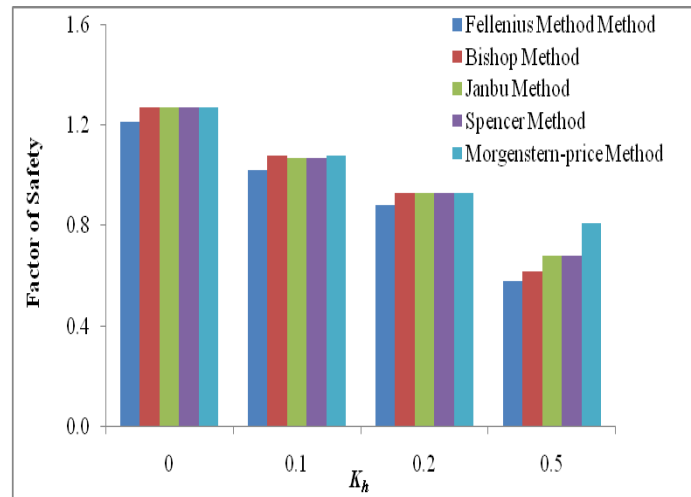


Fig. 7 Variation of factor of safety with  $k_h$  value at different types of soil.



(a)

(b)



(c)

Fig. 8 Variation of factor of safety with  $k_h$  value calculated by different methods of slope stability analysis for (a) Slope-A (b) Slope- B and (c) Slope- C

### Summary and Conclusions

From the analysis made and the results presented, following conclusions have been made:

- The factor of safety decreases with the increases of the earthquake loadings at both field density and maximum dry density conditions.
- The factor of safety increases with the increases of the density of soil.
- As cohesion of soil increases, the factor of safety increases.
- A soil type affects the factor of safety. At both field density and maximum dry density conditions, the static and pseudo static factor of safety changes in the following order:

Clayey sand > silty sand

- Slope stability analysis by Fellenius method gives the lowest factor of safety rather than all other different methods. Therefore, the design is more conservative or uneconomical.
- The factor of safety increases with the decrease of the angle of internal friction. Since the field soil is at the drier side of OMC, the deviator stress is more in field condition than in MDD condition (Figure 3). Hence, the angle of shearing resistance, which is the slope of the failure envelope, is higher in field condition than in MDD condition. But the overall shear strength of soil is more in

MDD than field. That is why the FOS is increasing in spite of decrease of the  $\phi$  value.

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