

Comparative Study of Optimization Approaches for the Issue of out of Band Power Emission in OFDM Systems

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Abstract— Out of band power emission is considered as a serious issue in multi carrier modulation techniques such as orthogonal frequency division multiplexing (OFDM). Precoding is a good solution for this issue. Precoder maps the original modulated signal vector to a new vector, whose elements are close to that of the original but the out of band power spectrum is minimal. In this paper, we made a comparative study of two recently introduced precoder formulations.

Keywords—OFDM; Out of band emission; Precoder

I. INTRODUCTION

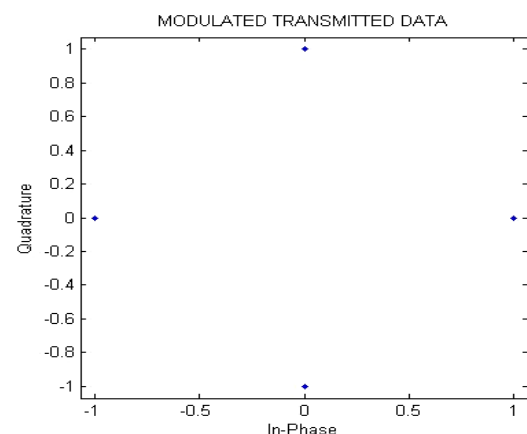
Wireless communication fields such as broadcasting and mobile cellular systems choose multicarrier modulation techniques in order to avoid multipath fading, inter symbol interference etc. This kind of modulation technique can be efficiently implemented by using Fourier bases and is called orthogonal frequency division multiplexing (OFDM). In this paper, we focus on the OFDM related issue known as out of band power emission.

Out of band power emission is leakage of power to adjacent band during multicarrier transmission. Generally OFDM systems do not assign any data on the subcarriers near the edges of the band of transmission. These unused subcarriers are named as guard bands [1]. There are predefined radio frequency masks defined by different standards which are kept as reference in order to achieve lower out of band power emission. Nulling of power at undesired part of the spectrum is achieved by the precoders. This operation is called shaping the power spectrum. This can reduce significant amount of power at the out of band region of the OFDM signal.

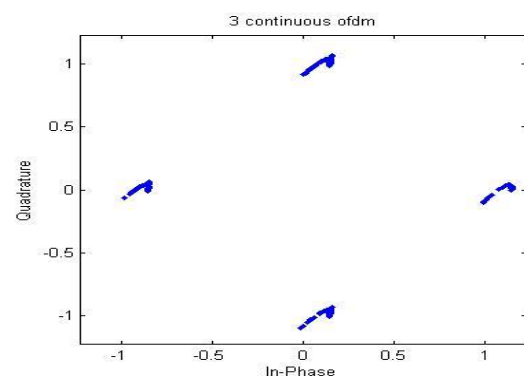
Many algorithms are proposed to solve the issue of out of band power emission. The aim is to attain low powers at the edge of the band by shaping the power spectrum of OFDM signal, without compromising the bit error rate (BER). There are different optimization formulation for the problem. Letter [2] proposes a method called subcarrier weighting to significantly suppress the OFDM sidelobes, by 10 dB. The precoder designed in [3] is based on changing the phase and amplitude of the transmitting signal. Also in that case, OFDM signals are taken such a way that they are consecutively

continuous. The phase rendering results in tilt in the constellation plots, which is shown in Fig. 1.

Another method specified in [4] is shaping the spectrum by introducing nulls at different frequencies. Thus it becomes the formulation of a least square problem. The proposed precoder in [5] finds that there is no need to make nulls in the spectrum, as in [4], instead it tries to reduce the out of band power under a relaxed mask.



(a) Constellation plot for qpsk modulated symbols



(b) Constellation plot for N-continuous OFDM with N=3

Fig. 1. Phase rendering specified in [3]

The precoder proposed in [6] is based on the problem of spectral leakage resulting in interference to licenced users in OFDM based cognitive radio system. This formulation can be compared with the optimization problem in [4].

All these proposals basically maps the modulated symbols to another vector closer to original one but with less power at the out of band region. That is, a precoding is performed prior to the IFFT modulation at the transmitting side.

II. INTERPRETATION

Single-carrier modulation techniques use only one sinusoidal wave at all times, while in the multi-carrier modulation techniques, several sinusoidal waves are used as carriers simultaneously [1]. Thus the high bit rates loaded on a single carrier is reduced to lower bit rates on each subcarrier. In order to make these sub carriers orthogonal, each sub carrier is an integer multiple of a fundamental subcarrier leading to the so called Orthogonal frequency division multiplexing (OFDM).

Essentially, an OFDM symbol in time domain can be visualized as a windowed version of subcarriers loaded with data. The window $s(t)$ is a rectangular pulse with duration T .

$$s(t) = \begin{cases} 1, & -T/2 \leq t \leq T/2 \\ 0, & \text{elsewhere} \end{cases} \quad (1)$$

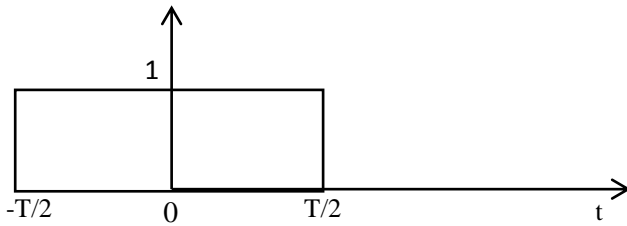


Fig. 2. Window function in (1)

Its Fourier transform is a real sinc function centered at origin.

$$S(\omega) = \frac{\sin(\omega T / 2)}{\omega T / 2} \quad (2)$$

Fig. 2 shows the time domain and Fig. 3 shows the frequency domain representation of rectangular pulse.

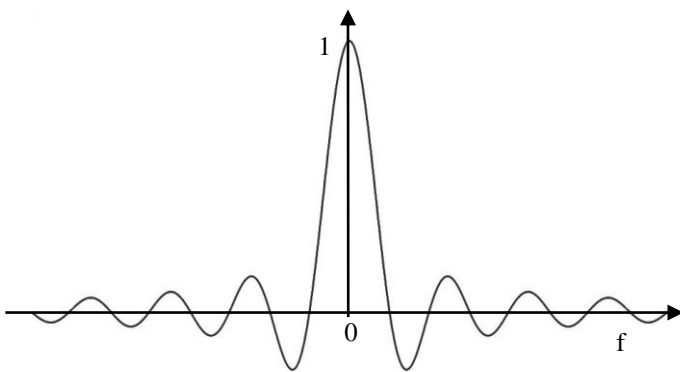
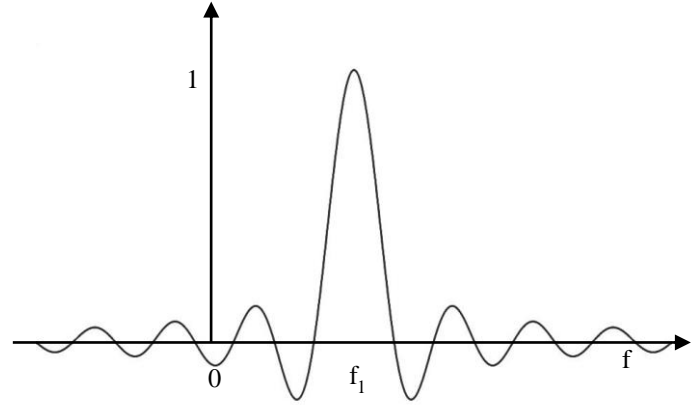
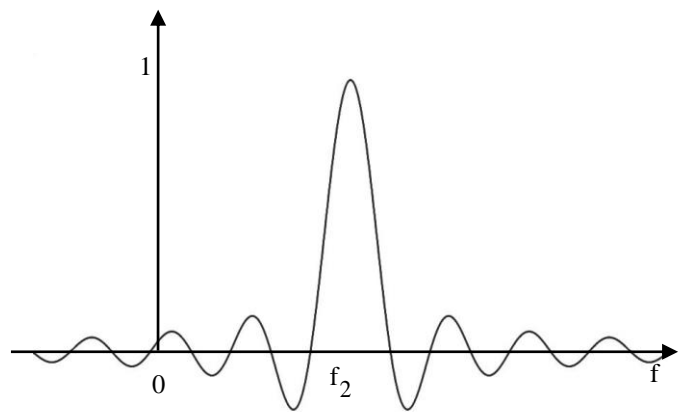


Fig. 3. Fourier transform of (1)

Let f_1, f_2, \dots, f_k be different subcarrier frequencies in an OFDM system. The Fourier transform of subcarriers in the OFDM system are fundamentally shifted version of sinc functions. Fig. 4 shows the fourier transform of two different subcarriers.



(a)



(b)

Fig. 4. Translated OFDM symbol with (a) subcarrier index f_1 (b) subcarrier index f_2

III. SYSTEM MODEL

Here We assume the complex fourier bases used as subcarriers in OFDM are of duration T . We visualize that, infinite duration fouries bases are windowed (or truncated) by the given rectangular pulse (as shown in section II). So the fourier transform of the fourier bases are obtained by convolving Fourier transform of a rectangular pulse (which is a sinc) and fourier transform of infinitely long fourier bases (which are Dirac deltas). This convolution results in shifted version of sinc functions.

Let $\{\omega_{c_1}, \omega_{c_2}, \dots, \omega_{c_m}\}$ be the subcarrier frequencies. Its fourier transform is again sinc function centred at locations $\omega_{c_1}, \omega_{c_2}, \dots, \omega_{c_m}$ and $\{\omega_1, \omega_2, \dots, \omega_n\}$ be

the out of band frequencies where we want the spectral sum to be zero. Let v_i (element of R^n) represent the spectral contribution by i th subcarrier at locations ω_1 to ω_n . It is given by,

$$v_1 = \begin{pmatrix} \text{Tsinc}((\omega_0 - \omega_{c_1})T/2) \\ \text{Tsinc}((\omega_1 - \omega_{c_1})T/2) \\ \vdots \\ \vdots \\ \text{Tsinc}((\omega_n - \omega_{c_1})T/2) \end{pmatrix} \quad (3)$$

The contribution becomes $s_i \cdot v_i$, if s_i is the precoded data, loaded onto i th subcarrier. Since we want at all n locations the sum of spectral contribution by all subcarriers to be zero, we have the relation,

$$s_1 \cdot v_1 + s_2 \cdot v_2 + \dots + s_m \cdot v_m = 0$$

In matrix form,
$$\begin{bmatrix} | & | & & & | \\ v_1 & v_2 & \dots & \dots & v_m \\ | & | & & & | \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \quad (4)$$

or, $As=0$

The optimization problem becomes, $\min_s \|d-s\|_2$ subject to

$As=0$. d be the original information vector given by, $d = [d_1, d_2, \dots, d_m]$.

In paper [4], Beek used accurate sinc function to obtain the matrix A . On the other hand, paper [6] approximates the sinc envelope by $\frac{1}{\omega_i - \omega_{c_k}}$, for $k = [1, 2, \dots, m]$ and $i = [1, 2, \dots, n]$, $n < m$. So they obtained, A matrix as follows,

$$A = \begin{pmatrix} \frac{1}{\omega_1 - \omega_{c_1}} & \frac{1}{\omega_1 - \omega_{c_2}} & \dots & \frac{1}{\omega_1 - \omega_{c_m}} \\ \frac{1}{\omega_2 - \omega_{c_1}} & \frac{1}{\omega_2 - \omega_{c_2}} & \dots & \frac{1}{\omega_2 - \omega_{c_m}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\omega_n - \omega_{c_1}} & \frac{1}{\omega_n - \omega_{c_2}} & \dots & \frac{1}{\omega_n - \omega_{c_m}} \end{pmatrix} \quad (5)$$

Section IV shows the matlab simulation code for obtaining the power spectrum of the OFDM signal with and without precoding. It uses (5) for forming the precoder.

IV. SIMULATION AND RESULTS

All simulations in papers [4],[5] and [6] are verified using matlab version 7.8.0, in a 32 bit windows 7 system. The code shown below is creating A matrix in (5), nulling is done at points $m = [-2999 -3000 -8999 -9000 2999 3000 8999 9000]$.

A. Matlab Simulation

```

clc;
close all;
clear all;

K=1680; %used subcarriers
carrier= [-840:-1 1:840];

wmj=[-2999 -3000 -8999 -9000 2999 3000 8999 9000];

C1=[];C2=[];

for i=1:8 %%%%%%%%% n=8
    C=[]
    for n=1:K %%%%%%%%% m=1680
        C(n)=[1/wmj(i)-carrier(n)]; % equation (15) in [6]
    end
    C1=[C1;C];
end

% A=[];

% for j=1:8
% f=fi(j);
% for k=-840:-1
% a(k+841)=T*exp(-i*pi*(Ts-Tg)*(f-k/Ts))*sinc(T*(f-
% k/Ts));
% end
% for k=1:840
% a(k+840)=T*exp(-i*pi*(Ts-Tg)*(f-k/Ts))*sinc(T*(f-
% k/Ts));
% end
% A=[A;a];
%end

I=eye(K);

P=I-(C1'*inv(C1*C1')*C1);

d=load('d4qam.mat');
x=d.d;

d_bar=P*x; %%%%%%%%% s=Pd

N = 840;
over_sample_factor = 2;
M = N*over_sample_factor;
Mod = 4;
symbol = 1;
bitlength = N*log2(Mod)*symbol;
itr_num = 200;
fft_len = 2*M;
    
```

```

signal_freq = zeros(itr_num,fft_len);

for itr = 1:itr_num
    after_zp = zeros(1,M);
    after_zp(1:N/2) = x(N/2+1:N);
    after_zp(M-N/2+1:M) = x(1:N/2);
    ofdm_symbol = ifft(after_zp);
    signal_freq(itr,:) = abs(fft(ofdm_symbol,fft_len)).^2;
end

PSD_mean = mean(signal_freq,1);

mean_sig_power = mean([PSD_mean(1:N/2)
PSD_mean(fft_len-N/2+1:fft_len)]); % mean power of data
subcarrierse
PSD_mean = fftshift(PSD_mean);
bin_length = 2;
num_bins = floor(fft_len / bin_length);
PSD_smooth = zeros(1, num_bins);
for k = 1:num_bins
    PSD_smooth(k) = mean(PSD_mean((k-1)*bin_length + 1 :
k*bin_length));
end
plot(linspace(10,20,num_bins),(10*log10(PSD_smooth./mean
n_sig_power)),'k','LineWidth',2);

hold on;

for itr = 1:itr_num
    after_zp = zeros(1,M);
    after_zp(1:N/2) = d_bar(N/2+1:N);
    after_zp(M-N/2+1:M) = d_bar(1:N/2);
    ofdm_symbol_new = ifft(after_zp);
    signal_freq(itr,:) =
abs(fft(ofdm_symbol_new,fft_len)).^2;
end

PSD_mean_new = mean(signal_freq,1);
% plot(fftshift(10*log10(PSD_mean_new)), 'r');

mean_sig_power = mean([PSD_mean_new(1:N/2)
PSD_mean_new(fft_len-N/2+1:fft_len)]); % mean power of
data subcarrierse
PSD_mean_new = fftshift(PSD_mean_new);
bin_length = 2;
num_bins = floor(fft_len / bin_length);
PSD_smooth = zeros(1, num_bins);
for k = 1:num_bins
    PSD_smooth(k) = mean(PSD_mean_new((k-1)*bin_length
+ 1 : k*bin_length));
end
plot(linspace(-
10,20,num_bins),(10*log10(PSD_smooth./mean_sig_power))
,'k-','LineWidth',2);

xlabel('frequency');
ylabel('power spectral density');
hold off;
grid on;

```

```

legend('without precoding', 'with
precoding','Location','Best');

% N = 1680;
% over_sample_factor = 2;
% M = N*over_sample_factor;
% Mod = 4;
% symbol = 1;
% bitlength = N*log2(Mod)*symbol;
% bit_data = randi([0,1],bitlength,1);
% h = modem.qammod('M', Mod, 'SymbolOrder', '
Binary', 'InputType', 'Bit');
% d = modulate(h,bit_data);
% save 'd4qam.mat' d;

```

B. Result

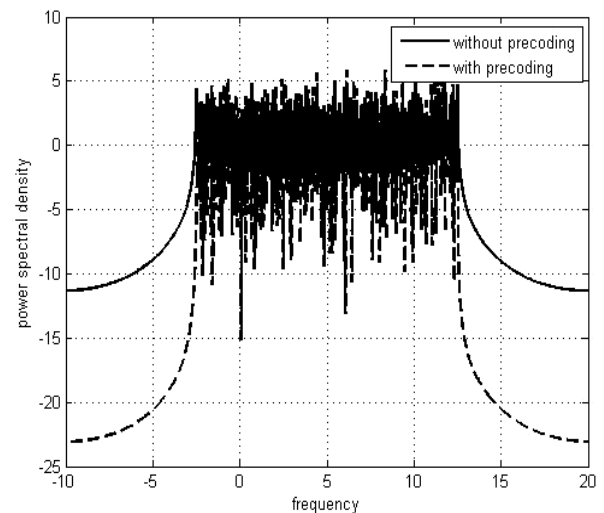


Fig. 5. Power spectral plot of OFDM symbol with (dashed line) and without precoding using [4]

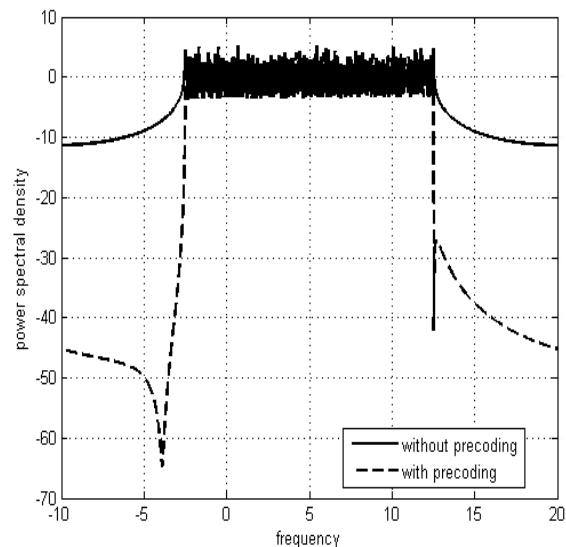


Fig. 6. Power spectral plot of OFDM symbol with (dashed line) and without precoding using [6]

Result shows that there is significant power decrease after precoding the OFDM symbol. The method explained in [4] gives about ten dB difference in power between original and precoded data, shown in Fig.5. (Note that the data is 4-QAM modulated). Fig. 6. Shows the power spectral plot (using same data) as per [6]. It gives better result by making about -35 dB power difference between the original data and precoded data.

V. CONCLUSION

A precoder is a useful and simple way to reduce out of band power emission in orthogonal frequency division multiplexing scheme based systems. Precoder can be designed by a variety of optimization approaches. The precoder proposed in [6] is found to be better than the one proposed in [4].

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