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Comparative Study and Complexity Analysis of Signal Detection in MIMO

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***Abstract*— There is ever growing demand of wireless services of higher data rates to increase the system capacity and spectral Efficiency but as we know wireless transmission is impaired by signal fading and interference. Also a conventional single input single output (SISO) system where the transmitter and receiver are equipped with single antenna could have limitations to support higher data services. The increasing requirement on data rate and quality of service for wireless system calls for new techniques to increase spectrum efficiency and the link reliability. The use of multiple antennas at both the ends of wireless link promises significant improvement in terms of spectrum efficiency and link reliability. This technology is known as multiple input and multiple output (MIMO) wireless system.**

MIMO offers diversity gain(spatial diversity) and increases the data rate by transmitting several information stream in parallel at the same transmit power. However, spatial demultiplexing or signal detection at the receiver side is a challenging task for Spatially Multiplex MIMO(SM MIMO) systems. To achieve this, wide range of algorithms offering various trade off between performance and computational complexities have been discussed by various researchers over the last decade. This paper gives the development of the various signal detection algorithms for SM MIMO , their performance and associated complexity.

Keywords-Detection,maximum like-hood (ML), spatial multiplexing, wireless communication, ZF, V-Blast,MMSE.

I. INTRODUCTION

In MIMO systems,it is usually required to detect signals jointly as multiple signals are transmitted through multiple signal paths between the transmitter and the receiver .This joint detection becomes the MIMO detection. The performance improvement resulting from MIMO wireless technology comes at the cost of increasing computational complexity in the receiver. The design of low complexity receiver is therefore one of the key problem in MIMO wireless system design.

Before describing the of the MIMO detection the system model for a narrow band MIMO link is introduced.

A. Narrowband MIMO system Model

The System model considered has M_T transmit and M_R receive antennas with $M_R \geq M_T$ denoted as M_R x M_T. The transmitted symbols are taken independently from a quadrature amplitude modulation (QAM) constellation of P points and the M_R dimensional received signal vector, using matrix notation is given by

$$\begin{bmatrix} y_{11} & . & . & . & y_{1T} \\ y_{21} & . & . & . & y_{2T} \\ . & . & . & . & . \\ . & . & . & . & . \\ y_{N1} & . & . & . & y_{NT} \end{bmatrix} = \begin{bmatrix} h_{11} & . & . & . & h_{1M} \\ h_{21} & . & . & . & y_{2M} \\ . & . & . & . & . \\ . & . & . & . & . \\ h_{N1} & . & . & . & y_{NM} \end{bmatrix} \begin{bmatrix} s_{11} & . & . & . & s_{1T} \\ s_{21} & . & . & . & s_{2T} \\ . & . & . & . & . \\ . & . & . & . & . \\ s_{M1} & . & . & . & s_{MT} \end{bmatrix} + \begin{bmatrix} n_{11} & . & . & . & n_{1T} \\ n_{21} & . & . & . & n_{2T} \\ . & . & . & . & . \\ . & . & . & . & . \\ n_{N1} & . & . & . & n_{NT} \end{bmatrix} \quad (1)$$

which is equivalent to

$$y = Hs + n \quad (2)$$

Where H denotes the M_R X M_T channel matrix, s = $[s_1\ s_2 \ldots\ldots\ s_{MT}]^T$ is the M_T dimensional transmit signal vector, and n stands for the M_R -dimensional additive independent identically distributed circularly symmetric complex Gaussian noise vector [1]. MIMO detection is to detect the transmit signal s from the received signal y under the knowledge of estimated channel state information (CSI) where CSI contains the information of H as well as stastical properties of n in equation 1.

B. Spatial Multiplexing

In a MIMO system both transmit and receive antenna combine to give a large diversity order. In which spatial diversity gain can be obtained when multiple antenna are present at either the transmit or the receive side. In a spatial multiplexing system [2,3] the data stream to be multiplexed into M_T lower rate stream which are then simultaneously sent from the M_T transmit antennas after coding and modulation and all the transmitted streams occupy the same frequency band(i.e. they are co-channel signals). At the receiver side, each receive antenna observes a superposition of the transmitted signals. The receiver then separates them into constituent data streams and remultiplexes them to recover the original data stream. And this separation step at the receiver needs to determine the performance and computational complexity of the receiver. Algorithms to separate the parallel data streams corresponding to the M_T transmit antenna can be divided into four categories. Next sections discuss these algorithms their performance and computational complexity .It includes both the linear and nonlinear receiver algorithms.

II. Linear Receiver Algorithms

In linear receivers, the received signal vector y is linearly transformed by a matrix equalizer that basically undoes the effect of the channel H to obtain an estimate of the transmitted symbol vector $\hat{s}$ .With this the received signal is filtered by a linear filter and each data symbol is detected separately and this filters are basically use to suppress the interfering signals. The matrix equalizer can be computed according to different criteria.

A. Zero Forcing (ZF) Receiver Algorithm

As given in [4, 5] a simple linear receiver is the zero-forcing (ZF) receiver which basically inverts the channel transfer matrix, i.e., assuming that H is invertible an estimate of the M_T x1 transmitted data symbol vector s is obtained as

$$\hat{s} = \boldsymbol{H}^{-1} y \tag{3}$$

It nullifies the Interference by following weight matrix

$$W_{ZF} = (H^H H)^{-1} y \tag{4}$$

where $(.)^H$denotes the Hermitian transpose operation.

In other words, it inverts the effect of channel as

$$\begin{aligned} \hat{s}_{ZF} &= W_{ZF}\, y \\ &= s + (H^H H)^{-1} H^H n \\ &= s + \bar{n}_{ZF} \end{aligned}$$

Where $$\bar{n}_{ZF} = W_{ZFn} = (H^H H)^{-1} H^H n \tag{5}$$

In equation (3) H is near singular and the term of noise

$(H^H H)^{-1} H^H n$, is enhanced. In this case, good performance cannot be guaranteed with an enhanced noise vector.

The ZF receiver hence perfectly separates the co-channel signals si (i= 0, 1,…, MT -1).For ill conditioned H, the ZF receiver performs well in high SNR regime, whereas in the low SNR regime there will be significant noise enhancement. The zero forcing criteria used in the receiver have the disadvantage that the inverse filter may excessively amplify noise at frequencies where folded channel spectrum has high attenuation. The ZF equalizer thus suffers from noise enhancement since it focuses on canceling the effects of the channel response at the expense of enhancing the noise, and is not often used for wireless link, and it also has poor bit error rate performance [5].

B. Minimum Mean Square Error (MMSE) Receiver Algorithm

An alternative linear receiver is a minimum mean square error (MMSE) receiver, which minimizes the overall error due to noise and mutual interference. For the MMSE the estimate of signal vector s is obtained according to

$$\hat{s} = \frac{\rho}{M_T} H^H \left(\sigma_n I_{MR} + \frac{\rho}{M_T} H H^T \right)^{-1} r \tag{6}$$

Where I_{MR} is the mutual Information between the transmitter and receiver.The MMSE receiver is less sensitive to noise at the cost of reduced signal separation quality. In the high SNR case the MMSE receiver converges to the ZF receiver [4, 5].

C. Performance Analysis and Associated Complexity of ZF and MMSE Algorithms

In [7]-[9], the asymptotic performance of linear receiver in MIMO fading channels had been discuss by considering two cases. First one is for fixed no. of antennas, the limit of error probability in the high–signal to noise ratio (SNR regime) in terms of the diversity–multiplexing tradeoff (DMT), second is the error probability for fixed SNR in the regime of large (but finite) number of antennas.

In comparison with the above two cases, as per as DMT is concerned, the ZF and MMSE receiver achieve the same (DMT), which is largely suboptimal even in the case where outer coding and decoding is performed across the antennas whereas behavior of the ZF and MMSE receivers at finite rate and non asymptotic SNR, is different. The ZF receiver achieves poor diversity at any finite rate, the MMSE receiver error curve slope flattens out progressively, as the coding rate increases. If the second case is considered i.e. when SNR is fixed and the numbers of antennas become large, the mutual information at the output of ZF and MMSE linear receiver has fluctuations that converge in distribution to a Gaussian random variable whose mean and variance can be characterized in closed form.

Based on the analysis of [7], [9] to achieve a required target spectral efficiency at a given block error rate and SNR operating point, an attractive design option may consist of increasing the no. of antennas (especially at the receiver) and using a low complexity linear receiver.

In [10], the authors consider a MMSE receiver and develop receive antenna selection algorithms to maximize the channel capacity, which again need not be optimal as far as error performance is concerned. In fact, such schemes perform only slightly better than deterministic antenna selection. Therefore, it is essential to model antenna selection problem with the aim of minimizing the error rate of a link, taking receiver processing into consideration this issue is appeared in [11,13]. In [12], authors proposed transmit antenna selection strategy for ZF receivers to mitigate the effect of transmit antenna correlation. The suggested algorithm pre-determines the subset of antennas to use, exploiting a priori knowledge of transmit correlation matrix at the transmitter. However, the proposed algorithm fails to exploit the transmit diversity gain that could be leveraged by selecting transmit antennas that exploit the current state of the fading channel. In [12] the authors also presented a transmit antenna selection algorithm for ZF receivers and later in [13] extended it to lattice-reduction-aided (LRA) ZF receivers, which have additional complexity, compared to traditional ZF receivers. The authors also proposed an approximate selection rule for LRA-MMSE receivers based on the minimization of maximum mean square error [Eq (24), 13]. The analysis in [11]-[13] is limited to single user and in [13], [14] antenna selection is considered only at the transmitter. Moreover, no exact analysis is provided for the MMSE receiver, which is known to have much superior performance than the ZF receiver at low and moderate SNR [15].

In [16], the authors consider the case in which the base station uses the linear minimum mean-square-error (MMSE) detector in order to detect the signals from the number of users. The performance objective for this receiver is the normalized MSE and main attempt is to extent the results of [17] for the MIMO transmission model and compute the normalized MSE as a function of the transmit covariance matrices and power allocation of the users and minimize the average MSE under a sum power constraint and under individual power constraints.In addition to this, the authors derive the optimization problem that balances the MSE requirements of users. The individual MSE are functions of the transmit covariance matrices and the achievable individual MSE region has been analyzed and It has been shown that the achievable MSE region is convex for the two user MIMO scenario.In [17] the linear MMSE multiuser receiver for synchronous code-division multiple-access (CDMA) systems was analyzed.

D. OSIC Signal Detection:

Generally ,the performance of the linear detection methods is worse but these methods requires a low complexity of hardware implementation .It is reported in the literature that the performance of these methods can be increase without increasing the complexity of hardware appreciably by an ordered successive interference cancellation (OSIC)method. It is bank of linear receivers ,each of which detects one of the parallel data streams, with the detected signal component successively canceled from each stage and the remaining signal with reduced interference can be used in the subsequent stage.[19]

III. NON LINEAR RECEIVER ALGORITHMS

In contrast to linear data detection, where all layers are detected jointly, a tree search approach is used in the non linear receivers. Following are the different algorithms which are use for Non linear data detection.

A. V-BLAST (Vertical Bell Laboratories layered space-time) Algorithm

An attractive alternative to ZF and MMSE receivers which in general yields improvement performance at the cost of increased computational complexity is called V-BLAST algorithm[18,19] .In V-BLAST rather than jointly decoding all the transmit signals, first decode the strongest signal ,then subtract this strongest signal from the received signal, proceed to decode the strongest signal of the remaining transmit signals, and so on .The optimum detection order in such a nulling and cancellation strategy is from the strongest to the weakest signal[18].Assuming that the channel H is known, the main steps of the V-Blast algorithm can be summarized as follows:

- Nulling: Nulling is used to find out the estimate of the strongest transmit signal by neglecting all the weaker transmit signals.(say using the zero forcing criterion).
- Slicing: Slicing is used to obtained the data bits in the estimated signals.
- Cancellation: These data bits are remodulated and the channel is applied to estimate its vector signal contribution at the receiver. The resulting vector is then subtracted from the received signal vector and the algorithm returns to the nulling step until all transmit signal are decode.

The V-BLAST algorithm show slightly better performance, but suffers from error propagation and is still suboptimal. In comparison with the linear data detection, where all layers are detected jointly, nulling and canceling (NC) uses a serial decision-feedback approach to detect each layer separately (e.g., [20]). When a layer has been detected, an estimate of the corresponding contribution to the received vector is subtracted from; the result is then used to detect the next layer, etc. In the absence of detection errors, NC progressively cleans from the interference corresponding to the layers already detected. To detect a specific layer, the layers that have not been detected yet are “nulled out” (equalized) according to the ZF or MMSE approach described above. Error propagation can be a problem because incorrect data decisions actually increase the interference when detecting subsequent layers. Thus, the order in which the layers are detected strongly influences the performance of NC and also increases the computational complexity [21].Therefore as per as V-Blast is concern it gives
Slightly better performance, but suffers from error propagation.

B. Maximum Likelihood Receiver Algorithm

The receiver which yields the best performance in terms of error rate is the maximum likelihood (ML) receiver. Maximum likelihood (ML) detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H and finds the one with the minimum distance. The ML detection can be carried out by exhaustively searching for all the candidate vectors and selecting the maximum likely one with the smallest error probability.

However, this receiver also has the highest computational complexity which moreover exhibits exponential growth in the number of transmit antennas. Assuming the suitable channel state information, the ML receiver computes the estimate of $\hat{s}$ according to

$$\hat{s}_{ML} = \arg_s \min \left\| y - Hs \right\|^2 \qquad (7)$$

Where the minimization is performed over all possible codeword vectors.ML detection is optimal in the sense of minimum error probability when all data vectors are equally likely. It is the optimum detection method and minimizes the BER. A straightforward approach to solve (4) is an exhaustive search. Unfortunately, the corresponding computational complexity grows exponentially with the transmission rate R, since the detector needs to examine 2R hypotheses for each received vector. While the implementation of exhaustive-search ML has been shown to be feasible in the low rate regime R $\leq$ 8 bpcu [22], complexity quickly becomes unmanageable as the rate increases [23], [24].

In MIMO system, Maximum-Likelihood (ML) decoding is equivalent to finding the closest lattice point in an N-dimensional complex space. In general, this problem is known to be NP hard. In [25], the authors proposed a quasi-maximum likelihood algorithm based on Semi-Definite Programming (SDP) and several SDP relaxation models for MIMO systems has been introduce, with increasing complexity. The authors use interior-point methods for solving the models and obtain a near-ML performance with polynomial computational complexity. Lattice basis reduction is applied to further reduce the computational complexity of solving these models.

To overcome the complexity issue, a variety of sub-optimum polynomial time algorithms are suggested in the literature for lattice decoding. However, unfortunately, these algorithms usually result in a noticeable degradation in the performance. Examples of such polynomial time algorithms include: Zero Forcing Detector (ZFD) [26], [27], Minimum Mean Squared Error Detector (MMSED) [28], [29], Decision Feedback Detector (DFD) and Vertical Bell Laboratories Layered Space-Time Nulling and Cancellation Detector (VBLAST Detector) [18], [21] Lattice basis reduction has been applied as a pre-processing step in sub-optimum decoding algorithms to reduce the complexity and achieve a better performance. Minkowski reduction [30], Korkin-Zolotarev reduction [31] and LLL reduction [32] have been successfully used for this purpose in [32–37].

For the Maximum Likelihood detection it can be say that the complexity of this approach is usually too high for complex constellation and for the large no of antennas, making it impossible to implement for large array sizes and high order
digital modulation schemes.

C. Sphere Decoding Algorithm

Sphere decoding (SD) method intends to find the transmitted signal vector with minimum ML metric, that is, to find the ML solution vector .However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors.SD adjusts the sphere radius until there exists no vectors within a sphere, and decreases the radius when there exist multiple vectors within the sphere.

This algorithm achieves the Maximum–likelihood performance with reduced complexity by only searching over the noiseless received signals that lie within a hyper sphere of radius R around the received signal y.

$$\hat{s}_{ml} = \arg\left\{\min_{s}\|y - Hs\|^2 \leq R^2\right\} \tag{8}$$

Normally, this algorithm is implemented as a depth first tree search, where each level in the search represents one transmit antenna's signal.If at a given level, a given branch exceeds the radius constraint, then that part of the tree can be removed from further consideration. Unfortunately, it is difficult to estimate how much of the tree needs to be searched in advance, since this depends on both the noise and the channel conditions. Therefore for Sphere Decoder it can be say that the complexity of the sphere decoder is not fixed, but typically vary with time [38].

The ML detection problem discuss in [39]–[41] has been solve by the sphere decoding approach. While the algorithm has a nondeterministic instantaneous throughput, its average complexity was shown to be polynomial in the rate [42] for moderate rates, but still exponential in the limit of high rates [43]. However, these asymptotic results do not properly reflect the true implementation complexity of the algorithm, which for most practical cases is still significantly lower than an exhaustive search. The algorithm is thus widely considered the most promising approach towards the realization of ML detection in high-rate MIMO systems. The sphere decoding algorithm is introduction in [39] and its application to wireless communications in [41], reduction of the computational complexity of the algorithm has received significant attention [44], [45], [46].

IV. CONCLUSION

This paper carries out detail study of various SM-MIMO signal detection techniques reported in the literature. In linear receivers, the received signal vector is linearly transformed by a matrix equalizer and all the layers are detected jointly whereas in the non linear receiver different tree search approaches are used. Among the two linear algorithms i.e. and MMSE, it has been found that MMSE is having better performance than the ZF in the ill condition SNR regime.

In the non linear receiver algorithms the V-BLAST gives slightly better performance than the linear MMSE algorithm, but suffers from error propagation. The Maximum likelihood receiver is one which yields the best performance in terms of error rate, but this receiver also has the highest computational complexity which moreover exhibits exponential growth in the number of transmit antennas. The Maximum-likelihood performance with reduced computational complexity is possible with the Sphere Decoding Algorithm. In sphere decoding it is very difficult to estimate the no. of trees needs to be searched in advance, since this depends on both the noise and the channel conditions. Therefore the complexity of the sphere decoder is not fixed, but typically varies with time.

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