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Comparative Performance Study of LS and MMSE Channel Estimation over Time Varying Channel in OFDM System

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***Abstract*- Orthogonal Frequency Division Multiplexing (OFDM) has been recently applied widely in wireless communication systems, due to high data rate, transmission capability with high bandwidth efficiency and its robustness to multipath delay. Channel estimation is an essential problem at the receiver, where the wireless channel is usually frequency selective and time varying. The estimation of channel at pilot subcarriers is based on Least Square (LS), Minimum Mean Square Error (MMSE) while interpolation is done using linear interpolation. For performance evaluation of LS and MMSE channel estimators in OFDM system, the previous works use the block type pilot arrangement, where the pilot tones are transmitted into all subcarriers but at specific period which is suitable for frequency selective fading. In order to keep track of the time-varying channel characteristic, the pilot tones symbols must be placed as the coherence time which causes data reduction. Aiming at this disadvantage, we propose to use the comb-type pilot arrangement where the pilot tones are transmitted at each time to track the rapid variation of the channel. The Clarke' model is used to perform the time varying channel and 16-QAM as the modulation scheme. The performance of the algorithms is measured in terms of Mean Square Error (MSE) and Symbol Error Rate (SER). Simulation results reveal that MMSE estimator provides better performance to track the time-varying channel .**

Keywords: OFDM System; LS Estimator; MMSE Estimator; comb-type pilot arrangement; Block-type pilot arrangement

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing OFDM) has received a broad attention in wireless communication system due to its resistance against multipath fading and high spectral efficiency. For these reasons, it has been accepted by many of the future generation system such as LTE, IEEE802.11 and WIMAX [1].

At high data rates, channel distortion to data is very significant; therefore channel estimation becomes essential before the demodulation of OFDM signals [2].

In OFDM system, channel estimation methods can be divided into two classes: blind channel estimation and pilot based channel estimation. The blind channel estimation is carried out by using the statistical properties of the received signals [3].

In pilot based channel estimation, pilot tones that are known at priori by the receiver are multiplexed along with the data for channel estimation [4].

The pilot-based channel estimation can be performed by either inserting pilot tones into all subcarriers of the OFDM symbol with a specific period (block type pilot arrangement) or inserting pilot tones into each OFDM symbols with a specific period of frequency(comb-type pilot arrangement).

By now, many channel estimation algorithms are used such as LS estimator and MMSE estimator. The previous performance comparison between the two channel estimation estimators are reported previously [5-7]. All use the block-type pilot arrangement, which is suitable for frequency selective fading. In practice, with the mobility between the transmitter and the receiver, the wireless channel is time-varying channel. In order to keep track of the time-varying channel characteristic, it might incur too much pilot tones which causes data reduction.

In this paper, our contribution is to compare the performances of LS and MMSE over time-varying channel using comb-type pilot arrangement. Monte Carlo matlab simulation is used to measure the performance in term of mean square error (MSE) and symbol error rate (SER).

This paper is organized as follows. In section II overview of pilot based OFDM system model is explained. Section III is divided into two subsections: III.A described LS estimator, III.B presented MMSE estimator. Section IV presented our proposed method. Section V explained linear interpolation. Simulation results are discussed in section VI. Conclusion of the work is presented in section VII.

II. OFDM SYSTEM

OFDM system model block diagram in Fig1 can be described as follow: at beginning the binary is mapped according to modulation followed by serial to parallel conversion. After that, pilot sub-carriers are multiplexed with data sub-carriers, which give $X(k)$ samples. Then IFFT is performed on $X(k)$ sample that transform frequency domain samples $X(k)$ into time domain $x(n)$, which can be shown as

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$$x(n) = \text{IFFT}\{X(k)\} = \sum_{n=0}^{N_{\text{FFT}}-1} X(k) e^{\frac{j2\pi kn}{N_{\text{FFT}}}} \quad (1)$$

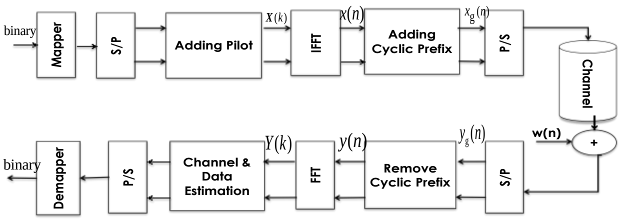

Figure 1 OFDM System

Where N_{FFT} represents the number of FFT points. To combat inter-symbol interference guard band interval denoted by N_G are added in each OFDM symbol and samples becomes $x_g(n)$, which can be expressed as:

$$x_g(n) = \begin{cases} x(N_{\text{FFT}} + n) & n = -N_G, -N_{G+1}, ..., -1 \\ x(n) & n = 0, 1, 2, \ldots, N_{\text{FFT}} - 1 \end{cases} \quad (2)$$

After passing through the time variant channel, the received signal $y_g(n)$ becomes

$$y_g(n) = x_g(n) \otimes h(n) + w(n) \quad (3)$$

Where $w(n)$ represents additive white Gaussian noise. The impulse response of the time variant channel $h(n)$ can be expressed using Clarke's model as:

$$h(n) = \sqrt{\frac{2}{M}} \sum_{r=1}^{M} e^{j[2\pi f_d \cos(\alpha_r) + \phi_r]} \quad (4)$$

Where M represents the number of propagation paths, $\phi_r \sim U[-\pi, \pi)$ and $\alpha_r \sim U[-\pi, \pi)$ are the random phase and angle of arrival of the r^{th} multipath component respectively, and f_d is the maximum Doppler frequency due to the relative motion between transmitter and receiver. After serial to parallel conversion, removal of guard interval takes place from $y_g(n)$

$$y(n) = y_g(n + N_G) \quad n = 0, 1, 2, \ldots, N_{\text{FFT}} - 1 \quad (5)$$

Then $y(n)$ is send to FFT block for the following operation

$$Y(k) = \text{FFT}\{y(n)\} = \frac{1}{N_{\text{FFT}}} \sum_{n=0}^{N_{\text{FFT}}-1} y(n) e^{\frac{-j2\pi kn}{N_{\text{FFT}}}}, n = 1, 2, \ldots, N_{\text{FFT}} \quad (6)$$

suppose that there is no ISI, because channel impulse response length is smaller than the guard band interval.The output of the FFT block is $Y(k)$, which can be represented as

$$Y(k) = X(k)H(k) + W(k), k = 1, 2, \cdots, N_{FFT} \quad (7)$$

where $H(k) = \text{FFT}\{h(n)\}$ and $W(k) = \text{FFT}\{w(n)\}$.

$W(k)$ and $H(k)$ are the Fast Fourier Transform of $w(n)$ and $h(n)$ respectively.

After that, the received pilots sub-carriers $Y_p(k)$ are obtained from $Y(k)$ and then $\hat{H}(k)$ is calculated from the information carried by $H_p(k)$.The transmitted data samples can be:

$$X(k) = \frac{Y(k)}{\hat{H}(k)} \quad (8)$$

where $\hat{H}(k)$ is the estimated channel transfer function and $Y(k)$ is the received signal. Finally de-mapping takes place, where binary data are obtained at the receiver output.

III. CHANNEL ESTIMATION

A. LS ESTIMATOR

Let $\boldsymbol{X}_p$ be the diagonal matrix of pilots given as $\boldsymbol{X}_p = diag\{X_p(1)\, X_p(2) \ldots X_p(N_p)\}$, N_p is the number of pilots in one OFDM symbols, h_p is the impulse response of the channel at pilot sub-carrier in one OFDM symbol, and $\boldsymbol{w}_p$ is the AWGN channel noise at pilot sub-carrier. If there is no Inter Symbol Interference (ISI), the signal received is written as

$$Y_p = X_p F h_p + W_p \quad (9)$$

Where $\boldsymbol{Y}_p$ the vector of output signal is after demodulation as $\boldsymbol{Y}_p = [Y_p(1)\, Y_p(2) \ldots Y_p(N_p)]^T$, T is transpose, F the Fourier Transform matrix. . The purpose of LS algorithm is to minimize the cost function K without noise.

$$K = \left| \boldsymbol{Y}_p - \boldsymbol{X}_p \mathbf{F} h_p \right|^2 \quad (10)$$

Let $\hat{\boldsymbol{H}}_{P,LS}$ is the estimate impulse response of the channel

$$\hat{\boldsymbol{H}}_{P,LS} = \boldsymbol{X}_p^{-1} \boldsymbol{Y}_p \quad (11)$$

LS has low computational complexity as compared to other channel estimators, but its major drawback is high MSE value.

B. MMSE ESTIMATOR

If the channel and AWGN are not correlated, MMSE estimate of H is given by [8]

$$\hat{\boldsymbol{H}}_{\text{MMSE}} = \mathbf{R}_{H_p Y_p} \mathbf{R}_{Y_p Y_p}^{-1} \boldsymbol{Y}_p \quad (12)$$

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Where
$$\mathbf{R}_{H_pY_p} = E\{\boldsymbol{H}_p\boldsymbol{Y}_p^H\} = \mathbf{R}_{H_pH_p}\boldsymbol{X}_p^H$$
$$\mathbf{R}_{Y_pY_p} = E\{\boldsymbol{Y}\boldsymbol{Y}_p^H\} = \boldsymbol{X}_p\mathbf{R}_{H_pH_p}\boldsymbol{X}_p^H + \sigma_w^2\boldsymbol{I}_{N_p}$$

are the cross covariance matrix between $\boldsymbol{H}_p$ and $\boldsymbol{Y}_p$, and auto-covariance matrix of $\boldsymbol{Y}_p$ respectively $\mathbf{R}_{H_pH_p}$ is auto-covariance matrix of $\boldsymbol{H}_p$. σ_w^2 is the noise-variance. If $\mathbf{R}_{H_pH_p}$ and σ_w^2 are known to the receiver, CIR could be calculated by MMSE estimator as below:

$$\begin{aligned}\hat{\boldsymbol{H}}_{MMSE} &= \mathbf{R}_{H_pY_p}\mathbf{R}_{Y_pY_p}^{-1}\boldsymbol{Y}_p \\ &= \mathbf{R}_{H_pH_p}\boldsymbol{X}_p^H(\boldsymbol{X}_p\mathbf{R}_{H_pH_p}\boldsymbol{X}_p^H + \sigma_w^2\mathbf{I}_{N_p})^{-1}\boldsymbol{X}_p\hat{\boldsymbol{H}}_{p,LS}^{-1} \\ &= \mathbf{R}_{H_pH_p}(\mathbf{R}_{H_pH_p} + \sigma_w^2(\boldsymbol{X}_p^H\boldsymbol{X}_p))^{-1}\ \hat{\boldsymbol{H}}_{p,LS}\end{aligned} \quad (13)$$

A major drawback of the MMSE estimator is its high computational complexity, especially the matrix inversions is needed each time the data change.

IV. PROPOSED METHOD

A block type pilot arrangement is depicted in Figure2. In this time type, OFDM symbols with pilot at all subcarriers are transmitted periodically for channel estimation. Using these pilots, a time-domain interpolation is performed to estimate the channel along the time axis. Let S_t denote the period of pilot symbols in time. In order to keep track of the time-varying channel characteristic, the pilot symbols must be placed as frequently as the coherence time is. As the coherence time is given in an inverse form of the Doppler frequency $f_{Doppler}$ in the channel, the pilot symbol period must satisfy the following inequality:

$$S_t \le \frac{1}{f_{\text{Doppler}}}$$

For fast fading, the block type is not suitable to track the rapid variation of the channel characteristic.

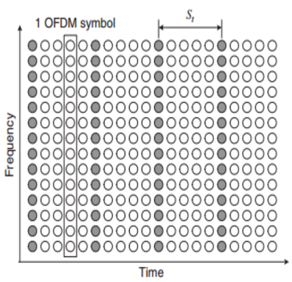

Figure 2 Block -type pilot arrangement

Aiming at this disadvantage, we propose to multiplex the pilot tones with data at each time but at specific subcarriers(Figure3). The channel is estimated at pilot subcarriers using the LS and MMSE estimators. With interpolation techniques the estimation of channel at data subcarrier can be obtained using channel estimation at pilot subcarriers. The N_p pilot signals are uniformly inserted into the subcarriers $X(k)$ according to the following equation.

$$X(k) = X(iL+l) \quad l = 0,1,2,\dots,L-1$$
$$= \begin{cases} X_p(k) & l = 0 \\ Data & l = 1,2,\dots,L-1 \end{cases} \quad (14)$$

Where L = number of subcarriers (N)/number of pilots (N_p), i = pilot carrier index.

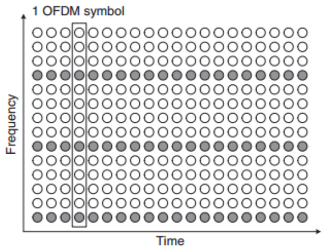

Figure 3 Comb-type pilot arrangement

V. LINEAR INTERPOLATION

In this method two consecutive pilots are used to estimate channel at data sub-carriers $k, iL < k < (i+1)L$ which are located in between the pilots, as showed below [9-10]:

$$\hat{H}(k) = \hat{H}(iL+l) \quad (15)$$

$$\hat{H}(k) = (\hat{H}(i+1) - \hat{H}(i))\frac{l}{L} + \hat{H}(i) \quad , i = 0,1,2,\dots,N_p - 1 \quad (16)$$

VI. SIMULATION AND RESULTS

This section discusses the results of the simulation using MATLAB that were performed based on the channel estimators discussed in the section III.

For the simulation of basic OFDM system, we used the following parameters as shown in Table.

Table: Simulation parameters

Parameters	Specification
No. of FFT points	128
Length of Cyclic Prefix	16
Total no. of Subcarriers	128
Total no. of Symbol	144
Pilot Spacing	2
Number of Pilots	64
Pilot Arrangement	Comb Type
Signal Constellation	16QAM
Interpolation	linear
Channel Model	Clarke' model
Number of Channel Taps	5
Channel Estimation Techniques	LS,MMSE
Doppler Frequency	80HZ

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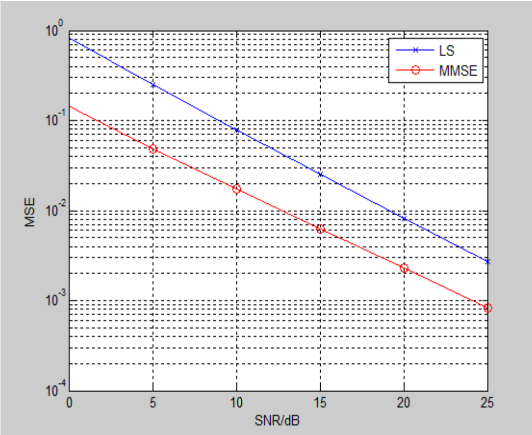

Figure 4 LS and the MMSE channel estimators for OFDM system based on the parameter of Mean square error

SNR	0	5	10	15	20	25
MSE_LS	0.779	0.247	0.079	0.024	0.008	0.002
MSE_MMSE	0.128	0.049	0.017	0.006	0.002	8e-04

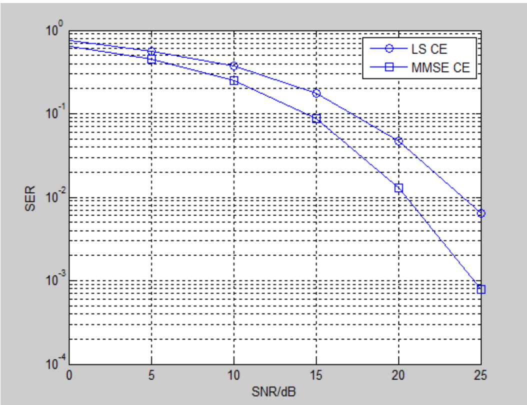

Figure 5 LS and the MMSE channel estimators for OFDM system based on the parameter of Symbol Error Rate

SNR	0	5	10	15	20	25
SER_LS	0.744	0.566	0.375	0.178	0.046	0.006
SER_MMSE	0.635	0.447	0.248	0.087	0.012	7e-04

Fig.4 shows the mean square error (MSE) of channel estimation at different SNR in dB. As SNR increases mean square error decreases for both LS and MMSE. Fig.5 shows the symbol error rate (SER) at different SNR in dB. As SNR increases symbol error rate decreases for both cases. For a given SNR, MMSE estimator performs better than LS estimator. The complexity of MMSE estimator is larger than LS estimator but gives better performance in comparison to LS.

VII. CONCLUSION

This paper highlights the comparative performance study between LS and MMSE estimators over time varying channel using comb-type pilot arrangement. The channel estimation is one of the fundamental issues of OFDM system design. The transmitted signal under goes many effects such reflection, refraction and diffraction. Also due to the mobility, the channel response can change rapidly over time. At the receiver these channel effects must be canceled to recover the original signal. From the present simulation based comparative study, it can be concluded that MMSE estimator provides better performance to track the time-varying channel.

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