Comparative Analysis of Load Flow Computational Methods Using MATLAB

Gaganpreet Kaur
Student, Power Engineering (M.Tech),
GZS PTU Campus, Bathinda

S. K Bath
Associate Prof., Electrical Engineering Deptt.
GZS PTU Campus, Bathinda

B. S Sidhu
Associate Prof., Electronics Engineering Deptt.
GZS PTU Campus, Bathinda

Abstract- The power system analysis and design is generally done by Using Load flow analysis. The main information attained from this study includes the magnitudes and phase angles of load bus voltages, reactive powers at generator buses, real and reactive power flow in transmission lines. This information is essential for the continuous monitoring of the current state of the system, Planning, operation, economic scheduling and exchange of power between utilities.

Different methods are used for load flow analysis. The objective of this study is to develop MATLAB programs for load flow study to calculate bus voltages, their phase angles, real power loss and reactive power loss in the power system, computational time, and number of iterations, accuracy and memory for IEEE 9, IEEE 14, IEEE 25 bus systems. The different methods of load flow study are analysed and compared with each other. Every method has advantages and disadvantages in different conditions. So, comparison of these methods can be useful to select the best method for a typical network. As a result, some suggestions are proposed to apply the methods.


I. INTRODUCTION

Load flow studies [9] are used to ensure that electrical power transfer from generators to consumers through the grid system is stable, reliable and economic. Load flow analysis is fundamental to the study of power systems. This analysis is at the heart of contingency analysis and the implementation of real-time monitoring systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle.

II. LOAD FLOW ANALYSIS

The complex power ($S_i$) injected by the source into the $i^{th}$ bus of a power system is:

$$ S_i = P_i + jQ_i $$

Where $P_i$ is the real power and $Q_i$ is injected power into the $i^{th}$ bus.

$$ P_i = |V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) $$

$$ Q_i = -|V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) $$

Where $V_i$ is the voltage at the $i^{th}$ bus, $V_k$ is the voltage at the $k^{th}$ bus, $Y_{ik}$ is the mutual admittance between nodes $i$ and $k$, $\theta_{ik}$ is the angle of $Y_{ik}$, $\delta_k$ is the angle of $V_k$ and $\delta_i$ is angle of $V_i$.

Bus Classification: A bus is a node at which one or many lines, one or many loads and generators are connected. In a power system each node or bus is associated with 4 quantities, such as magnitude of voltage, phase angle of voltage, active or true power and reactive power in load flow problem two out of these 4 quantities are specified and remaining 2 are required to be determined through the solution of equation. Depending on the quantities that have been specified, the buses are classified into 3 categories:

Load Buses: In these buses no generators are connected. At this type of bus, the net power $P_i$ and $Q_i$ are specified whereas $|V_i|$ and $\delta_i$ are unspecified.

Voltage Controlled Buses: These are the buses where generators are connected. At this type of bus, the net power $P_G$ and $|V_i|$ are specified whereas $Q_i$ and $\delta_i$ are unspecified.

Slack or Swing Buses: Usually this bus is numbered as 1. This bus is distinguished from other two types of buses by the fact that real and reactive powers at this bus are not specified. Instead, voltage magnitude and phase angle are specified.

III. APPROXIMATE (APPROX.) LOAD FLOW

In this method [7] following assumptions and approximations are made in the load flow equations:
Line resistances being small are neglected i.e. $P_L$, the active power loss of the system is zero. Thus $\delta_i \approx 90^\circ$ and $\delta_k \approx -90^\circ$. 

(\delta_i - \delta_k) is small ($< n/6$) so that $\sin(\delta_i - \delta_k) \approx (\delta_i - \delta_k)$. This is justified from considerations of stability.

All buses other than the slack bus are PV buses i.e. voltage magnitudes at all the buses including the slack bus are specified.

Equations then reduce to

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| (\delta_i - \delta_k); \quad (4)$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |V_k|^2 |\cos(\delta_i - \delta_k)| + |V_i|^2 |V_k| \quad (5)$$

Since $|V_i|$’s are specified, Eq. (4) represents a set of linear algebraic equations in $\delta$’s which are (n-1) in number as $\delta_i$ is specified at the slack bus. The $n^{th}$ equation corresponding to slack bus (n=1) is redundant as the real power injected at this bus is now fully specified as

$$P_i = \sum_{k=2}^{n} P_{ki} - \sum_{k=2}^{n} P_{ki}$$

Equation (2) can be solved explicitly for $\delta_2, \delta_3, \ldots \delta_n$ when substituted in Eq. (5) yields $Q_i$’s, in the reactive power bus injections.

IV. GAUSS SEIDEL (GS) LOAD FLOW

The GS method [4] is an iterative algorithm for solving nonlinear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At all subsequent iteration, the solution is updated till convergence is reached.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus.

$$S_i = V_i^2 \sum_{k=1}^{n} Y_{ik} V_k^*$$

This can be written as

$$P_i - jQ_i = V_i^2 \sum_{k=1}^{n} Y_{ik} V_k$$

So that,

$$V_i = \frac{1}{V_i^2} \left[ P_i - jQ_i - \sum_{k=1}^{n} Y_{ik} V_k \right] \quad (6)$$

Whereas $i = 2, 3, \ldots \ldots \ldots \ldots \ldots n$

Equation (6) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. In Gauss Seidel method, the value of the updated voltages is used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value.

Algorithm for GS method

Step1. Prepare data for the given system as required.

Step2. Formulate the bus admittance matrix $Y_{BUS}$. This is generally done by the rule of inspection.

Step3. Assume initial voltages for all buses =2, 3, \ldots \ldots n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0 \angle 0^\circ$. This is normally referred as the flat start solution.

Step4. Update the voltages. In any $(r+1)^{th}$ iteration, from (6) the voltages are given by

$$V_i^{(r+1)} = \frac{1}{V_i^2} \left[ P_i - jQ_i - \sum_{k=1}^{n} Y_{ik} V_k^{(r+1)} - \sum_{k=1}^{n} Y_{ik} V_k^{(r)} \right] \quad (7)$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2, 3, \ldots \ldots (i-1) in the current $(r+1)^{th}$ iteration. Hence these values are used. For buses $(i+1)$, \ldots \ldots n, values from previous, $r^{th}$ iteration are used.

Step5. Continue iterations till

$$|\Delta V_i^{(r+1)}| = \left| V_i^{(r+1)} - V_i^{(r)} \right| \leq \varepsilon$$

Where, $\varepsilon$ is the tolerance value. Generally, it is customary to use a value of 0.0001 p.u.

Step6. Compute slack bus power after voltages have converged Using,

$$S_i^* = P_i - jQ_i \quad [Assuming \text{ bus 1 is slack bus}]$$

Step7. Compute all line flows.

$$S_{ik} = V_i^{*} \left( V_k^{*} - V_k \right) + V_i^2 V_k^{*}$$

Step8. The complex power loss in the line is given by $S_{ik} + S_{ki}$. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of $Q_i$ to be used in (7). From (0) we have

$$Q_i = -Im \left( V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right)$$

Where, $Im$ stands for the imaginary part. At any $(r+1)^{th}$ iteration, at the PV bus-i,

$$Q_i^{(r+1)} = -Im \left( V_i^{(r+1)*} \sum_{k=1}^{n} Y_{ik} V_k^{(r+1)} + \left( V_i^{(r)} \right)^* \sum_{k=1}^{n} Y_{ik} V_k^{(r)} \right)$$
The steps for \(i\)th PV bus are as follows:

1. Compute \(Q_i^{(r+1)}\) Using (10).
2. Calculate \(V_i\) Using (7) with \(Q_i = Q_i^{(r+1)}\).
3. Since \(|V_i|\) is specified at the PV bus, the magnitude of \(V_i\) obtained in step 2 has to be modified and set to the specified value \(|V_{ii}^{sp}|\). Therefore,
\[
V_i^{(r+1)} = |V_{ii}^{sp}| < \delta_i^{(r+1)} \tag{11}
\]

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e. \(Q_i^{(r+1)}\) is computed Using (10) is either less than \(Q_{i,\min}\) or greater than \(Q_{i,\max}\), it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the \((r+1)\)th iteration and the voltage is calculated with the value of \(Q_i\) set as follows:

If \(Q_i < Q_{i,\min}\) Then \(Q_i = Q_{i,\min}\).
If \(Q_i > Q_{i,\max}\) Then \(Q_i = Q_{i,\max}\). \tag{12}

If in the subsequent iteration, if \(Q_i\) falls within the limits, then the bus can be switched back to PV status.

**Acceleration of convergence**

It is found that in GS method of load flow, the number of iterations increases with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant \(\alpha\), called the acceleration factor. In the \((r+1)\)th iteration we can let
\[
V_i^{(r+1)}(\text{accelerate by}) = V_i^{(r+1)} + \alpha (V_i^{(r+1)} - V_i^{(r)}) \tag{13}
\]

Where \(\alpha\) is a real number. When \(\alpha = 1\), the value of \(V_i^{(r+1)}\) is the computed value. If \(1 < \alpha < 2\), then the value computed is extrapolated. Generally \(\alpha\) is taken between 1.2 to 1.6, for GS load flow procedure.

**V. NEWTON RAPHSON (NR) LOAD FLOW**

Newton Raphson (NR) [7] method is used to solve a system of nonlinear algebraic equations of the form \(f(x) = 0\). Consider a set of \(n\) nonlinear algebraic equations given by
\[
f_i(x_1, x_2, ..., x_n) = 0 \quad i = 1, 2, ..., n \tag{14}
\]

Let \(x_1^0, x_2^0, ..., x_n^0\) be the initial guess of unknown variables and \(\Delta x_1, \Delta x_2, ..., \Delta x_n\) be the respective corrections. Therefore,
\[
f_i(x_1 + \Delta x_1, x_2 + \Delta x_2, ..., x_n + \Delta x_n) = 0 \tag{15}
\]

The above equation can be expanded Using Taylor’s series to give
\[
f_i(x_1^0, x_2^0, ..., x_n^0) + \left[\frac{\partial f_i}{\partial x_1} \Delta x_1^0 + \frac{\partial f_i}{\partial x_2} \Delta x_2^0 + \cdots + \frac{\partial f_i}{\partial x_n} \Delta x_n^0\right] + \text{Higher order terms} = 0 \tag{16}
\]

Where \(i = 1, 2, ..., n\).

Where, \(\frac{\partial f_i}{\partial x_1}, \frac{\partial f_i}{\partial x_2}, ..., \frac{\partial f_i}{\partial x_n}\) are the partial derivatives of \(f_i\) with respect to \((x_1, x_2, ..., x_n)\) respectively, evaluated at \((x_1^0, x_2^0, ..., x_n^0)\). If the higher order terms are neglected, then (16) can be written in matrix form as
\[
\begin{bmatrix}
f_1^0 & \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & f_2^0 & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & f_n^0 & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}
\begin{bmatrix}
\Delta x_1^0 \\
\Delta x_2^0 \\
\vdots \\
\Delta x_n^0
\end{bmatrix} = 0 \tag{17}
\]

In vector form (17) can be written as
\[
f^0 + j \Delta x = 0 \tag{18}
\]

\(j \Delta x\) is known as the Jacobian matrix equation (18) can be written as
\[
f^0 = [j] \Delta x \tag{19}
\]

Approximate values of corrections \(\Delta x^0\) can be obtained from equation (19).These being a set of linear algebraic equations can be solved efficiently by triangularisation and back substitution. Updated values of \(x\) are then
\[
x^{(r+1)} = x^r + \Delta x^r \tag{20}
\]

Iterations are continued till equation (14) is satisfied to any desired accuracy i.e.
\[
|f_i(x^{(r)})| < \varepsilon \tag{21}
\]

Where \(i=1, 2, ..., n\).

**NEWTON RAPHSON Algorithm**

First, assume that all buses are PQ buses. At any PQ bus the load flow solution must satisfy the following non-linear algebraic equations

\[
f_{1f}(V, \delta) = P_i(\text{specified}) - P_i = 0 \tag{21a}
\]
\[
f_{1q}(V, \delta) = Q_i(\text{specified}) - Q_i = 0 \tag{21b}
\]

Where expressions for \(P_i\) and \(Q_i\) are given in equations. For a trial set of variables \[|V_i|, \delta_i\] the vector of residuals \(f^0\) of equation (18) corresponds to \[f_{1f} = P_i(\text{specified}) - P_i(\text{cal}) \tag{22a}\]
\[ f_{ij} = Q_{i(\text{specified})} - Q_{i(\text{cal})} = \Delta Q_i \]  

(22b)

While the vector of corrections \( \Delta X^0 \) corresponds to \( \Delta \| V \|, \delta \) Equation (18) for obtaining the approximate corrections vector can be written for the load flow case as

\[
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i
\end{bmatrix} = 
\begin{bmatrix}
H_{im} & N_{im} \\
J_{im} & L_{im}
\end{bmatrix} \\
\begin{bmatrix}
\Delta \delta \\
\Delta \| V \|
\end{bmatrix} \\
\begin{bmatrix}
\Delta \| V \|
\end{bmatrix} = 
\begin{bmatrix}
\Delta \delta \\
\Delta \| V \|
\end{bmatrix}
\]

(23)

Whereas,

\[
H_{im} = \frac{\partial P_i}{\partial \| V \|}, N_{im} = \frac{\partial Q_i}{\partial \| V \|}, J_{im} = \frac{\partial Q_i}{\partial \delta}, L_{im} = \frac{\partial Q_i}{\partial \| V \|}
\]

(21a)

It is to be observed that the jacobian elements corresponding to the \( i^{th} \) bus residuals and \( m^{th} \) bus corrections are 2x2 matrix enclosed in the box in equation (21a) where \( i \) and \( m \) are both PQ buses. Since at the slack bus \( P_i \) and \( Q_i \) are unspecified and \( \| V \| \) and \( \delta_i \) are fixed. Consider now the presence of PV buses. If the \( i^{th} \) bus is a PV bus, \( Q_i \) is unspecified so that there is no equation corresponding to equation (21b). Therefore the jacobian elements are

\[
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i
\end{bmatrix} = 
\begin{bmatrix}
H_{im} & N_{im} \\
J_{im} & L_{im}
\end{bmatrix} \\
\begin{bmatrix}
\Delta \| V \|
\end{bmatrix} = 
\begin{bmatrix}
\Delta \| V \|
\end{bmatrix}
\]

(21b)

If the \( m^{th} \) bus is also a PV bus \( \| V \| \) becomes fixed so that \( \Delta \| V \| = 0 \) and \( \delta \) are fixed. Therefore the jacobian elements are

\[
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i
\end{bmatrix} = 
\begin{bmatrix}
H_{im} & N_{im} \\
J_{im} & L_{im}
\end{bmatrix} \\
\begin{bmatrix}
\Delta \delta
\end{bmatrix} = 
\begin{bmatrix}
\Delta \delta
\end{bmatrix}
\]

(22b)

If the \( m^{th} \) bus is a PQ bus while \( m^{th} \) bus is a PV bus, then elements are

\[
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i
\end{bmatrix} = 
\begin{bmatrix}
H_{im} & N_{im} \\
J_{im} & L_{im}
\end{bmatrix} \\
\begin{bmatrix}
\Delta \| V \|
\end{bmatrix} = 
\begin{bmatrix}
\Delta \| V \|
\end{bmatrix}
\]

(22b)

It is convenient for numerical solution to normalize the voltage corrections

\[ \Delta \| V \| \]

As a consequence of which, the corresponding jacobian elements become

\[
N_{im} = \frac{\partial Q_i}{\partial \| V \|}, L_{im} = \frac{\partial Q_i}{\partial \| V \|}
\]

VI. FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of Fast Decoupled Load Flow (FDLF) [7]. Memory requirement of Newton-Raphson is resulted in development of FDLF [7]. Memory requirement of Newton-Raphson is reduced by this method. The property of weak coupling between \( P-\delta \) and Q-V variables gave the necessary motivation in developing the fast decoupled load flow method. In which \( P-\delta \) and Q-V problems are solved separately. The elements to be neglected are submatrices [N] and [J]

\[ [\Delta P] = [H][\Delta \delta] \]

(24)

Here, certain assumptions, the entries of the [H] and [L] submatrices will become considerably simplified

\[ H_{ij} = L_{ij} = -|V_i||V_j|B_{ij} \quad \text{for} \ i \neq j \]

\[ H_{ii} = L_{ii} = -|V_i|^2 \quad \text{for} \ i=j \]

Matrices [H] and [L] are square matrices with dimension \( (n_{PQ} + n_{PV}) \) and \( n_{PQ} \) respectively. Equations (24) and (25) can now be written as

\[ [\Delta P] = [V_i][V_j][B'_{ij}][\Delta \delta] \]

(26)

\[ [\Delta Q] = [V_i][V_j][B^*_{ij}][\Delta \| V \|] \]

(27)

Where \( B'_{ij}, B^*_{ij} \) are elements of \([-B]\) matrix.

Fast decoupled load flow algorithm

Step1. Omitting from \([B']\) the representation of those network elements that predominantly affect reactive power flows.

Step2. Neglecting from \([B^*]\) the angle shifting effects of phase shifters

Step3. Dividing each of the equation (26) and (27) by \( |V| \) and setting \( |V| = 1 \) p.u in the equations.

Step4. Ignoring series resistance in calculating the elements of \([B']\) which then becomes the dc approximation power flow matrix.

With above modifications, the resultant simplified FDLF equations become

\[ \frac{[\Delta P]}{|V|} = [B'][\Delta \delta] \]

(28)

\[ \frac{[\Delta Q]}{|V|} = [B^*][\Delta \| V \|] \]

(29)

In Equation (28) and (29) both \([B']\) and \([B^*]\) are real, sparse and have the structures of [H] and [L] respectively. Since they have contains only admittances. Equations (28) and (29) are solved alternatively always employing the most recent voltage value. Single iteration implies one solution for \([\Delta \delta]\) to update \([6]\) and then one solution for \([\Delta \| V \|]\) to update \([|V|]\) to be called 1-\( \delta \) and 1-V iteration. Separate convergence tests are applied for the real and reactive power mismatches as follows: \( \text{max} \ [\Delta P] \leq 6 \varepsilon_p \); \( \text{max} \ [\Delta Q] \leq 6 \varepsilon_q \). Where \( \varepsilon_p \) and \( \varepsilon_q \) are the tolerances.

VII. RESULTS AND DISCUSSION

Above discussed load flow methods have been implemented by using MATLAB on sample test systems of IEEE 9-Bus System, 14- Bus System and 25- Bus power System. Performance of these methods have been studied in terms of number of iterations taken for a given accuracy, computational time, convergence obtained, requirement of computer storage memory etc. As discussed in the following sections.
**Computational Time (in seconds)**

From Fig. 1 it is clear that the time per iteration in An Approximate, Gauss Seidel and Newton Raphson methods increases almost directly as the number of buses of the system while the elapsed time of the Fast Decoupled is less than the Newton Raphson method. But as accuracy increase from 0.01 to 0.000001 computational time of Newton Raphson method is quite less than other methods.

![Graph showing computational time](image)

**Number of Iteration**

It is clear from Fig. 6 the Gauss Seidel method requires larger number of iterations to converge to given voltage mismatch tolerance, compare with other methods. Approximate method is non-iterative and Fast decoupled methods takes more number of iterations to converge.

![Graph showing number of iterations](image)

**Memory (in Bytes)**

It is clear from Fig. 14 that Newton-Raphson requires more memory than Gauss Seidel, Approximate method, and Fast decoupled method.
**Convergence Characteristics**

The convergence characteristic are described by plotting the bar graphs of change in voltage magnitudes during successive iterations as a function of required tolerances as shown in Figs. 10, 11 and 12.

It is clear from below bar graphs that as accuracy increases absolute voltage mismatch of Newton Raphson method decreases i.e. this method is best to achieve the convergence.

**Real Power Losses (in MW)**

As we seen from bar graphs, it is clear that real power loss obtained from Newton Raphson method are constant i.e. does not vary as accuracy increases, where as in other methods losses increases as accuracy level increase.
Reactive Power Losses (MVAR)

As seen from bar graphs, it is clear that reactive power loss obtained from Newton Raphson method are constant, whereas in other methods losses increases as accuracy level increase.
VIII. CONCLUSION

From above results it indicates that Newton Raphson method is more reliable because it converges faster and it takes least number of iterations when compared with the other methods. In general the Newton Raphson algorithm takes the least number of iteration to converge despite its longer computing time but as accuracy increases computing time of Newton Raphson is quite less than other methods. The number of iteration for the Gauss-Seidel increases directly as the number of the buses of the network, whereas the number of iterations for the Newton Raphson method remains practically constant, independent of the system size and approximate method is a non-iterative method. It
is based on approximations. However, since the convergence characteristics of the Fast decouple method is geometric compare to the quadratic convergence of the Newton Raphson, thus it has more number of iteration. Therefore because of high accuracies obtained in only a few iterations, the Newton Raphson method is important for use and more reliable than any of the methods.

IX. REFERENCES

APPENDIX
IEEE 9-BUS SYSTEM INPUT DATA

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<th>To bus</th>
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### IEEE 14-Bus System Input Data

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### IEEE 25-BUS SYSTEM INPUT DATA

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