

# Comparative Analysis of Different Wavelets for Image Denoising

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**Abstract—** In this paper Comparative Analysis of different wavelets have been presented for image denoising. Wavelets Transform are used to overcome the drawbacks of Fourier Transform & short Fourier Transform. Fourier Transform can perform frequency analysis but cannot provide time analysis. Short time Fourier Transform can provide frequency & time analysis but selection of window size is a big problem. Wavelets are used to overcome these problems and are capable of providing efficient time and frequency analysis. So different wavelets are used to determine an image like Haar, Sym and Coif. All wavelets have been compared and analyzed based on Mean, Median and S.D. All wavelets have been designed and simulated using methods. The comparison results have been shown in simulated and tabular form.

**Keywords—** Wavelets, Transform, Short Time Fourier Transform, Continuous wavelet Transform.

## I. INTRODUCTION

Wavelet is a small wave and its transform convert a signal into a series of wavelets that provide a way for analyzing waveforms, bounded in both frequency and duration. It also allows signal to be stored more efficiently than by Fourier transform. It is being able to provide better approximate real world signals and well suited for approximately data with sharp discontinuities. Fourier Transformation is the most popular transformation. Fourier transform is an integral from  $t = -\infty$  to  $t = +\infty$ . So there is no time localization. Even if you would transform a wavelet to its frequency domain, still the relative phase relation of different contributing frequencies determines the position in time of the transformed wavelet. Of course a Fourier transform can be performed on a certain time interval  $t$ , but keep in mind that, when transforming back to time domain, the transformed signal will repeat itself every time interval  $t$ . This again will give no localization in time for a Fourier transform. Frequency spectrum is basically the frequency components of that signal and also show what frequencies exist in the signal. Fourier transform is the one way to find frequency content and it tells how much of each frequency exists in a signal. Stationary signal are those signal whose frequency content unchanged in time and all frequency components exist at all times and Non Stationary signal frequency

changes with time. The most famous example is the "chirp" signal. Fourier Transform only gives what frequency component exist in the signal. The time and frequency information cannot be seen at the same time and time frequency representation of the signal is needed. But cannot provide simultaneous time and frequency localization and Not useful for analyzing time-variant, non-stationary signals. It is also not efficient for representing discontinuities or sharp corners (i.e., requires a large number of Fourier components to represent discontinuities). So need a *local* analysis scheme for a time-frequency representation (TFR). Windowed Fourier transform or Short Time Fourier transform (STFT) Segmenting the signal into narrow time intervals (i.e., narrow enough to be considered stationary) and then Take the Fourier transform of each segment. Dennis Gabor in 1946 used short time Fourier transform to analyze only a small section of the signal at a time and this technique is called windowing the signal and mainly the segment of signal is assumed stationary. Instead of transforming the whole signal  $s(t)$  all at once using the Fourier Transform, it is transformed on a block-by-block basis using a moving time-window, centred at the time instant  $t$ , The Short Time Fourier Transform is defined in equation (1) :

$$P(t, f) = \int_{-\infty}^{\infty} s(\lambda)h(\lambda - t)e^{-j2\pi f\lambda} d\lambda = \underset{\lambda \rightarrow f}{FT} \{s(\lambda)h(\lambda - t)\} \quad \text{--(1)}$$

Where  $h(t)$  is a suitable time-window such as a hamming window.

## II. WAVELETS

A wavelet is a wave-like oscillation with an amplitude that starts out at zero(0), increases, and then decreases back to zero. Thus a wavelet can be defined within a certain time span, starting at  $f(t_0) = 0$  and ending at  $f(t_{end})$  at 0. This then gives us time localization. It can typically be visualized as a "brief oscillation" like one might see recorded by a seismograph or heart monitor. The word *wavelet* has been

used for decades in digital signal processing and exploration geophysics. The equivalent French word *ondelette* meaning "small wave" was used by Morlet and Grossmann in the early 1980s. Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. Wavelets can be combined, using a "reverse, shift, multiply and integrate" technique called convolution, with portions of a known signal to extract information from the unknown signal. As a mathematical tool, wavelets can be used to extract information from many different kinds of data, including – but certainly not limited to – audio signals and images. Sets of wavelets are generally needed to analyze data fully. A set of "complementary" wavelets will decompose data without gaps or overlap so that the decomposition process is mathematically reversible. Thus, sets of complementary wavelets are useful in wavelet based compression/ decompression algorithms where it is desirable to recover the original information with minimal loss. Wavelet theory and its application have grown tremendously during the last decade. It has become a cutting edge technology in computer and human vision, neural network detecting self-similar behaviour in a time-series area of image processing. Fourier Transform is not suitable for non-stationary time varying signal and Short Time Fourier Transform lags in resolution but Wavelet Transform has the ability to analyse any time varying signal and processes data at different resolution [1]. Wavelet Transform is better choice than traditional Fourier Transform because of its ability to localise in frequency and time domain simultaneously. It uses its multi resolution capability to decompose the image into multiple frequency bands. Also it provides very low mean square error, high compression ratio and good image quality. It is suitable for application where scalability and tolerable degradation are important for its inherent multi resolution nature wavelet coding schemes. In addition, they are better matched to the human visual system [2-3]. Wavelet theory is applicable to several subjects. All wavelet transforms may be considered forms of time-frequency representation for continuous-time (analog) signals and so are related to harmonic analysis. Almost all practically useful discrete wavelet transforms use discrete-time filter banks. These filter banks are called the wavelet and scaling coefficients in wavelets nomenclature. These filter banks may contain either finite impulse response (FIR) or infinite impulse response (IIR) filters. The wavelets forming a continuous wavelet transform (CWT) are subject to the uncertainty principle of Fourier analysis respective sampling theory: Given a signal with some event in it, one cannot assign simultaneously an exact time and frequency response scale to that event. The product of the uncertainties of time and frequency response scale has a lower bound. Thus, in the scaleo gram of a continuous wavelet transform of this signal, such an event marks an entire region in the time-scale plane, instead of just one point. Also, discrete wavelet bases may be considered in the context of other forms of the uncertainty principle. Wavelet transforms are broadly divided into three classes: continuous, discrete and multi resolution-based. An image is often corrupted by noise in its acquisition or transmission. The goal of de noising method is to remove the noise while retaining as much as possible of the important

image features. One of the fundamental challenges in the field of image processing and computer vision is image de noising, where the underlying goal is to estimate the original image by suppressing noise from a noise-contaminated version of the image. Image noise may be caused by different intrinsic (i.e., sensor) and extrinsic (i.e., environment) conditions which are often not possible to avoid in practical situations. Therefore, image de noising plays an important role in a wide range of applications such as image restoration, visual tracking, image registration, image segmentation, and image classification, where obtaining the original image content is crucial for strong performance. While many algorithms have been proposed for the purpose of image de noising, the problem of image noise suppression remains an open challenge, especially in situations where the images are acquired under poor conditions where the noise level is very high. In fact, in the 2-D case and for Haar, Coif and Sym wavelets, de noising part has been shown with the residuals. Here we can analyse on the basis of mean, median and standard deviation which are the important parameters. Here in this selecting thresholding method is fixed from threshold and un scaled white noise is selected for noise structure. And we can analyse the whole details in horizontal details coefficient. Wavelet transforms treats both continuous and discrete time cases. The choice depends upon orthogonality, compactness, width and shape of wavelet function [4]. Haar, sym and coif SWT denoising in 2D is wavelet family used for image denoising. Haar, Daublets (db), sym and coif are orthogonal wavelets from which haar wavelet is simplest and fastest to implement. Unlike haar, symmetry and asymmetry is not possible for real compactly supported orthogonal wavelets. Biorthogonal wavelets (B-Spline and V-Spline) are symmetric and exhibit the property of linear phase which is needed for signal and image compression [5-6].

### III. IMAGE DENOISING USING WAVELETS

In mathematics, the Haar wavelet is a certain sequence of rescaled "square-shaped" functions which together form a wavelet family or basis. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal function basis. The Haar sequence is now recognised as the first known wavelet basis and extensively used as a teaching example in the theory of wavelets. The Haar sequence was proposed in 1909 by Alfréd Haar. Haar used these functions to give an example of a countable orthonormal system for the space of square-integrable functions on the real line. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. The original image for each wavelet is shown in Fig.1.

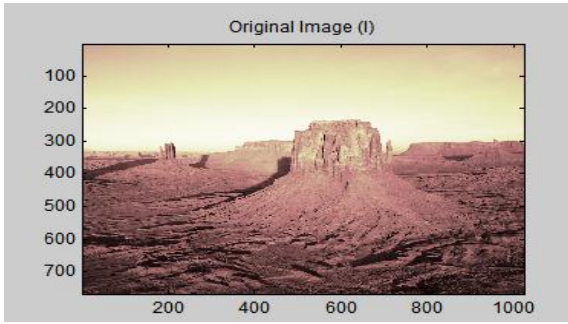


Fig. 1 Original image

It is known that the Haar wavelet has the most compact spatial support of all wavelets and is also an optimal edge matching filter. This method filters the gradient data into several reduced resolution gradient data sets. The Haar decomposition is determined from this data. This method, as would be expected from using the Haar wavelets, generates blocky errors when noise is present. To remove this we can analyse it on various parameters. The denoise image is also shown in Fig.2.

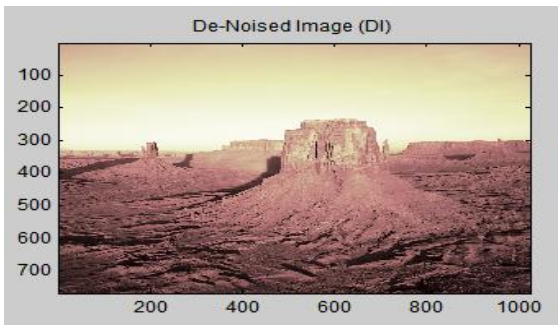


Fig. 2 De noised image

The computation cost of this method is on the order of 20 flops per pixel. This is the total cost as there are no iterations. In functional analysis, the Haar systems denotes the set of Haar wavelets as written in equation (2).

$$\{t \mapsto \psi_{n,k}(t) = \psi(2^n t - k); n \in \mathbb{N}, 0 \leq k < 2^n\} \quad (2)$$

Here is a plot of the residuals (I)-(DI) IN Fig.3.

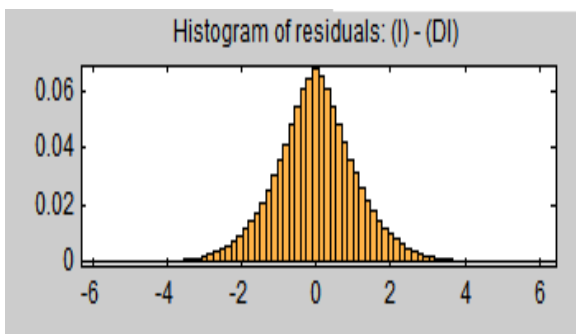


Fig. 3 Histogram of residuals

Coiflets are discrete wavelets designed by Ingrid Daubechies, at the request of Ronald Coifman, to have scaling functions with vanishing moments. The wavelet is near symmetric, their wavelet functions have N/3 vanishing moments and scaling functions N/3-1. Denoise image of coif is shown in Fig.4.

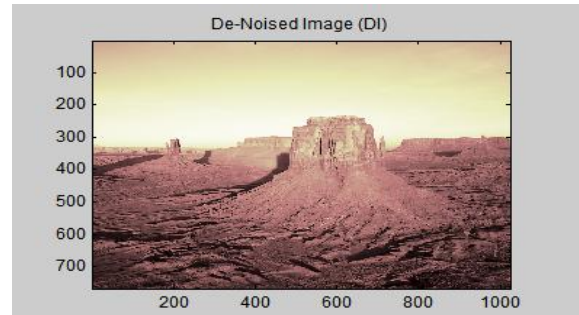


Fig. 4 De noised image of coiflet

Both the scaling function (low-pass filter) and the wavelet function (High-Pass Filter) must be normalised by a factor  $1/\sqrt{2}$  Mathematically, this looks like  $B_k = (-1)^k C_{N-1-k}$  where  $k$  is the coefficient index,  $B$  is a wavelet coefficient and  $C$  a scaling function coefficient.  $N$  is the wavelet index, i.e. 6 for C6. And its histogram for residuals is shown in Fig.5.

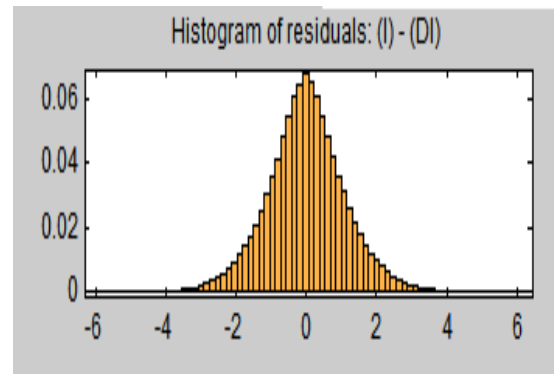


Fig. 5 Histogram of residuals

Sym proves better in Maximum Normalization. So ‘Sym’ wavelet type is performed to obtain the least asymmetry with shift invariance. Its denoised image is shown over here in Fig.6.

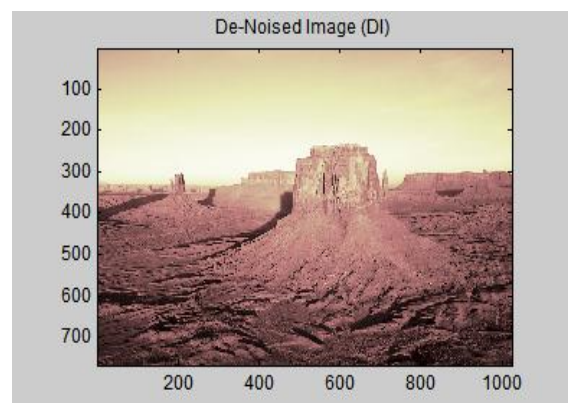


Fig. 6 De noised image of coif

They do have the combined features of Discrete Wavelet transform and Continuous Wavelet Transform. Its histogram is also shown here in Fig.7.

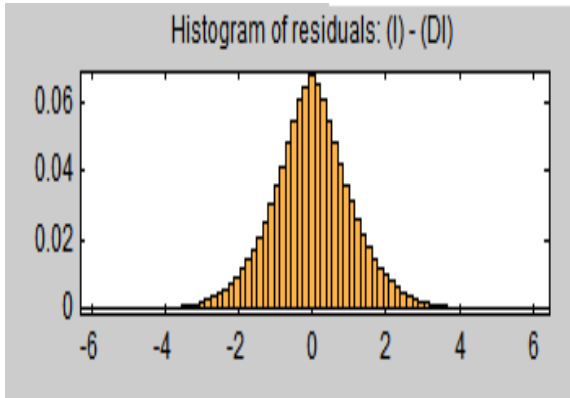


Fig. 7 Histogram of residuals

### III. RESULT ANALYSIS

The Comparison of various wavelets is shown through the table 1. & the histogram in between mean, median and standard deviation is shown in Fig. 8.

Table1. Shows comparison in given wavelets

	HAAR Wavelet	COIF Wavelet	SYM Wavelet
MEAN	4.723e-006	2.798e-006	-1653e-006
MEDIAN	0	-0.004974	-0.005827
STANDARD DEVIATION	1.045	1.103	1.104

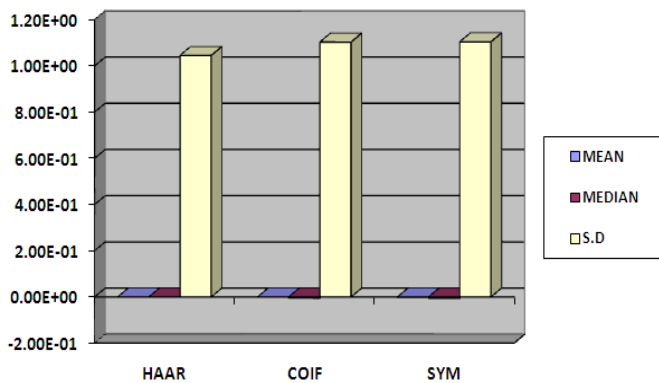


Fig 8 shows histogram in between mean, median and standard deviation

### IV. CONCLUSION

Image denoising has been performed in this paper using different wavelet techniques. A real image has been denoised using Haar , Coif & Sym wavelets. Mean , Median and S. D of denoised images have been compared to analyze the performance. The results shows in haar - mean ,median and S.D are 4.723e-006 , 0 1.045 respectively , in coif -mean ,median and S.D are 2.798e-006 , -0.004974, 1.103 respectively , in sym -mean ,median and S.D are -1.653e-006 , -0.005827, 1.104 respectively.

### V. REFERENCES

- [1] Lotfi A. A., Hazrati M. M., Sharei M., Saeb Azhang ,” CDF(2,2) Wavelet Lossy Image Compression on Primitive FPGA”, IEEE , 2005,pp. 445-448.
- [2] Uytterhoeven G., “Wavelets: Software and Applications”, K.U. Leuven Celestijnenlaan, Department of Computer Science, Belgium, 1999.
- [3] Rao R. and Bopardika A. S, “Wavelet Transform, Introduction to Theory and Applications,” Pearson Education Asia,2002.
- [4] Rioul O. and Vetterli , “Wavelets and Signal Processing,” IEEE Signal Processing Magazine, 1991, Vol.91, pp. 14-34.
- [5] Thong Nguyen and Dadang Gunawan , “Wavelets and Wavelets-Design Issues”, IEEE, ICCS Singapore, 1994, pp. 188-194.
- [6] Young R. K. (1993) “Wavelet theory and its applications “, Kluwer Academic Publishers; Boston/Dordrecht/London.