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Comparative Analysis of Chemical Propulsion Orbit Transfer Methods and Contribution of Gravity Assist for a Space Mission to Jupiter

Naga Bharath Gundrati[1]
[1]Mechanical and Aerospace Engineering,
University at Buffalo,
Buffalo, USA

Meghana Rachamallu[2]
[2]Industrial and Systems Engineering,
Virginia Polytechnic Institute and State University,
Blacksburg, USA

***Abstract*— The selection of an appropriate orbit transfer method is one of the major tasks in planning a spacecraft mission. This paper provides an analysis of three major chemical propulsion orbit transfer mechanisms that can be applied to the transfer of a spacecraft from Lower Earth Orbit to Jupiter's orbit; and consequentially the time is taken for the transfer, including the energy and fuel requirements. Lagrange points which form a gravitationally determined path to the orbit of Jupiter are calculated. The juxtaposition of all the energy and the requirements for each of the orbit transfer method and Interplanetary Transfer Network are presented. The Hyperbolic Excess and Capture velocities required to escape the Gravitational force of Earth and get drawn into an orbit about a particular altitude above Jupiter respectively are calculated. Finally, the velocity boost from the gravitational force of Mars, due to a flyby is also computed and the effect of orientation angle on the velocity gain is shown.**

Keywords*— Gravity assist, Hohmann transfer, interplanetary transfer network, Jupiter, orbit transfer.

I. INTRODUCTION

The primary motivation behind this particular topic is the spacecraft mission by NASA called the Juno. The orbital transfer is an integral part of orbital mechanics dealing with the trajectory of the spacecraft involving orbital maneuvers, plane changes, and interplanetary transfers.

There are chiefly three types of chemical propulsion orbit transfer methods which are unique in their approach trajectories, the Delta-V and time required to reach the final orbit. The orbital transfer chosen for a satellite is a compromise between the mission requirements, the capabilities of the rocket used to launch the satellite and orbital mechanics. In addition to the orbit transfer, the Delta-V required for the spacecraft to escape the gravitational pull of the source planet and to hook on to the destination orbit are overlooked. In the effort to be more accurate for the total fuel required and time required for the mission it is essential to calculate the hyperbolic escape and capture velocities as well.

Often in the space travel, the presence and motion of astronomical bodies are exploited to provide a predetermined path or to gain velocity to aid in the travel of the spacecraft to the desired planned or an orbit quickly. The Interplanetary Transport Network is a low energy transfer alternative which requires little to no energy to traverse from one planet to another. It connects the Lagrange points of the planets and is a result of the gravitational pull of these planets. The another useful application of the planets' gravitational force and relative movement, is the flyby or gravity assist.

To be able to bring all the orbit transfer methods to a common platform and evaluate their relevance and quantify their contribution to the overall mission is appreciated.

This paper compares the different types of orbit transfers to come out with the optimal method for interplanetary transfer to Jupiter based on the different types of mission requirement.

II. INITIAL SETTING

We consider a spacecraft mission involving traversing from earth to reach Jupiter. To achieve this, we make a few assumptions to set a background for the comparison.

A. Assumptions

1) Circular Orbits

There are no circular orbits in the solar system. The orbits of every planet have some eccentricity. But in an effort to simplify the calculations we assumed all the planets to have circular orbits. Although orbit transfer mechanisms can also be applied to the transfer between elliptical orbits and circular and elliptical orbits, our focus is only on the transfer between two circular orbits.

2) Two-body problem

To focus on the orbit transfer mechanisms alone we eliminate the external stimuli such as the effect of the other celestial bodies, space debris etc. This is accomplished by treating the orbit transfer as a two-body problem. This effectively simplifies our approach and prevents the consideration of the gravitation forces of the other planets and space objects.

3) The velocity changes due to propulsion are considered to occur instantaneously.

4) Origin of the launch

The spacecraft is launched from a location at the equator and is parked in the Lower Earth orbit (LEO). The lower earth orbit (LEO) is the orbit where the spacecraft is usually parked for assembly and other purposes. It is assumed to be at an altitude of 350 km from the earth.

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5) Inclination of the orbits

The spacecraft from LEO is transferred to the Jupiter's orbit which is at an inclination of 1.03 degrees.

6) Symbols and Notations

TABLE 1 Symbols and Notations

v_{esc}	Escape velocity	v_f	Final Orbital Velocity
μ	Gravitational constant	v_t	Velocity at Transfer
r	Radius of the planet	τ	Time taken for the transfer
Δv_e	Hyperbolic escape velocity	e_t	Eccentricity of transfer ellipse
Δv_c	Hyperbolic capture velocity	ϕ	Flight path angle
v_{enc}	Encounter velocity	E	Eccentric anomaly
Δv_{inc}	Delta-V for inclination	y	Ratio of mass of planet and sun
v_i	Initial orbital velocity	z	Ratio of the distance from to sun and planet
inc	Inclination in degrees	a	Semi-major axis
m	Mass of fuel	e	Eccentricity
Isp	Specific impulse	r_p	Closest point of approach to the planet
g_o	9.8m/s^2	v_p	Velocity at the closest point to the planet
r_p	Initial orbital radius	f	True anomaly
r_f	Final orbital radius	v_B	Velocity of the flyby body
a_t	Semi-major axis of the transfer orbit	h	Momentum
v_i	Initial orbital velocity	v_∞	Speed of approach from infinity

B. Entry and Exit Velocities

The Delta-V calculated for the orbit transfer does not incorporate the velocity change required to overcome the gravitational force of the corresponding planets. If the spacecraft is put into a transfer orbit alone it would keep shuttling between the two planet orbits because the orbital period of the spacecraft does not equal to either of the planets orbital periods.

1) Hyperbolic Escape Velocity

The concept of escape velocity is such that the spacecraft just barely escapes the gravitational field of the planet. It is essential that we provide a certain amount of energy for the spacecraft to not only escape the gravity of Earth but have a finite constant value [1]. Hence we choose the LEO as our burn-out point where we intend to initiate the orbit transfer.

$$v_{esc} = \sqrt{\frac{2\mu}{r}} \quad (1)$$

$$\Delta v_e = \sqrt{{v_{enc}}^2 + {v_{esc}}^2} - v_{esc} \quad (2)$$

The hyperbolic escape velocity is computed to be 13.57 km/s and the corresponding Delta-V required to gain the velocity beyond the escape velocity of earth is 2.39 km/s.

2) Hyperbolic Capture Velocity

The subsequent step after the orbit transfer is to capture the spacecraft into the Jupiter's orbit. This requires an additional velocity for the spacecraft to intersect the planets orbit at a certain elevation which is taken as 500 km for calculation purposes.

$$\Delta v_c = \sqrt{{v_{enc}}^2 + {v_{esc}}^2} - v_{esc} \quad (3)$$

This results in a hyperbolic capture velocity to be 60.77 km/s which is higher than the escape velocity of Jupiter by 1.42 km/s.

C. Inclination Change

The LEO is out of plane with the Jupiter's zero degrees orbit by 1.03 degrees. To change the orbital plane of the spacecraft the velocity vector is adjusted thereby inclination of the spacecraft is remodeled [2].

$$\Delta v_{inc} = 2v_i \sin\left(\frac{inc}{2}\right) \quad (4)$$

Utilizing the law of cosines, the thrust in a direction perpendicular to the orbital plane of the spacecraft is determined.

The Delta-V is computed to be 0.53 km/s.

D. Fuel Consumption

The Delta-V budget and the minor changes in it are better understood if their repercussions on the fuel mass that needs to be carried are studied. The specific impulse of the fuel is assumed to be 500 secs; this value reflects the amount of thrust produced when a unit of propellant is consumed [3]. The dry mass of the spacecraft i.e. excluding the fuel mass is assumed to be 500 kg.

$$m = dry\ mass(e^{\frac{\Delta v}{Isp * g_o}} - 1) \quad (5)$$

III. HOHMANN TRANSFER

Hohmann transfer is credited to be the fundamental orbit transfer for the interplanetary space missions. Hohmann transfer works on the assumption that the source and the destination orbits are circular.

The Hohmann transfer strategy uses an elliptical trajectory between the initial orbit and the final orbit. This transfer mechanism is known to be one of the low energy transfer, involving two burns. The two burns take place one at the perigee and the final burn at the apogee of the transfer [4].

$$v_i = \sqrt{\frac{\mu}{r_i}} \quad (6)$$

$$v_f = \sqrt{\frac{\mu}{r_f}} \quad (7)$$

$$v_{t_a} = \sqrt{\left(\frac{2\mu}{r_i}\right) - \left(\frac{\mu}{a_t}\right)} \quad (8)$$

$$v_{t_b} = \sqrt{\left(\frac{2\mu}{r_f}\right) - \left(\frac{\mu}{a_t}\right)} \quad (9)$$

$$\Delta v_a = v_{t_a} - v_i \quad (10)$$

$$\Delta v_b = v_f - v_{t_b} \quad (11)$$

$$\Delta v = |\Delta v_a| - |\Delta v_b| \quad (12)$$

$$\tau_t = \pi\sqrt{\frac{{a_t}^3}{\mu}} \quad (13)$$

The Delta-V required for both the burn including an inclination change prior to the transfer is 14.97 km/s. The time required for the transfer is 2.73 years and the fuel needed to carry out the transfer is 10114 kg.

There is a possibility that the inclination changes before beginning the orbit transfer are not possible. This could be due to various reasons, one of them being that the desired inclination path is unattainable due to space debris and other unfavorable conditions.

The Hohmann strategy provides a trajectory wherein the transfer orbit can originate from the initial orbit and reach the final orbit at a different inclination.

The Delta-V requirement for this transfer is calculated to be 14.74 km/s, taking 2.73 years. The fuel mass necessary for this transfer is 9623.4.

IV. BI-ELLIPTIC TRANSFER

Bi-elliptic orbit transfer is another strategy which assumes that the orbits are circular. This transfer usually takes a longer time that the Hohmann transfer but the Delta-V requirement is smaller.

The Bi-elliptic transfer is a slow transfer. The trajectory followed in this transfer is an ellipse, wherein the apoapsis of the transfer ellipse ranges further than the final orbit [4]. This can be an indication as to how slow can this transfer be.

$$a_{t2} = \frac{r_i + r_t}{2} \quad (14)$$

$$a_{t2} = \frac{r_t + r_f}{2} \quad (15)$$

$$v_i = \sqrt{\frac{\mu}{r_i}} \quad (16)$$

$$v_{t1_a} = \sqrt{\left(\frac{2\mu}{r_i}\right) - \left(\frac{\mu}{a_{t1}}\right)} \quad (17)$$

$$v_{t1_b} = \sqrt{\left(\frac{2\mu}{r_t}\right) - \left(\frac{\mu}{a_{t1}}\right)} \quad (18)$$

$$v_{t2_b} = \sqrt{\left(\frac{2\mu}{r_t}\right) - \left(\frac{\mu}{a_{t2}}\right)} \quad (19)$$

$$v_{t2_b} = \sqrt{\left(\frac{2\mu}{r_t}\right) - \left(\frac{\mu}{a_{t2}}\right)} \quad (20)$$

$$v_{t2_c} = \sqrt{\left(\frac{2\mu}{r_f}\right) - \left(\frac{\mu}{a_{t2}}\right)} \quad (21)$$

$$v_f = \sqrt{\frac{\mu}{r_f}} \quad (22)$$

$$\Delta v_a = v_{t1_a} - v_i \quad (23)$$

$$\Delta v_b = v_{t2_b} - v_{t1_b} \quad (24)$$

$$\Delta v_c = v_f - v_{t2_{cb}} \quad (25)$$

$$\Delta v = |\Delta v_a| + |\Delta v_b| + |\Delta v_c| \quad (26)$$

$$\tau_t = \pi\sqrt{\frac{{a_{t1}}^3}{\mu}} + \pi\sqrt{\frac{{a_{t2}}^3}{\mu}} \quad (27)$$

This transfer mechanism is a three burn method. The first two velocity changes occur at the periapsis and the apoapsis of the transfer ellipse and the last burn occurs at when the transfer ellipse intersects with the final orbit.

This strategy provides a flexibility where we can control the time taken for the transfer and the Delta-V required. This can be done by altering the apoapsis of the transfer ellipse to suit our mission requirements. However, it is done within a certain range due to the obvious trade off that when the apoapsis is increased to conserve Delta-V the time increases too.

The extension of the apoapsis of the transfer orbit is varied to study how the extent of the variation impacts the delta-v and the time was taken. Let us assume that the apoapsis varies from the required apoapsis to 4 times the required apoapsis.

The variation of the apoapsis value impact on the delta-v and time taken are plotted:

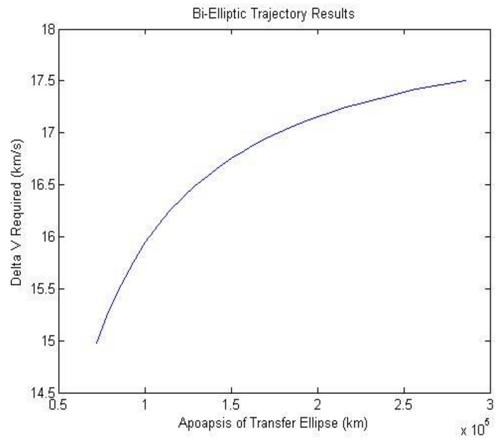

Fig. 1: Delta-V required for corresponding Apoapsis of the transfer ellipse

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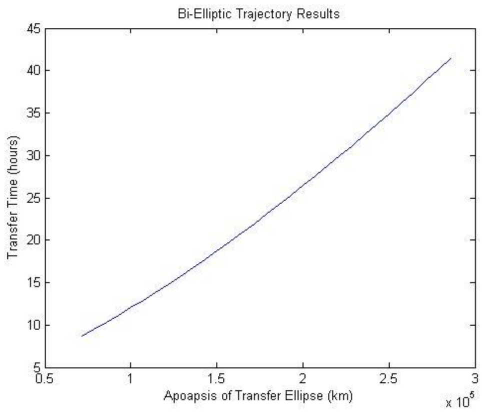

Fig. 2: Transfer time required for corresponding Apoapsis of the transfer ellipse

For the comparison purposes let us choose apoapsis of the value twice than the desired orbit. The time it takes for the transfer to complete is 17.72 years, requiring a total surge in the velocity of 16.67 km/sec, burning a fuel of mass 14532 kg.

V. ONE TANGENT BURN TRANSFER

One tangent burn transfer is essentially used when a quick transfer is necessary. One of the perks of this transfer is that it can be employed between circular and coaxial elliptical orbits. This method stands out from the orbit transfers discussed so far by the shape of trajectory the transfer orbit follows, which is a hyperbola [4].

In One tangent burn transfer, the spacecraft leaves the initial orbit tangential to the source orbit. It then traverses a route in the shape of a hyperbola intersecting the final orbit at an angle equal to the flight path angle of the transfer orbit.

It is possible for a curve to have an infinite number of tangents; therefore, an infinite number of transfer orbits are possible. this orbit transfer involves two burns out of which only one is a tangential burn, from which it got its name. To locate the second burn, we need to know the true anomaly of the trajectory.

$$R = \frac{r_i}{r_f} \quad (28)$$

$$e_t = \frac{R-1}{\cos(v_{t_b}) - R} \quad (29)$$

$$a_t = \frac{r_i}{1-e} \quad (30)$$

$$\tan\emptyset = \frac{e_t \sin(v_{t_b})}{1 + e_t\cos(v_{t_b})} \quad (31)$$

$$\Delta v_b = \sqrt{\left(v_{t_b}{}^2 + v_f{}^2 - 2v_{t_b}v_f\cos\emptyset\right)} \quad (32)$$

$$\Delta v = |\Delta v_a| - |\Delta v_b| \quad (33)$$

$$cosE = \frac{e_t + \cos(v_{t_b})}{1 + e_t\cos(v_{t_b})} \quad (34)$$

$$\tau = \sqrt{\frac{a_t{}^3}{\mu}}\{2k\pi + (E - e_t sinE) - (E_o - e_t sinE_o)\} \quad (35)$$

Thus we can design the transfer orbit from an array of available trajectories depending on the mission requirements. When the true anomaly of the orbit is changed the size and flight path angle are modified. This, in turn, affects the Delta-V and time taken for the spacecraft to complete the transfer.On varying, the angle of arrival or the true anomaly, in this case, the impact on the delta-v and transfer time are plotted.

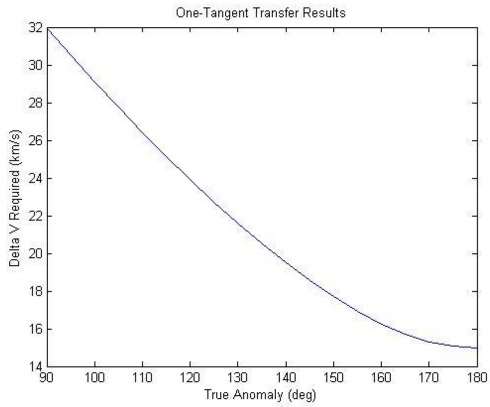

Fig. 3: Delta-V required for corresponding True Anomaly of the transfer ellipse

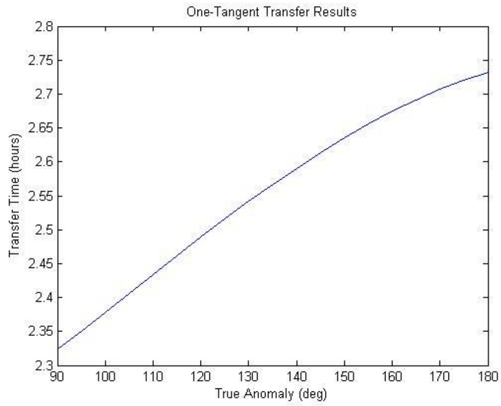

Fig. 4: Transfer time required for corresponding True Anomaly of the transfer ellipse

Including the initial change in the inclination the total Delta-V that the orbit transfer requires is 19.51 km/s. It consumes of fuel mass 26291 kg reaching the final orbit 2.59 years after the beginning of the transfer.

VI. INTERPLANETARY TRANSPORT NETWORK

The solar system has certain locations near the planets, which are the gravity sweet spots. The spacecraft can orbit the planet at these locations with little or no fuel [5]. We can exploit these points in space and conserve fuel.

Although it is not a straightforward approach in reality. Here we are tackling these issues as two body problems. Whereas, in space, there are far more complications and many gravitational wells that influence the path of the spacecraft.

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Vol. 5 Issue 11, November-2016

The special points around the points where the planets exert their gravitational force to hold the spacecraft, for that matter any object are known as Lagrange points [6]. There are five Lagrange points. The Lagrange point between the planet and the Sun is L1, and the point which lies behind the planet when looked from the sun is L2.

For the comparisons sake lets shoot a spacecraft which escapes the Earth's gravity to reach the L2 point of Earth as its burnout point.

The L2 point of Earth is calculated from

$$\frac{1}{(1+z)^3} + \frac{y}{z^2(1+z)} = 1 \quad (36)$$

Therefore, the L2 of Earth is 1.5 million km from Earth.

The hyperbolic excess velocity required to reach L2 is 0.012 km/s.

The spacecraft when in a Lagrange point orbits the planet and the falls off track and travels to the next planet. Since we are restricting our comparison to a two body problem calculations we will transfer the spacecraft from the Earth's L2 point to Mars L1 point using Hohmann Transfer.

$$\frac{1}{(1-z)^3} - \frac{y}{z^2(1-z)} = 1 \quad (37)$$

The L1 point of Mars is at 1.08 million km from Mars.

The Delta-V required for this transfer is 5.39 km/s and the transfer completes in 0.71 years.

As the spacecraft reaches the L1 point of Mars. we can calculate the velocity boost from the gravitational force of Mars [7].

One of the factors which affect the gravity assist phenomenon besides the parameters of the planet such as the gravitational constant and the mass; is the altitude at which we intersect the planet. The closer we are the more is the increase in the velocity. Here the altitude is the L1 point.

The velocity of the increases with the true anomaly decreasing from the initial value till it reaches zero and then drops till the true anomaly is regained in the opposite direction. This just implies the approach and departing of the spacecraft.

$$a = \frac{-\mu}{{v_\infty}^2} \quad (38)$$

$$e = 1 + \left(\frac{r_p v}{\mu}\right) \quad (39)$$

$$p = r_p(1+e) \quad (40)$$

$$f_\infty = cos^{-1}\left(\frac{-1}{e}\right) \quad (41)$$

$$v_p = \sqrt{\frac{2\mu}{r_p} + {v_\infty}^2} \quad (42)$$

$$h = v_p r_p \quad (43)$$

$$r = \frac{p}{1+ecosf} \quad (44)$$

$$v = \sqrt{\frac{2\mu}{r} + {v_\infty}^2} \quad (45)$$

$$\beta = f_\infty + f \quad (46)$$

$$\gamma = cos^{-1}\left(\frac{h}{rv}\right) \quad (47)$$

$$\delta = \beta - \gamma - 90° \quad (48)$$

$$V = \sqrt{v^2 + {v_B}^2 - 2vv_B\cos(\theta + \delta)} \quad (49)$$

The reduction in the velocity does not completely retract the velocity boost. It provides a residual surge in the velocity.

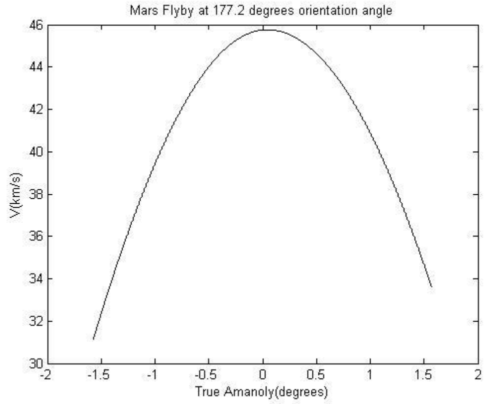

Fig. 5: The change in the velocity as the spacecraft approaches and leaves Mars

The orientation angle i.e., the angle between the velocity of the flyby body which is the speed of Mars vector and the velocity at infinity which is the speed at the apoapsis during the Hohmann transfer vector impacts the velocity gain from the gravity assist.

The velocity gain against the orientation angle is plotted to further illustrate this claim.

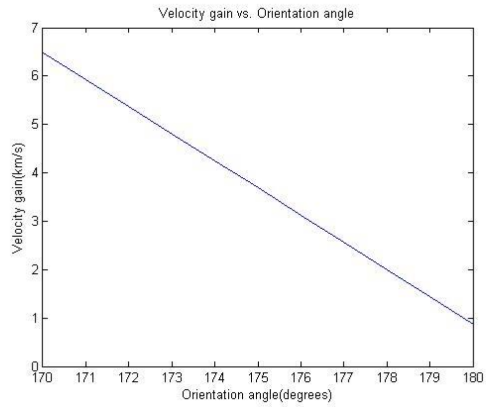

Fig. 6: The velocity gain from Mars Flyby at different orientation angles

To move from L1 point to L2 point the spacecraft requires a boost of 2.44 km/s, which is provided by the gravity assist.

From the L2 point of Mars, the spacecraft is then transferred to L1 of Jupiter which is at 51.89 million km from Jupiter.

The Delta-V for this Hohmann transfer to the L1point of Jupiter is 9.77 km/s which is completed in 2.85 years.

The spacecraft is finally captured from the L1 point into the Jupiter's orbit at an elevation of 500 km consuming a Delta-V of 11.4 km/s.

The Time take for this overall space mission is 3.57 years. The 4176.56 kg of fuel is burned to produce a total Delta-V of 15.16 km/s.

VII. CONCLUSION

The values of Delta-V, fuel mass and the time taken for the transfer excluding the hyperbolic escape and capture velocities is tabulated. The Hohmann transfer is mentioned twice, once where the inclination change is done prior to the transfer and once where the transfer is performed along with the inclination change.

TABLE 2 Comparison of Orbit Transfer Methods

Orbit Transfer	**Delta -V (km/s)**	**Time (years)**	**Fuel mass (kg)**
Hohmann(separate inclination)	14.97	2.73	10,114
Hohmann(including inclination)	14.74	2.73	9,623
Bi-elliptic	16.67	17.72	14,532
One Tangent	19.57	2.59	26,291
Interplanetary Transfer Network	15.16	3.56	4,175

The Hohman transfer is seemingly the better option when all the parameters are considered. A large Delta-V is observed at periapsis of the transfer orbit for the Hohmann transfer. This might be a problem because if the spacecraft needs to deviate from the path due to unfavorable conditions then the correction burn will be enormous.

The Bi-elliptic transfer which is effectively two Hohmann transfers fused together will be a good option to prevent such penalties. The intermediate burn absorbs the huge surge in the Delta-V of the final burn. Given that every little equipment on the spacecraft is very expensive, it is a good idea to avoid a large final burn. For the Bi-elliptical transfer to be beneficial the final and the initial orbits have to be in a ratio greater than 12:1 [8]. In addition, the increasing the size of the transfer orbit by increasing the apoapsis has a direct impact on the transfer time and the Delta-V. The larger the trajectory that a spacecraft follows the longer is the transfer time. However, the Delta-V for a leisure transfer is smaller.

When we look at these transfers from a purely time constrained point of view, the One tangent transfer is the better option. The propulsion initially to reach the final orbit provides a velocity larger than required and therefore additional Delta-V is necessary, furthermore this phenomenon is higher for smaller true anomaly value consuming little time.

The Interplanetary transfer network is rather not so straight forward. In this paper, we prompt the orbit transfer from the Lagrange points of the planets. Whereas, in reality, the spacecraft orbits the planet from that distance and then moves to the next planets following the gravity wells of the surrounding bodies. It uses the gravity assist of the planets to navigate to next planet which is accurately described in the paper. Then again the fuel mass is half when compared to the fuel mass required for the Hohmann transfer. This is due to splitting the transfer into two separate burns with a velocity boost from the flyby maneuver and the distance between the Lagrange points is smaller than the actual orbits. Further studies to calculate the exact energy spent for using the Interplanetary transport network and the time that it requires for an arbitrary orbit are required.

For future, this analysis can be extended to electric propulsion systems. So far we studied the orbit transfer strategies that utilize chemical propulsion. However, Low thrust transfer is an electric propulsion.

The thrusters in an electric propulsion transfer give a low thrust [1]. However, the fuel has a high specific impulse value. This low thrust in spite of a high specific is due to the high power requirement. Nevertheless, due to the high specific impulse value, the overall fuel requirement is greatly mitigated. Low thrust transfer is a slow transfer which requires less Delta-V. Nonetheless, it requires continuous thrust to keep it in the desired trajectory. It follows a spiral trajectory to reach the destination orbit. Since it is extremely expensive in terms of time it is generally used to transfer the spacecraft between the lower orbits.

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