Challenges in Application of Pulse Current Gas Metal Arc Welding Process for Preparation of Weld Joint with Superior Quality

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Abstract— Quality of weld joint in the form of its mechanical and metallurgical properties produced using gas metal arc welding process is governed by combination of parameters employed. These parameters also affect arc stability, bead appearance and thermal and metal transfer behaviour. For a given shielding gas, diameter and composition of filler wire, different modes of metal transfer that can be achieved are short circuit, globular, spray and pulsed spray. The spray metal transfer provides better ease of operation due to controlled mode of it but at relatively higher welding current above transition level. This may give rise to increase in heat input and temperature of weld pool along with wider weld isotherm which may adversely affect quality weld joint. This problem to some extent can be minimized by pulsing of welding current resulting in application of comparatively lower heat input and temperature of weld pool. But application of pulse current gas metal arc welding involves additional pulse parameters of pulse current Ip, base current, Ib, pulse on time, tp and pulse off time tb simultaneously interacting in nature. Together, there can be infinite combination of possible parameters at a given heat input. Therefore, the major challenges in application of pulse current gas metal arc welding process are to select an appropriate combination of pulse parameters that will be able to produce a weld joint with superior quality. Considering all these, detail study of methods of selection of pulse parameters in pulsed current gas metal arc welding has been discussed.

Keywords — Pulse current; Pulse current duration; Base current; Base current duration; Pulse frequency.

I. INTRODUCTION

The increasing population of the world is demanding higher use of structures such as ships, offshore structures, steel bridges, and pressure vessels [1]. Fusion welding is the most widely used manufacturing process employed in fabrication of these structures. Because of requirement by the customers of high quality involving joining and competition from global market, the selection of joining process has become critical. This aspect forces the organization to go towards the automation of the process involved in fabrication. There are several choices of the fusion welding processes, such as common conventional shielded metal arc welding, gas tungsten arc welding, submerged arc welding and gas metal arc welding [2, 14]. These welding processes influence severity of weld thermal cycle in different manner depending upon amount of weld deposition, welding parameters and shielding environment. However, SMAW suffers the

drawback of requirement of lower angle of attack by a skilled welder especially in case of relatively higher thickness with narrow gap and the process automation is comparatively critical. SMAW has also limitation of slag entrapment [3]. In case of GTAW process, the welding speed is considerably lower necessitating use of higher time in preparation of the weld joint. The submerged arc welding process can be successfully used for thick section welding but it requires rather higher heat inputs which may adversely affect the mechanical and metallurgical properties of the weld joint. These limitations can be overcome by application of GMAW process due to its ability to produce fast and continuous weld at any position even with lower heat input than submerged arc welding process.

In GMAW process a continuous consumable solid wire electrode is used along with an externally supplied inert shielding gas. The consumable wire electrode produces an arc with the work pieces to be joined, made as a part of the electric circuit and also provides filler metal to the weld joint. The externally supplied shielding gas plays double role in GMAW, first it protects the arc and the molten or hot, cooling weld metal from the atmospheric air. Second, it provides desired arc characteristics through its effect on ionization. A distinct advantage of GMAW is the mode of molten metal transfer. There are four possible modes in which the molten metal in the form of droplets can be transferred from electrode to the work pieces. These are short circuiting, globular, spray and pulse spray type metal transfer. In GMAW process, characteristics of metal transfer, wire melting rate and size of the weld pool primarily dictates the weld thermal cycle and bead geometry, depending upon various welding parameters. Out of several modes of metal transfer of GMAW process, the spray mode of metal transfer offers better ease of operation primarily due to dominating electromagnetic force resulting in projected transfer of the droplet. In GMAW process utilizing continuous current, depending upon filler wire material, filler wire size and shielding environment, spray transfer can be achieved only at significantly higher welding current above transition level, which increases heat input to the weld [4, 47]. This, increase in heat input consequently enhances temperature of the weld pool and adversely affects quality of the weld joint in the form of its mechanical and metallurgical properties through its influence on weld thermal cycle, weld isotherm and geometry of the weld bead. The adverse conditions generated in weld by using higher heat input in

GMA welding can be considerably controlled by modification of thermal and solidification behaviour of weld deposit by pulsing current in gas metal arc welding process popularly known as pulse current gas metal arc welding (P-GMAW) [5, 6]. Pulsing of the current in P-GMAW process gives rise to low work piece heating along with high arc stiffness and strong mechanism of metal transfer depending upon pulse parameters allowing improved fusion and penetration in groove wall. However, the application of the pulse current GMAW results into involvement of relatively more number of parameters. The additional parameters created because of pulsing the current in pulse current GMAW are pulse or peak current (I_n), base current (I_b), pulse current duration (t_n), base current duration (t_b) and pulse frequency (f) at a given mean current (I_m) of P-GMAW process [7, 8, 46]. Because of simultaneous interaction of pulse parameters, the selection of appropriate pulse parameters becomes major challenge before manufacturing engineer that produce a weld joint with desired quality to fulfill the given objective. At a given heat input also, there can be infinite combination of possible pulse parameters. But hardly any literature is readily available which discusses the selection of pulse parameters in case of pulse current gas metal arc welding. Therefore, the selection of pulse parameters in case of pulse current gas metal arc welding has been discussed in detail which may be useful to the engineers and industry in producing a weld joint with desired quality for the application.

II. ASPECTS OF PULSE CURRENT GMAW

The pulse current GMAW is associated with many aspects which are discussed in the following paragraph.

- 1. In P-GMAW process two heat sources of different nature act simultaneously on weld pool when metal transfer primarily takes place at the peak current (I_P). One is continuous heat source known as arc heat source of double ellipsoidal nature, which melts and produces an initial weld pool in the base metal. The other one is an interrupted heat source supplying superheated filler metal to the weld, considered as a point heat source dictating the size and geometry of weld pool over that initially developed by the arc heating.
- Pulse current GMAW uses pulsing of current to melt the filler wire and parameters can be selected and applied to allow one small molten droplet to fall with each pulse as well as multi drop per pulse making it possible to weld thin as well as thick work pieces.
- 3. The pulsing of current allow spray type metal transfer with low mean current, decreasing the overall heat input and thereby reducing the size of the weld pool and heat affected zone thus avoiding adverse effect of relatively higher heat input.
- 4. The pulsing of current also provides a stable arc and little or no spatter, since no short-circuiting takes place.
- 5. This process is suitable for almost all metals and positions through suitable control of pulse parameters at a given heat input.

 This process provides facility to transfer heat of superheated molten droplets at required location by control of the depth of heat transfer through proper selection of its pulse parameters.

III. PARAMETERS OF PULSE CURRENT GMAW

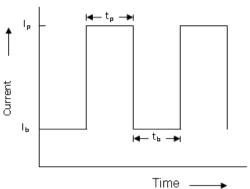


Fig.1 Welding current characteristics in pulsed current GMAW

In pulse current GMAW, the current is pulsed from high level to low level. Therefore, it gives rise to additional pulse parameters of (a) pulse current, I_p (b) base current, I_b (c) pulse current duration, t_p (d) base current duration, t_b and (e) pulse frequency, f.

- a) Pulse current (I_p): Higher level of current during pulsing as per welding current characteristics (Fig. 1). This current is kept above transition current for a given material, diameter of filler wire and shielding gas to take advantages of spray mode of metal transfer [9].
- b) Base current (I_b): Lower level of current during pulsing as per welding current characteristics (Fig. 1). This current is kept as low as possible provided that the arc should be maintained for a given material, diameter of filler wire and shielding gas.
- c) Pulse current duration (t_p): This is the time interval from beginning of current rise to the beginning of current fall during a particular pulse [10].
- d) Base current duration (t_b): This is duration of maintenance of lower current level.
- e) Pulse frequency (*f*): Pulse frequency is the number of pulse current per second and can be found by the inverse of the cycle time, *T* in seconds [11].

IV. CHALLENGES IN APPLICATION OF PULSE PARAMETERS FOR SUPERIOR QUALITY

The continuous current GMAW involves parameters of welding speed (S), welding current (I), arc voltage (V) and heat input (Ω). But the pulse current GMAW gives rise to involvement of additional parameters of I_p , I_b , t_p , t_b and f at a given mean current (I_m) and heat input (Ω) of P-GMAW process. These parameters are simultaneously interactive during deposition of weld bead. Therefore, the selection of pulse parameters is the major challenges to get a weld with superior quality. For efficient employment of pulsed current GMAW process to obtain weld joint with superior quality, understanding influence of pulse parameters on various aspects of weldment is very much required. Smooth spray

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type of metal transfer resulting in sound weld joint with better quality for the application can be achieved by careful selection of pulsed current GMAW parameters [12]. At the same time, selecting suitable combinations of pulse parameters for use with pulsed current GMAW may require a lot of trial and error with considerable loss of time, energy and money [13]. However, in actual practice, finding of optimum pulse parameters by trial and error to use for preparation of weld joint is very difficult because at a given heat input there are infinite number of possible pulse parameters. The complexity of selection of pulse parameters for a desired operation of P-GMAW process to some extent has been taken care by considering a summarized influence of pulse parameters, the effectiveness of which is amply justified in earlier works [15-19]. It is defined by a dimensionless

hypothetical factor $\phi = \left(\frac{I_b}{I_p}\right) \times ft_b$ derived on the basis of energy

balance concept [20] where, $t_b = \left(\frac{1}{f}\right) - t_p$.

It has been reported [21] that under identical conditions of the welding, transition current for P-GMAW is comparatively higher than that of GMAW. It may be due to formation of stationary pendant droplet at the tip of the electrode during base current which requires additional current for its deposition in order to overcome inertia of droplet. The transition current for 1.2 mm diameter steel electrode under the shielding of 80% Ar + 20% CO₂ was found to be 380A in P-GMAW as compared to 275A in GMAW process [21].

A. Mean current, Im

In P-GMAW process, mean current, I_m , for a square wave pulse can be obtained using expression [22, 23, 24] of

$$I_{m} = \frac{\left(I_{p} t_{p} + I_{b} t_{b}\right)}{\left(t_{p} + t_{b}\right)} \tag{1}$$

Where, t_b is base current duration and t_p is peak current duration, I_p and I_b are peak current and base current respectively. In this expression, there is involvement of four parameters which represents pulsed current characteristics [25, 44]. The average current has to be always below the transition current, as it will lower the heat input. The higher heat input to the weld adversely affects the properties of the weld as well as heat affected zone [26, 45].

It has been reported [21] that at a given wire feed rate (V_w) and droplet volume (V_d) , a stable parametric zone containing all feasible combinations of pulse parameters such as I_p , I_b , t_p and f can be obtained by evaluating the Eq. [1] under following constraints

$$I_{m} = m V_{w} + C \tag{2}$$

Where, m is the slope, and C is the intercept and V_w is wire feed rate (m/min). The restriction of required droplet volume, V_d , estimated by considering transfer of one drop per pulse (Number of droplets transferred per second equal to the frequency of pulse current) can be expressed as,

$$V_{d} = \frac{A_{w} \cdot V_{w}}{f} \tag{3}$$

Pulse frequency, f, may also be expressed as

$$f = \frac{1}{t_{pul}} \tag{4}$$

Where, $t_{pul} = t_b + t_p$ by rearrangement, Eq. [4] gives,

$$t_{pul} = \frac{V_d}{A_w.V_w} \tag{5}$$

Further, the condition of optimisation of drop detachment parameter has to be imposed to obtain smooth, reproducible spray transfer at all operating conditions. Based on the experimental observations, optimum condition for a given filler wire diameter and material ensuring transfer of one or multi drop per pulse can be achieved over a range of I_p and t_p combinations defined as,

$$I_p^2 t_p = D_n \tag{6}$$

Where, D_n is detachment parameter denoting number of drops detached per pulse as D_1 , D_2 , and D_3 represents 1, 2 or 3 drops per pulse duration respectively.

Condition for minimum base current for stable arc, below which are extinguished, can be given as, $I_b \ge \beta$ (7)

Where, β is minimum base current limit for stable arc. The range of possible combinations of pulse parameters is quite restricted at low wire feed rate but it extends with the increase in it [22]. At a specific arc length an approximately linear relationship exists between the amplitude of peak and base current for a given combination of wire feed rate, V_w , pulse frequency, f, and peak current duration, t_p , [22].

$$I_{p} \propto I_{b}$$
 (8)

At a given peak current duration and peak current in excess over the base current, I_e, both the base current and pulse frequency vary in direct proportion to the wire feed rate,

$$I_{\rm b}$$
 and $f \propto V_{\rm w}$ (9)

$$I_e$$
 can be expressed as $I_e = I_p - I_b$ (10)

At a given pulse frequency and I_e, both the base current and peak current duration vary directly with the wire feed rate,

$$I_{\rm b}$$
 and $t_{\rm p} \propto V_{\rm w}$ (11)

It is clear from Eqs. [9] and [10] that at a given I_e , both the base current and either pulse frequency or peak current duration vary directly with wire feed rate. This relationship has been found valid for the entire operating range of wire feed rate from 0.5 m/min to about 10 m/min with high degree of arc stability and stable mode of metal transfer for mild steel welding using 1.2 mm diameter filler wire under shielding of Ar + 5% CO_2 .

Eqn. [1] can also be expressed as,

$$I_{m} = I_{b} + I_{e} t_{p} f \tag{12}$$

By combining Eqns. [2] & [12],

$$m.V_w + C = I_b + I_e t_p f$$
 (13)

Eqn. [13] implies that for a synergic operation both the base current and product of peak current in excess over the base current, peak current duration and pulse frequency must vary directly with the wire feed rate [23].

B. Pulse current, Ip

In pulse current GMAW the peak current, Ip is generally kept above transition current for the given combination of electrode material, diameter and shielding gas to take advantages of spray mode of metal transfer. The transition current for some commonly used material for fabrication has been shown in Table-1. Higher peak currents will result in greater axial force due to magnetic field generated by the current carried through the electrode and if numerous drops of relatively smaller diameter develop, the arc forces may cause them to be fragmented or propelled laterally [27, 28, 29]. Hence, to ensure appropriate amplitude and duration of peak current, power law relations (I_pⁿ t_p = constant, n=2) have been used for determining the amplitude of peak current and duration [28, 29, 30]. Assuming all other parameters to remain constant the equation $I_p^n t_p = constant$, may be used to obtain the range of pulse amplitudes and durations to detach a single droplet per pulse. Minimum pulse amplitude has also been found out by using following expression for mild steel filler wire [31]

$$I_{p.\min} = \frac{2.85 \times 10^4 \,\sigma^{0.5} d_e f^{0.245}}{I_{eff}^{0.465} t_p^{0.6}} \tag{14}$$

Here σ is the surface coefficient, in N/m, d_e is electrode diameter, in mm, f is pulse frequency, in pulses/sec, I_{eff} is effective current given as $I_{eff}^2 = \left\{k_p I_p^2 + (1-k_p) I_b^2\right\}^{1/2}$ wherein k_p is pulse duty cycle defined as t_p/t_{pul} , in A, t_p is pulse time, in sec.

Table-1: Transition current for different materials

Wire Dia	Transition Current			
(mm)	Aluminium	Mild steel	Stainless Steel	
1.0	110	120	180	
1.2	140	200	230	
1.6	190	260	285	

C. Pulse current duration, t_p

The pulse duration is an important operating parameter as neck formation and elongation of the pendent drop occurs mostly during this period. If the pulse duration is kept too short, the elongated drop would recoil back to a spherical shape after the pulse and if the duration is too long, several drops might get detached during the period. Hence peak duration should be selected appropriately to detach one

molten drop per pulse [29, 32, 30] or multiple droplet detachment per pulse [33, 34] depending upon the situation whether the process is used for preparation of weld joint of thin or thick section. The process of one drop transfer per pulse with relatively low rate of metal transfer is popularly used in joining of thin section, whereas the multiple droplet transfer per pulse with proper control of arc characteristics and the behaviour of metal transfer resulting in high deposition rate finds wide spread application of P-GMAW process in weld fabrication of different materials of varied section size.

Subramaniam et al. [32] have used experiments to characterize the pulse current and pulse current duration to achieve desirable metal transfer for aluminum. An equation as a function of pulse current and its duration, base current and its duration was developed.

$$T_{p} = \left[\left(496.1 \left(1 - e^{-0.002 \, I_{b} T_{b}} \right) \right) + \frac{270.1}{\left(I_{b} T_{b} - 188.2 \right)^{2}} \right] / I_{p}$$
 (15)

Where, T_p and T_b are pulse current and background current durations respectively. This equation gives minimum pulse current duration required for detachment of droplets.

D. Base current, Ib

The base current is required to maintain the welding arc between the pulses [25, 27, 35,]. By the use of appropriate base current level, control of the weld pool and weld bead shape can be greatly increased. To minimize the heat input, the base current is often kept at minimum level, which may result in a high crowned weld bead with poor side wall fusion. It has been observed that drop detachment is little influenced by wire melting phenomenon during base current duration. The dynamic effects developed during peak current duration are responsible for metal transfer. No metal transfer will take place during maintenance of base current. Allum and Quintino [28], had chosen the background current on the basis of a 50Hz per 100A. Background current levels can vary significantly, but this will be of the order of 30 to 50A for mild steel [36], 50A for austenitic stainless steel [37] and 20A for aluminum alloys [38].

E. Base current duration, t_b

Volume of the droplet detached from the filler wire is strongly dependent on base current level and its duration. However, an appropriate value of peak current and its duration set a condition for a certain form of drop detachment [27]. During variation in the mean current with the change of pulse frequency at a given peak current and its duration, the droplet volume may be fixed by maintaining constant electrode heating at base current duration. It can be made constant with the enhancement of base current as its duration decreases with the increase of pulse frequency [27]. At a given peak current and its duration, increase of base current duration increases the detachment time of a droplet, whereas magnitude of base current has only a small effect on detachment time. Effect of base current duration on drop detachment time is more at low peak current level therefore, by employing the higher value of peak current influence of base current conditions on droplet detachment may be reduced [27]. During aluminium welding using 1.2 mm filler wire diameter and argon shielding, it is found that the detachment of one drop during base current can be achieved successfully with smooth weld metal transfer and excellent bead appearance [39].

F. Pulse frequency

The pulse frequency is having significant effect on both the minimum base current and minimum electrode melting rate [28, 40, 32] to obtain a constant arc length. At a given base current the increase in frequency of sinusoidal pulse enhances the mean and effective currents and electrode melting rate. However in case of variation in pulse frequency by keeping the mean current constant the base current should remain free to vary by following the energy balance criteria to achieve constant arc length. It is in general reported that [39] at a low pulse frequency, the metal transfers as large drops or lumps where the weld pool appears viscous and the arc becomes erratic. Whereas, with the increase in pulse frequency, the metal transfer takes place as small axial droplets with size corresponding to the balance between detaching forces at the peak current and retaining surface tension force. Hence, the appropriate frequency, which is primarily a function of mean current, can be pre-selected at a given condition. Theoretical frequency as a function of electrode melting rate with current pulsing, m_{pulse} , the volume of the drop at the peak current $V_{\text{drop}}(I_p)$ and density of the drop ρ_d [41] can be found out

Theoretical frequency =
$$\frac{m_{pulse}}{V_{drop}I_{p}\rho_{d}}$$
 (16)

The average melting rate considering square wave current may be estimated as the weighted sum of the melting rate at the I_p and I_b , using expression of

$$m_{vulse} = D.m.I_{p} + (1-D).m.I_{b}$$
 (17)

Where, D is the load duty cycle, m (I_p) the melting rate at peak current and m (I_b) is the melting rate at base current.

G. Ratio of peak current to base current (I_p/I_b)

During continuous current GMA welding, the increase in welding current reduces the duration of presence of molten droplet in arc cavern. Thus, it reduces the absorption of gases and consequently the porosity content of weld deposit. Whereas in case of pulsed current welding, the variation in arc pressure, which is primarily governed by the ratio of peak current to base current, leads to formation of vortex in inert jacket resulting air aspiration into shielding atmosphere causing enhancement of porosity content in weld metal. However it is also observed that a combination of highest I_p and the lowest t_p provides more uniform arc length (stable arc) and droplet detachment when compared to the combination of the lowest Ip and the highest tp. Thus the ratio of peak current to base current (Ip/ Ib) should preferably be kept less than 10 in case of pulse current MIG welding [33] by suitable adjustment of pulse frequency, pulse duration and wire feed speed to avoid difficulties such as rotating arc, heavy tapering of the liquid tip and disturbance in the shielding caused by the fluctuation in arc pressure. Earlier investigation [42, 43] carried out on single pass pulsed

current MIG of Al-Zn-Mg alloy have shown the influence of the ratio of I_p/I_b on the porosity content of weld deposit and suggested that the I_p/I_b ratio should be maintained in the range of 7.5 at a mean current of 150 A and around 4 at a mean current of 220 A to achieve a significant reduction in porosity content of the weld deposit. It has also been observed that by keeping the mean current in P-GMAW process similar to the welding current of continuous current deposition, the porosity content of weld metal becomes comparatively lower than that observed in case of later one.

H. Feed rate

The value of current to be applied for weld metal deposition is governed by wire feed rate in case of GMAW [48, 49]. Low wire feed rate causes melt back and a high feed rate can result in extinguishing the arc through short-circuiting [50]. The wire feed rate is determined by the size of droplet to be transferred and the transfer frequency, i.e. at low feed rates, the frequency should be low [25]. Therefore, for successful arc operation of GMAW process, the burn off rate must be equal to the wire feed rate to maintain constant arc length. Therefore, the following energy balance per unit time has been proposed [24, 40].

$$A_{w}V_{w(cc)}\rho_{w}Q_{m} = \left[V_{a} + \xi + \frac{3kT}{2e}\right]I + \frac{R_{0}E_{w}I^{2}}{A_{w}}$$
(18)

Where A_w is the cross sectional area of the filler wire (m²), V_w is the wire feed rate (m/s²) for continuous current welding, ρ_w is the density (kg/m³) of the filler wire, Q_m is heat per unit mass (J/kg) required for melting the filler wire, V_a is anode fall voltage (V), ξ is work function of metal

surface (V),
$$\frac{3kT}{2e}$$
 is thermal energy of electrons (V), R_0 is

resistivity (Ω m) of the filler wire, E_W is electrode extension (m) and I is welding current (A). Eq.18 may be rewritten as,

$$V_{w(cc)} = \frac{\psi I}{A_w V_{w(cc)} Q_m} + \frac{R_0 E_w I^2}{A_w^2 \rho_w Q_m}$$
 (19)

Where,
$$\psi = \left[V_a + \xi + \frac{3kT}{2e} \right]$$
 is the equivalent melting

potential at anode (Work function + anode voltage fall + thermal energy of electrons).

Eq. 19 may also be expressed as,

$$V_{w(cc)} = AI + BE_w I^2 \tag{20}$$

Where, A and B are constants representing wire melting due to arc heating and resistive heating respectively expressed as

$$A = \frac{\psi I}{A_{w}V_{w}Q_{m}} \tag{21}$$

$$B = \frac{R_0 E_w I^2}{A_w^2 \rho_w Q_m}$$
 (22)

The reported values of physical constants for different materials of the filler wire [51, 52, 53] are given in Table-2.

The wire burn off rate/melting rate for P-GMAW is expressed [54, 40] as

$$V_{w(pc)} = \int_{0}^{t_{pul}} V_{w} t_{pul} dt_{pul}$$
 (23)

Table-2: Physical properties of different materials of filler wire [51, 52, 53]

Property	Mild steel	Stainless steel
Melting Point, T _m , (K)	1750	1728
Specific heat, C _{p(s)} , (J/kg/K)	686	500
Specific heat, C _{p(l)} , (J/kg/K)	855	760
Latent heat of fusion, L, (J/kg)	2.76e ⁵	2.84e ⁵
Density of the solid metal, ρ_w , (kg/m^3)	7870	7750
Density of the molten metal, ρ_w , (kg/m^3)	6500	7507
Coefficient of surface tension, γ , (N/m)	1.03	1.35
Resistivity at melting point, R_0 , (Ωm)	8.2E ⁻⁷	1.3E ⁻⁶
Emissivity of molten droplet, ε	025	0.25

For a square pulsed current waveform it gives

$$V_{w(pc)} = (V_{wp}t_p + V_{wb}t_b)t_{pul}^{-1}$$
(24)

Where, $V_{w(pc)}$ is wire burn off rate and V_{wp} and V_{wb} are wire burn off rates during t_p and t_b respectively.

The V_{wp} and V_{wb} may be expressed as

$$\begin{split} V_{wp} &= A.I_p + B.E_w.I_p^2 \\ V_{wb} &= A.I_b + B.E_w.I_b^2 \end{split} \tag{25}$$

(26)

Therefore, Eq. 24 may be rewritten as

$$V_{w(pc)} = \left[\frac{(A.I_{p} + B.E_{w}.I_{p}^{2})t_{p} + (A.I_{b} + B.E_{w}.I_{b}^{2})t_{b})}{t_{pul}} \right]$$
(27)

Eq. 27 may be rewritten as

$$V_{w(pc)} = \left[\frac{A(I_{p}t_{p} + I_{b}t_{b}) + BE_{w}(I_{p}^{2}t_{p} + I_{b}^{2}t_{b})}{t_{pul}} \right]$$
(28)

Using $I_{m} = [(I_{p} t_{n} + I_{h} t_{h})/t]$ in eq. 28

$$V_{w(pc)} = AI_m + BE_w (I_p^2 t_p + I_b^2 t_b) t_{pul}^{-1}$$
(29)

As $I_p^2 t_p \gg I_b^2 t_b$ neglecting ohmic heating during base current

period, Eq. 29 reduces to

$$V_{w(pc)} = AI_m + BE_w I_p^2 t_p t_{pul}^{-1}$$
(30)

Considering $f = 1/t_{pul}$ and $I_p{}^2t_p = D_n$ Eq. 30 gives

$$V_{w(pc)} = AI_m + BE_w D_n f (31)$$

By considering $V_d=A_wV_w/f$ in Eq. 31 a linear relationship has been obtained as follows,

$$\frac{f}{I_m} = \frac{AA_w}{V_d - BDA_w E_w} \tag{32}$$

Solving eq. 32 for droplet volume V_d,

$$V_d = AA_w \left(\frac{I_m}{f}\right) + A_w E_w BD \tag{33}$$

Eq. 33 shows that for any wire feed rate, by fixing (I_m/f) , the droplet size can be held constant for a given wire diameter, electrode extension and detachment parameter.

It has been observed that for 1.2 mm diameter steel filler wire [28] for a desired combination of peak current and its duration, giving one drop per pulse, the base current and its duration may be evaluated using frequency value of 50Hz for 100A mean current. It gives satisfactory droplet transfer as well as volume of droplet remains insensitive to $I_{\rm m}.$ It can be expressed as

$$I_{\rm m}/f = 2 \tag{34}$$

By combining Eqs. 31 and 34

$$V_{w(pc)} = AI_m + \frac{BE_wDI_m}{2} = I_m \left(A + \frac{BE_wD}{2} \right) = \overline{A}I_m$$
 (35)

Where,
$$\overline{A} = \left(A + \frac{BE_wD}{2}\right)$$
 is modified burn off factor

In P-GMAW process, by considering ohmic heating during base current duration, Eq. 33 reduces to

$$V_{w(pc)} = AI_m + BE_w I_{eff}^2$$
(36)

Where, $I_{eff}^2 = \left\{k_p I_p^2 + (1-k_p) I_b^2\right\}^{1/2}$ and k_p is pulse duty cycle defined as t_p/t_{pul}

I_{eff} may also be expressed as,

$$I_{eff}^{2} = I_{m}^{2} + k_{n}(1 - k_{n})I_{e}^{2}$$
(37)

By combining Eqs. 36 and 37,

$$V_{w(pc)} = AI_m + BE_w (I_m^2 + k_p (1 - k_p) I_e^2)$$
(38)

In consideration of Eq. 20, Eq. 38 may be expressed as

$$V_{w(pc)} = V_{w(cc)equiv.} + BE_{w}k_{p}(1 - k_{p})I_{e}^{2}$$
(39)

Eq. 39 reveals the following aspects of P-GMAW [Alum 1983].

The burn off rate of pulsed current welding is higher than continuous current welding under the same equivalent welding current. Pulsed structure influences burn off rate for a given mean current and maximum burn off rate can be achieved when $k_p = 1/2$ i.e., equal peak and base current time and at largest peak current in excess over the base current.

I. Shielding gas

The role of shielding gas in arc welding process is to protect the electrode and the work piece from harmful atmospheric contaminants and act as a medium in which current can flow to sustain the arc [55]. In welding area where molten droplets are transferred across the arc into the weld pool, protection from atmospheric contaminants can be provided successfully by suitable shielding gas or gases [56, 57]. To provide protection from atmospheric contamination the main gases used are argon, helium, carbon dioxide and combination of these in different proportions. In addition to these a small amount of oxygen, nitrogen and hydrogen may be added to take advantages of benefits gained from these additions. The argon and helium are inert gases whereas the carbon dioxide is active gas. Therefore, the argon and helium are relatively more widely used for most of the applications. The selection of a gas or a mixture of gases is also primarily guided by the physical and chemical properties of the gas, the operating characteristics that each gas imparts to a particular process and the kind of metal or alloy that the gas is suppose to protect. The basic properties of a shielding gas that governs its right selection to improve weld quality at reduced overall cost of the welding operation are listed in Table 3. Argon and carbon dioxide due to their relatively higher gas density than air (Table-3) requires lower flow rates in use than do the lighter gases as they can easily displace air from the electrode region to ensure adequate protection of the weld puddle [1]. Whereas gases such as hydrogen and helium which are 7 and 14 times less dense than air are prone to turbulent flow at the exit from the blow pipe nozzle due to thermal buoyancy [56]. Ionization potential [1] is the amount of energy required to remove an electron from a gas atom and make it an ion or an electrically charged gas atom. The importance of ionization potential of a gas in welding process is from the welding arc, arc power and energy distribution point of view [57].

Energy distribution in axial and radial direction in an arc is also affected by the thermal conductivity of a gas [1] which varies with temperature [56]. But for welding, the radial energy distribution is much more important because the radial energy distribution affects the axial one. Energy flow in radial direction has lower path length than the axial one i.e. path of least resistance, hence gas with higher thermal conductivity will have energy distribution across the arc in radial direction resulting in comparatively lower energy availability in axial direction [1, 56]. Inert gas helium which has higher thermal conductivity than argon will produce lanticular shape of penetration in comparison with wine glass shaped penetration in steel material. The thermal conductivity of multi atom gases such as carbon dioxide, hydrogen and oxygen in the temperature range between 3000 and 4500 K is much higher than that of argon and helium [58]. When heated to high temperatures within the arc plasma, these gases break down or dissociate into their component atoms which get partially ionized, producing free electrons and current flow. As the dissociated gas comes in contact with relatively cool work surface, the atoms recombine and release heat at that point. This heat of recombination causes multi atomic gases to behave as if they have a higher thermal conductivity [1].

Table 3: Basic physical and chemical characteristics of the gases.

Gas	Atomic weight (Kg/ kmol)	Rel gas density at 273K and 1.013 bar	Ion potential (eV)	Reaction in arc
Argon (Ar)	39.948	1.380	15.7	Inert
Helium (He)	4.002	0.138	24.5	Inert
Carbon Dioxide (CO ₂)	44.011	1.529	14.4	Oxidizing
Oxygen (O ₂)	31.998	1.105	13.2	Oxidizing
Nitrogen (N ₂)	28.013	0.968	14.5	Reactive
Hydrogen (H ₂)	2.016	0.070	13.5	Reducing

By far the gas blends developed can be roughly divided into three categories: pure gases, two gas blends and three part gas blends composed of argon, helium, oxygen, carbon dioxide, or hydrogen [1]. However there are number of other factors which influence the desirability of a gas for arc shielding. Some of these are the influence of the shielding gas on the arcing and metal transfer characteristics during welding, weld penetration, width of fusion and surface shape patterns, speed of welding and undercut tendency. Hence selection of pure or combination of gases will depend upon the function it has to perform in the desired application.

The operation of pulsed current GMAW is limited by the shielding gases used. Spray mode of metal transfer can be obtained when argon gas is used as shielding gas. Therefore, this is used as base gas for making various gas mixtures which may be employed for pulsed current GMAW [36]. For example, argon and argon-oxygen or argon-carbon dioxide mixtures with low levels of active gas (for CO2 up to about 18%) can be used [59]. Commercial argon with small addition of oxygen is largely used for welding of aluminum, steel and stainless steel. The argon and oxygen gas mixture produces a constrictive arc, which makes it ideal for stabilizing the spray mode of metal transfer. In general, additions of CO₂ are preferred over O₂ as the arc is less constricted and the resulting weld bead has a better profile. For thick sections, higher CO₂ can be employed reduce the problem of lack of sidewall fusion, but this results in increase of spatter. For this argon 15% CO₂, 2%O₂ has given better results. Ninety nine percent pure argon,1%O2 mixture is mainly used with the high alloy ferrous materials.

Scotti has studied modes of metal transfer for stainless steel GMAW with Ar and Ar and O_2 mixtures. It is observed that increase in the oxygen content in the shielding gas reduces the transition current and droplet size. It has also seen that addition of 30% helium gives rise to improvement of fluidity and arc stability and allows higher welding speed reducing time of preparation of weld joint [60].

J. Weld isotherm

The weld isotherm dictates the size and geometry of the weld pool governing the superiority of weld joint. The weld isotherm depending on pulse parameters of pulse current gas metal arc welding may be estimated by superimposing the analytical solution of the distributed heat source of the arc on that of the point heat source of transferred superheated droplets of molten filler metal [6]. The expressions for the temperature, T, at any point $(x(\xi), y, z)$ in the weld fusion zone at a distance R with respect to central axis of the welding arc using combined heat source technique can be expressed [6, 61, 62]as

$$T = \frac{Q_{\rm f}}{2.\pi.k} e^{-\lambda.v.\xi} \cdot \left[\frac{e^{-\lambda.v.R}}{R} + \sum_{n=1}^{n=\infty} \left(\frac{e^{-\lambda.v.R_n}}{R_n} + \frac{e^{-\lambda.v.R_n'}}{R_n'} \right) \right] + T_d (40)$$

Where R, R_n and R'_n are
$$R = \sqrt{\xi^2 + y^2 + (z - h)^2}$$
 (41)

$$R_{n} = \sqrt{(2.n.d - (z - h))^{2} + \xi^{2} + y^{2}}$$
(41)

$$R'_{n} = \sqrt{(2.n.d + (z - h))^{2} + \xi^{2} + y^{2}}$$
 (43)

This expression is used for plotting the weld isotherm of any given temperature. Q_f is the heat transferred to the weld pool by the droplets of super heated filler metal and T_d is the temperature of the point $(x(\xi), y, z)$ due to arc heating using the distributed heat source expressed as follows [6, 61, 62]:

$$T_{d} = \frac{3\sqrt{3} \cdot Q_{AW}}{\rho \cdot c \cdot \pi \sqrt{\pi}} \int_{0}^{t} \left[\frac{\frac{dt}{\sqrt{\left(12a(t-t') + a_{h}^{2}\right)} \cdot \sqrt{\left(12a(t-t') + b_{h}^{2}\right)}}}{\frac{A'}{\sqrt{\left(12a(t-t') + c_{hf}^{2}\right)}} + \frac{B'}{\sqrt{\left(12a(t-t') + c_{hb}^{2}\right)}}} \right] + T_{0}$$

$$(44)$$

Where, Q_{AW} is the arc heat transferred to the weld pool.

V. CONCLUSIONS

The observations on various aspects of present study may be primarily concluded as follows.

- The pulse parameters of pulse current gas metal arc welding affect the quality of the weld joint prepared to a great extent. Among various pulse parameters, the peak current plays a dominant role in governing the quality in the form of properties of weld joint.
- The complexity of selection of pulse parameters for a desired operation of P-GMAW process to some extent can be solved by considering a summarized influence of pulse parameters of φ the expression of which has been found based on energy balance of the system.
- 3. It is possible to get both one drop per pulse or multi drop per pulse of metal transfer primarily taking place during pulse current duration, thereby allowing the effective use of pulse current GMAW in preparation of weld joint of thin as well as thick sections.
- 4. The objective of the selection of pulse parameters may be obtaining effective bead geometry, melting rate, stability of the arc or application of proper weld isotherm. By application of proper weld isotherm desired properties of the weld joint can be achieved.
- 5. The melting rate of electrode in pulsed current welding is higher than continuous current welding under the same equivalent welding current. This may result in requirement of application of relatively less time in preparation of weld joint.
- Through proper selection of pulse parameters the heat of super heated metal droplets can be concentrated to the desired location in the weld pool.

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