Cascade System Reliability with Stress and Strength Follow Lindley Distribution

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***Abstract*:- The cascade reliability model is a special type of redundancy of stress-strength model. The n-cascade system is a hierarchical standby redundancy system, where the standby component taking the place of the failed component for the decreased value of stress and independently distributed strength and it is the cold standby system. Lindley distribution belongs to an exponential family and it can be written as a mixture of an exponential and a gamma distribution with shape parameter 2. In this paper, the general expression for the reliability of $n-$cascade system was derived when stress and strength follow lindley distribution and the numerical values $R(1), R(2), R(3)$ *and* R_3 have been computed for some specific values of the parameters.**

Key words: Lindley distribution, $n-$cascade system, standby redundancy, stress – strength model, Reliability.

INTRODUCTION:

If X denotes the strength of the component and Y is the stress imposed on it, then the reliability of the component is given by [1],

$$R = P(X > Y)$$
$$= \int_{-\infty}^{\infty} \left\{ \int_{0}^{\infty} g(y)dy \right\} f(x)dx$$
(or)
$$= \int_{-\infty}^{\infty} \left\{ \int_{y}^{\infty} f(x)dx \right\} g(y)dy$$

The $n-$cascade system is defined as a special type of standby system with n components by Sriwastav et al [2]. The cascade redundancy is defined as a hierarchical standby redundancy where a standby component takes the place of a failed component with a changed stress. This changed stress is k_i times the preceeding stress. k_i is the attenuation factor of i^{th} stress y_i. Lindley introduced lindley distribution in the year1958. Lindley distribution belongs to an exponential family and it can be written as a mixture of an exponential and a gamma distribution with shape parameter 2.

Sriwastav and Pandit[2] derived expressions for the reliability of $n-$cascade system when stress and strength follow exponential distribution. They computed reliability values for a 2-cascade system with gamma and normal stress and strength distributions. Raghavachar et al [3] studied the reliability of a cascade system with normal stress and strength distribution. Uma Maheswari et al [4] studied the reliability of single strength under $n-$stresses with stress and strength follow exponential, normal and gamma distributions. They concluded that when $n-$ stresses acted on a single strength component with stress and strength follow exponential distribution, then the reliability of the system is same as the series system. Uma Maheswari et al [5] studied the reliability comparison of $n-$cascade system with the addition of an $n-$strengths system when stress and strength follow exponential distribution. Uma Maheswari et al [6] studied the reliability of a cascade system with normal stress and exponential strength. Uma Maheswari et al [7] studied the reliability of single stress under $n-$ strengths with stress and strength follow exponential, gamma and normal distributions. They concluded that when $n-$ strengths acted on a single stress component with stress and strength follow exponential distribution, then the reliability of the system is same as the parallel system. Tirumala Devi et al [8] studied the reliability of a system with m stresses and n strengths. Chumchum et al [9] studied the cascade system with $\Pr(X < Y < Z)$. Ghitany et al [10] studied the properties and applications of lindley distribution.

STATISTICAL MODEL:

Let $X_1, X_2, X_3, \ldots, X_n$ be the strengths of the components $C_1, C_2, C_3, \ldots, C_n$ as arranged in order of activation respectively. All the $X_i's$ are independent and non-identically distributed random variables with probability density

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 09, September-2016

functions$f_i(X_i)$; $i = 1,2, \ldots, n$. Also let Y_1 be the stress on the first component which is also randomly distributed with the density function $g(Y_1)$.

If $Y_1 < X_1$, the first component which is also randomly distributed stress varies with the density function $g(Y_1)$. If $Y_1 < X_1$, the first component C_1 works and hence the system survives. $Y_1 \geq X_1$ leads to the failure of C_1; thus the second component in line viz., C_2, takes its place and has a strength X_2. However, the stress Y_2 on C_2 will normally be different from Y_1. Let $Y_2 = K_2^* Y_1$, where K_2^* is the cumulative attenuation factor on the second component and $K_2^* = K_1 K_2$ where by definition $K_1 = 1$.Although the system has suffered the loss of one component, it survives if $Y_2 < X_2$ and so on.

In general, if the $(i-1)^{th}$ component C_{i-1} fails then the i^{th} component C_i, with the strength X_i, gets activated and will be subjected to stress.

$$Y_i = K_i Y_{i-1} = K_i^* Y_1 \qquad (1)$$

$$where \qquad K_i^* = K_1 K_2 \ldots \ldots K_i \qquad (2)$$

represents the cumulative attenuation factor on the i^{th} component C_i.

The system could survive with a loss of the first $(n-1)$ components if and only if $X_i \leq Y_i$; $i = 1,2,3, \ldots \ldots, n-1$ and $X_n > Y_n$. The system totally fails if all the components fail when $X_i \leq Y_i$; $i = 1,2, \ldots \ldots n$.

The probability $R(n)$ of the system to survive with the first $(n-1)$ components failed and the n^{th} component active is

$$R(n) = P\left[\left\{\bigcap_{i=1}^{n-1}(X_i \leq Y_i)\right\} \cap (X_n > Y_n)\right] \qquad (3)$$

$R(2), R(3), \ldots \ldots \ldots, R(n)$ are increments in reliability due to the addition of the $2^{nd}, 3^{rd}, \ldots \ldots, n^{th}$ components respectively.
Then

$$R(n) = P[X_1 \leq K_1^* Y_1, X_2 \leq K_2^* Y_1, \ldots \ldots \ldots, X_{n-2} \leq K_{n-2}^* Y_1, X_{n-1} \leq K_{n-1}^* Y_1, X_n > K_n^* Y_1] \qquad (4)$$

we can obviously associate the n^{th} component attenuation factor with Y_1.

Since the stress is attenuated, the subsequent stresses $Y_2, Y_3 \ldots \ldots$ are described in terms of Y_1. Hence it is necessary to specify the distribution of Y_1. Let $g(Y_1)$ and $f_i(X_i)$ be the probability density function of Y_1 and X_i $(i = 1,2, \ldots, n)$ respectively.

The equation (4) can now be written as

$$R(n) = \int_0^\infty \left[\int_0^{K_1^* y_1} f_1(x_1)dx_1 \times \int_0^{K_2^* y_1} f_2(x_2)dx_2 \times \ldots \ldots \int_0^{K_{n-1}^* y_1} f_{n-1}(x_{n-1})dx_{n-1} \times \int_{K_n^* y_1}^\infty f_n(x_n)dx_n\right] g(y_1)dy_1 \qquad (5)$$

(or)

$$= \int_0^\infty [F_1(K_1^* y_1) F_2(K_2^* y_1) \ldots \ldots \ldots F_{n-1}(K_{n-1}^* y_1) \bar{F}_n(K_n^* y_1)] g(y_1)dy_1 \qquad (6)$$

$$where \quad F_i(K_i^* y_1) = \int_0^{K_i^* y_1} f_i(x_i)dx_i \qquad and$$

$$\bar{F}_i(K_i^* y_1) = 1 - F_i(K_i^* y_1) \qquad (7)$$

Then the system reliability of n −cascade model is

$$R_n = \sum_{i=1}^{n} R(i)$$

Reliability computations:

Let X be the strength and Y be the stress of a system p.d.f’s and c.d.f’s are

$$f(x) = \frac{\lambda^2}{1+\lambda}(1+x)e^{-\lambda x} \quad , \lambda, x > 0$$

$$g(y) = \frac{\mu^2}{1+\mu}(1+y)e^{-\mu y} \quad , \mu, y > 0$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 09, September-2016

$$F(x) = 1 - e^{-\lambda x}\left(1 + \frac{\lambda x}{\lambda + 1}\right), \lambda, x > 0$$

$$G(y) = 1 - e^{-\mu y}\left(1 + \frac{\mu y}{\mu + 1}\right), \mu, y > 0$$

Reliability for stress – strength models:

$$R = P(X > Y)$$

$$= \int_0^\infty F(y) g(y) dy$$

$$= \int_0^\infty \left(1 - e^{-\lambda y}\left(1 + \frac{\lambda y}{\lambda + 1}\right)\right)\frac{\mu^2}{1+\mu}(1+y)e^{-\mu y}dy$$

$$= \int_0^\infty \frac{\mu^2}{1+\mu}(1+y)e^{-\mu y}dy - \int_0^\infty \frac{\mu^2}{1+\mu}\left(1 + \frac{\lambda y}{\lambda+1}\right)(1+y)e^{-(\mu+\lambda)y}dy$$

$$= 1 - \frac{\mu^2}{1+\mu}\left[\frac{1}{\mu+\lambda} + \frac{1}{(\mu+\lambda)^2} + \frac{\lambda}{(1+\lambda)(\mu+\lambda)^2} + \frac{2\lambda}{(1+\lambda)(\mu+\lambda)^3}\right] \quad (8)$$

Reliability for cascade model:

Marginal reliability of the i^{th} component for $i = 1, 2, 3$ is

$$R(1) = 1 - \frac{\mu^2}{1+\mu}\left[\frac{1}{\mu+\lambda} + \frac{1}{(\mu+\lambda)^2} + \frac{\lambda}{(1+\lambda)(\mu+\lambda)^2} + \frac{2\lambda}{(1+\lambda)(\mu+\lambda)^3}\right] \quad (9)$$

$$R(2) = \int_0^\infty F_1(y_1)\{1 - F_2(k_2^* y_1)\} g(y_1) dy_1$$

$$= \int_0^\infty \left[\begin{matrix}\left[1 - e^{-\lambda_1 y_1}\left(1 + \frac{\lambda_1 y_1}{\lambda_1 + 1}\right)\right]\left\{1 - \left[1 - e^{-k_2^*\lambda_2 y_1}\left(1 + \frac{k_2^*\lambda_2 y_1}{\lambda_2 + 1}\right)\right]\right\} X \\ \frac{\mu_1^2}{1+\mu_1}(1+y_1)e^{-\mu_1 y_1}\end{matrix}\right] dy_1$$

$$\begin{aligned}= \frac{\mu_1^2}{1+\mu_1}\int_0^\infty \Bigg[&\left(1 + y_1 + \frac{k_2^*\lambda_2 y_1}{\lambda_2+1} + \frac{k_2^*\lambda_2 y_1^2}{\lambda_2+1}\right)e^{-(k_2^*\lambda_2+\mu_1)y_1} \\ &+ \left(-e^{-\lambda_1 y_1} - y_1 e^{-\lambda_1 y_1} - \frac{k_2^*\lambda_2}{\lambda_2+1}y_1 e^{-\lambda_1 y_1} - \frac{k_2^*\lambda_2}{\lambda_2+1}y_1^2 e^{-\lambda_1 y_1}\right)e^{-(k_2^*\lambda_2+\mu_1)y_1} \\ &+ \left(\frac{-\lambda_1}{\lambda_1+1}y_1 e^{-\lambda_1 y_1} - \frac{\lambda_1}{\lambda_1+1}y_1^2 e^{-\lambda_1 y_1} - \frac{\lambda_1}{\lambda_1+1}\frac{k_2^*\lambda_2}{\lambda_2+1}y_1^2 e^{-\lambda_1 y_1} \right. \\ &\left. - \frac{\lambda_1}{\lambda_1+1}\frac{k_2^*\lambda_2}{\lambda_2+1}y_1^3 e^{-\lambda_1 y_1}\right)e^{-(k_2^*\lambda_2+\mu_1)y_1}\Bigg] dy_1\end{aligned}$$

$$\begin{aligned}= \frac{\mu_1^2}{1+\mu_1}\Bigg[&\frac{1}{(k_2^*\lambda_2+\mu_1)} + \frac{1}{(k_2^*\lambda_2+\mu_1)^2} - \frac{1}{(k_2^*\lambda_2+\mu_1+\lambda_1)} - \frac{1}{(k_2^*\lambda_2+\mu_1+\lambda_1)^2} + \frac{k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+\mu_1)^2} \\ &- \frac{k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+\mu_1+\lambda_1)^2} + \frac{2k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+\mu_1)^3} - \frac{2k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+\mu_1+\lambda_1)^3} \\ &- \frac{\lambda_1}{(\lambda_1+1)(k_2^*\lambda_2+\mu_1+\lambda_1)^2} - \frac{2\lambda_1}{(\lambda_1+1)(k_2^*\lambda_2+\mu_1+\lambda_1)^3} - \frac{2k_2^*\lambda_1\lambda_2}{(\lambda_1+1)(\lambda_2+1)(k_2^*\lambda_2+\mu_1+\lambda_1)^3} \\ &- \frac{6k_2^*\lambda_1\lambda_2}{(\lambda_1+1)(\lambda_2+1)(k_2^*\lambda_2+\mu_1+\lambda_1)^4}\Bigg] \quad (10)\end{aligned}$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 09, September-2016

$$R(3)=\int_0^\infty F_1(y_1)F_2(k_2^*y_1)\{1-F_3(k_3^*y_1)\}g(y_1)dy_1$$

$$=\int_0^\infty\left[\begin{matrix}\left[1-e^{-\lambda_1y_1}\left(1+\frac{\lambda_1y_1}{\lambda_1+1}\right)\right]\left[1-e^{-k_2^*\lambda_2y_1}\left(1+\frac{k_2^*\lambda_2y_1}{\lambda_2+1}\right)\right]\left\{1-\left[1-e^{-k_3^*\lambda_3y_1}\left(1+\frac{k_3^*\lambda_3y_1}{\lambda_3+1}\right)\right]\right\}X\\ \frac{\mu_1^2}{1+\mu_1}(1+y_1)e^{-\mu_1y_1}\end{matrix}\right]dy_1$$

$$=\frac{\mu_1^2}{1+\mu_1}\int_0^\infty\left\{\left[1+y_1+\frac{k_3^*\lambda_3}{\lambda_3+1}y_1+\frac{k_3^*\lambda_3}{\lambda_3+1}y_1^2\right]e^{-(k_3^*\lambda_3+\mu_1)y_1}\left[1-e^{-\lambda_1y_1}-\frac{\lambda_1}{\lambda_1+1}y_1e^{-\lambda_1y_1}\right]\left[1-e^{-k_2^*\lambda_2y_1}-\frac{k_2^*\lambda_2}{\lambda_2+1}y_1e^{-k_2^*\lambda_2y_1}\right]\right\}dy_1$$

$$\begin{aligned}
=\frac{\mu_1^2}{1+\mu_1}\Bigg[&\frac{1}{(k_3^*\lambda_3+\mu_1)}+\frac{1}{(k_3^*\lambda_3+\mu_1)^2}-\frac{1}{(k_3^*\lambda_3+\mu_1+\lambda_1)}-\frac{1}{(k_3^*\lambda_3+\mu_1+\lambda_1)^2}-\frac{1}{(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)}\\
&-\frac{1}{(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^2}+\frac{1}{(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)}+\frac{1}{(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^2}+\frac{k_3^*\lambda_3}{(\lambda_3+1)(k_3^*\lambda_3+\mu_1)^2}\\
&-\frac{k_3^*\lambda_3}{(\lambda_3+1)(k_3^*\lambda_3+\mu_1+\lambda_1)^2}-\frac{k_3^*\lambda_3}{(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^2}+\frac{k_3^*\lambda_3}{(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^2}\\
&-\frac{k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^2}+\frac{k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^2}+\frac{2k_3^*\lambda_3}{(\lambda_3+1)(k_3^*\lambda_3+\mu_1)^3}\\
&-\frac{2k_3^*\lambda_3}{(\lambda_3+1)(k_3^*\lambda_3+\mu_1+\lambda_1)^3}-\frac{2k_3^*\lambda_3}{(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^3}+\frac{2k_3^*\lambda_3}{(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^3}\\
&-\frac{2k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^3}+\frac{2k_2^*\lambda_2}{(\lambda_2+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^3}-\frac{\lambda_1}{(\lambda_1+1)(k_3^*\lambda_3+\mu_1+\lambda_1)^2}\\
&-\frac{2\lambda_1}{(\lambda_1+1)(k_3^*\lambda_3+\mu_1+\lambda_1)^3}+\frac{\lambda_1}{(\lambda_1+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^2}+\frac{2\lambda_1}{(\lambda_1+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^3}\\
&-\frac{2k_3^*\lambda_1\lambda_3}{(\lambda_1+1)(\lambda_3+1)(k_3^*\lambda_3+\mu_1+\lambda_1)^3}+\frac{2k_3^*\lambda_1\lambda_3}{(\lambda_1+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^3}\\
&+\frac{2k_2^*\lambda_1\lambda_2}{(\lambda_1+1)(\lambda_2+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^3}-\frac{2k_2^*k_3^*\lambda_2\lambda_3}{(\lambda_2+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^3}\\
&+\frac{2k_2^*k_3^*\lambda_2\lambda_3}{(\lambda_2+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^3}-\frac{6k_3^*\lambda_1\lambda_3}{(\lambda_1+1)(\lambda_3+1)(k_3^*\lambda_3+\mu_1+\lambda_1)^4}\\
&+\frac{6k_3^*\lambda_1\lambda_3}{(\lambda_1+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^4}+\frac{6k_2^*\lambda_1\lambda_2}{(\lambda_1+1)(\lambda_2+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^4}\\
&-\frac{6k_2^*k_3^*\lambda_2\lambda_3}{(\lambda_2+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1)^4}+\frac{6k_2^*k_3^*\lambda_2\lambda_3}{(\lambda_2+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^4}\\
&+\frac{6k_2^*k_3^*\lambda_1\lambda_2\lambda_3}{(\lambda_1+1)(\lambda_2+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^4}\\
&+\frac{24k_2^*k_3^*\lambda_1\lambda_2\lambda_3}{(\lambda_1+1)(\lambda_2+1)(\lambda_3+1)(k_2^*\lambda_2+k_3^*\lambda_3+\mu_1+\lambda_1)^5}\Bigg]\quad(11)
\end{aligned}$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 09, September-2016

In general

$$R(n)=\frac{\mu_1^2}{1+\mu_1}\Bigg[\frac{1}{(k_n^*\lambda_n+\mu_1)}+\frac{1}{(k_n^*\lambda_n+\mu_1)^2}-\frac{1}{(k_n^*\lambda_n+\mu_1+\lambda_1)}-\frac{1}{(k_n^*\lambda_n+\mu_1+\lambda_1)^2}-\sum_{i=2}^{n-1}\frac{1}{(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)}$$
$$-\sum_{i=2}^{n-1}\frac{1}{(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^2}+\sum_{i=2}^{n-1}\frac{1}{(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)}+\sum_{i=2}^{n-1}\frac{1}{(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^2}$$
$$+\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+\mu_1)^2}-\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+\mu_1+\lambda_1)^2}-\sum_{i=2}^{n-1}\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^2}$$
$$+\sum_{i=2}^{n-1}\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^2}-\sum_{i=2}^{n-1}\frac{k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^2}$$
$$+\sum_{i=2}^{n-1}\frac{k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^2}+\frac{2k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+\mu_1)^3}-\frac{2k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+\mu_1+\lambda_1)^3}$$
$$-\sum_{i=2}^{n-1}\frac{2k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^3}+\sum_{i=2}^{n-1}\frac{2k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^3}$$
$$-\sum_{i=2}^{n-1}\frac{2k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^3}+\sum_{i=2}^{n-1}\frac{2k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^3}-\frac{\lambda_1}{(\lambda_1+1)(k_n^*\lambda_n+\mu_1+\lambda_1)^2}$$
$$-\frac{2\lambda_1}{(\lambda_1+1)(k_n^*\lambda_n+\mu_1+\lambda_1)^3}+\frac{\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{1}{(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^2}$$
$$+\frac{2\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{1}{(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^3}-\frac{2\lambda_1}{(\lambda_1+1)}\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+\mu_1+\lambda_1)^3}$$
$$+\frac{2\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^3}+\frac{2\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^3}$$
$$-\sum_{i=2}^{n-1}\frac{2k_n^*\lambda_nk_i^*\lambda_i}{(\lambda_n+1)(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^3}+\sum_{i=2}^{n-1}\frac{2k_n^*\lambda_nk_i^*\lambda_i}{(\lambda_n+1)(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^3}$$
$$-\frac{6\lambda_1}{(\lambda_1+1)}\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+\mu_1+\lambda_1)^4}+\frac{6\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{k_n^*\lambda_n}{(\lambda_n+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^4}$$
$$+\frac{6\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^4}-6\sum_{i=2}^{n-1}\frac{k_n^*\lambda_nk_i^*\lambda_i}{(\lambda_n+1)(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1)^4}$$
$$+6\sum_{i=2}^{n-1}\frac{k_n^*\lambda_nk_i^*\lambda_i}{(\lambda_n+1)(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^4}$$
$$+\frac{6\lambda_1}{(\lambda_1+1)}\sum_{i=2}^{n-1}\frac{k_n^*\lambda_nk_i^*\lambda_i}{(\lambda_n+1)(\lambda_i+1)(k_n^*\lambda_n+k_i^*\lambda_i+\mu_1+\lambda_1)^4}+\cdots$$
$$+\frac{\lambda_1}{(\lambda_1+1)}(n+1)!\prod_{i=2}^{n}\frac{k_i^*\lambda_i}{(\lambda_i+1)(k_n^*\lambda_n+k_{n-1}^*\lambda_{n-1}+\cdots+k_2^*\lambda_2+\mu_1+\lambda_1)^{n+2}}\Bigg]\quad(12)$$

Total reliability of the n −cascade system is

$$R_n=R(1)+R(2)+R(3)+\cdots+R(n)\quad(13)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 09, September-2016

Numerical Calculations:

Table 1 ($k^*_i = \frac{1}{i}$)

μ_1	λ_1	λ_2	λ_3	R(1)	R(2)	R(3)	R_3
0.3	0.01	0.1	5	0.003121	0.002621	0.008866	0.014608
0.4	0.01	0.1	5	0.00182	0.001624	0.017581	0.021025
0.5	0.01	0.1	5	0.001199	0.001106	0.030321	0.032626
0.6	0.01	0.1	5	0.000855	0.000805	0.047809	0.049469
0.7	0.01	0.1	5	0.000645	0.000614	0.070803	0.072062
0.8	0.01	0.1	5	0.000506	0.000487	0.100101	0.101094
0.9	0.01	0.1	5	0.000409	0.000396	0.136545	0.13735
1	0.01	0.1	5	0.00034	0.000331	0.18102	0.181691

Table 2($k^*_i = \frac{1}{i}$)

μ_1	λ_1	λ_2	λ_3	R(1)	R(2)	R(3)	R_3
0.5	0.01	0.1	5	0.001199	0.001106	0.030321	0.032626
0.5	0.01	0.2	5	0.001199	0.000939	0.027745	0.027851
0.5	0.01	0.3	5	0.001199	0.000774	0.025713	0.027686
0.5	0.01	0.4	5	0.001199	0.000632	0.024095	0.027642
0.5	0.01	0.5	5	0.001199	0.000517	0.022799	0.024515
0.5	0.01	0.6	5	0.001199	0.000424	0.021754	0.023377
0.5	0.01	0.7	5	0.001199	0.00035	0.020911	0.02246
0.5	0.01	0.8	5	0.001199	0.000291	0.020228	0.021718
0.5	0.01	0.9	5	0.001199	0.000244	0.019677	0.02112
0.5	0.01	1	5	0.001199	0.000206	0.019232	0.020637

CONCLUSION:

The general expression for the reliability of n –cascade system was derived when stress and strength follow lindley distribution and the numerical values $R(1), R(2), R(3)$ *and* R_3 was computed for some specific values of the parameters. From the tables, it is observed that as the stress parameter increases the system reliability increases and the system reliability decreases if the strength parameter increases when stress and strength follow lindley distribution.

REFERENCES:

[1] K C Kapur and L R Lamberson, "Reliability in Engineering Design", John Wiley and Sons, Inc., New York, 1977.

[2] S.N.Narahari Pandit and G.L.Sriwastav, "Studies in Cascade Reliability – I", IEEE Transactions on Reliability, R-28, Iss 5, 1979, pp 411-414.

[3] A.C.N.Raghava Char, B.Kesava Rao and S.N. Narahari Pandit, "The Reliability of a Cascade system with Normal Stress and Strength distribution", ASR, vol. no.2, 1987, pp 49-54.
[4] T.S.Uma Maheswari, "Reliability of single strength under $n-$stresses", Micro Electron Reliability, Pergamon Press, OXFORD, vol 32, no 10, 1992, pp 1475-1478.
[5] T.S.Uma Maheswari, "Reliability comparison of $n-$cascade system with the addition of an $n-$strengths system", Micro Electron Reliability, Pergamon Press, OXFORD, vol 33, no 4, 1993, pp 477-479.
[6] T.S.Uma Maheswari, "Reliability of a cascade system with normal stress and exponential strength", Micro Electron Reliability, Pergamon Press, OXFORD, vol 33, no 4, 1993, pp 927-936.
[7] T.S.Uma Maheswari, "Reliability of single stress under $n-$strengths of life distribution", Micro Electron Reliability, Pergamon Press, OXFORD, vol 34, no 3, 1994, pp 569-572.
[8] M.Tirumala Devi, K.Sandhya and T.S.Uma Maheswari, "Reliability of a system with $m-$ stresses and $n-$ strengths", OPSEARCH, 49(4), 2012, pp 463-481.
[9] Chumchum Doloi, Munindra Borah and Jonali Gogoi, "Cascade system with $\Pr(X < Y < Z)$", Journal of Informatics and Mathematical Sciences, vol 5, no 1, 2013, pp 37-47.
[10] M.E.Ghitany, B.Atieh and S.Nadarajah, "Lindley distribution and its application", Mathematics and Computers in Simulation, 78, 2008, pp 493-506.