Capacity Of Channel MIMO

Ando Nirina Andriamanalina, Andry Auguste Randriamitantsoa , Tahina Ezéchiel Rakotondraina

[123] **University of Telecommunication, High School polytechnic of Antananarivo, University of Antananarivo Antananarivo, BP 7710, Madagascar**

Abstract

In radio system, to optimize the transmission, we can use diversity technology. We then met the frequency diversity, temporal, spatial and polarization diversity. Regarding the spatial diversity, it allows to consider several categories radio channel namely SIMO systems, MISO, SISO and MIMO. For a SIMO channel or Single Input Multiple Output, we have a system where the transmitter is composed of a single antenna while the receiver is composed of several antennas. For MISO or Multiple Input Single Output is a system where the transmitter is composed of several antennas channel while the receiver is composed single antenna. The use of spatial diversity at both the transmitting and receiving system is called MIMO.

Keywords: *Author Channel, diversity, MIMO, SNR.*

1. Introduction

The beginnings of spatial diversity became implanted to the stations of bases that use several antennas to connect to the users.

To the level of the terminal, it will be possible to transmit different data simultaneously. This technique is called MIMO or "Multiple Input-Multiple Output". This process permitted to exploit two types of gains therefore of which the one of the struggle against the unconsciousness of the channel and the one of the improvement of the debit or the capacity of the channel.

2. MIMO channel modelisation

MIMO channel can be channel represented by a complex matrix H translating the spatial dimension. For MIMO channel with N_T transmit antennas and N_R receive antennas, and $B \sim N(0, {\sigma_b}^2)$ Gaussian noise which be supposed independently and identically distributed, the output is defined by :

$$Y = HX + B \qquad (1)$$

$$H = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1N_T} \\ h_{21} & h_{22} & h_{23} & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R1} & h_{12} & \dots & h_{N_RN_T} \end{pmatrix} \qquad (2)$$

Where, $h_{kl} = \alpha_{kl} e^{j\phi_{kl}}$ as $1 \leq k \leq N_R$; $1 \leq l \leq N_T$ represent the complex gain between the $l^{i\text{-ème}}$ transmit antenna and the $k^{i\text{-ème}}$ receive antenna where α_{kl} and ϕ_{kl} are the amplitude and the phase of the complex coefficient.

3. Capacity of channel MIMO

Channel capacity measures the maximum amount of information that could be transmitted through a channel and received with negligible error. It is defined then by [1] [2]:

$$C = \max_{p(x)} I\,(X; Y\,) \qquad (3)$$

For a SISO link with a constant channel gain h, the capacity is expressed as [1]:

$$C = log_2(1 + \gamma|h|^2) \qquad (4)$$

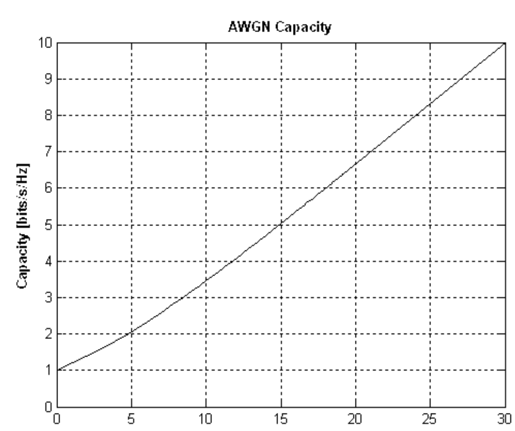

Fig. 1 Awgn SISO capacity.

3.1 Mutual information

The mutual information between the input and output signal, X and Y is expressed as [1] [2]:

$$I\,(X; Y\,) = H(Y) - H(Y|X) \qquad (5)$$

$$I\,(X; Y\,) = H(Y) - H(B) \qquad (6)$$

The capacity is obtained by maximizing the received signal entropy $H(Y)$. For a circular symmetric gaussian random vector $X = \{X_1, \dots, X_d\} \in \mathbb{C}^d$ with positive hermitian covariance K_X. The entropy $H(Y)$ of Y is expressed as[1] [2] :

$$H(Y) = log_2\, det(\pi e R_y) \qquad (7)$$

The mutual information is defined by [1] :

$$I(X;Y) = log_2\left(det\{I_{N_R} + HR_x H^H (R_b)^{-1}\}\right) \ (8)$$

We supposed that the maximal mutual information is gotten under the constraint that the power given out total P_T is finished and constant then the capacity of a MIMO channel is defined by [1] :

$$C_{MIMO} = \max_{p(x):E[X^H X]\leq P_T} I\,(X;Y) \qquad (9)$$

3.2 MIMO channel when CSI is know

When no Channel State Information is available at the transmitter, equal power allocation is adopted. Then the matrix of covariance of the signal given out is defined by [1] :

$$R_x = \frac{P_T}{N_T} I_{N_T} \qquad (10)$$

The capacity of a MIMO channel is expressed as [1]:

$$C_{MIMO} = log_2\left(det\left\{I_{N_R} + \frac{\gamma}{N_T} HH^H\right\}\right) \qquad (11)$$

Where γ represent the report signal on noise.

3.3 MIMO channel by SVD decomposition

When SVD factorization is used, the MIMO channel H is expressed as:

$$H = USV^H \qquad (12)$$

Where, the matrix diagonal $S(N_R \times N_T)$ are square roots of eigen values from HH^H or $H^H H$. The rank of matrix channel H is defined by:

$$r = rank(H) = min(N_R, N_T) \qquad (13)$$

With SVD factorization, MIMO capacity is expressed as :

$$C_{SVD} = \sum_{i=1}^{r} log_2\left(1 + \frac{\gamma}{N_T}\lambda_i\right) \qquad (14)$$

Simulation results of capacity of (11) and (14) are depicted in the Fig. 2.

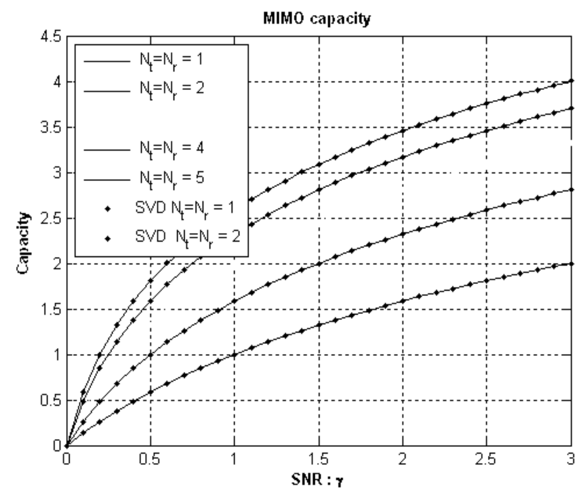

Fig. 2 MIMO Capacity.

Simulation results of $C_{MIMO} - C_{SVD}$, (11) - (14) is depicted in the Fig. 3.

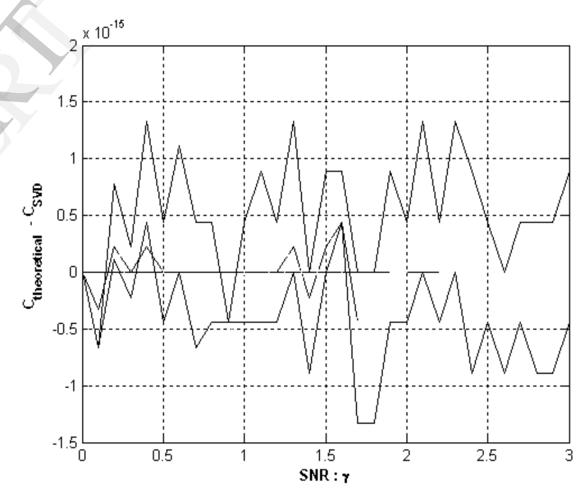

Fig. 3 $C_{MIMO} - C_{SVD}$

Let's note that r represents the number of independent channel in the transmission or the number of the lines and columns of the matrix H that are linearly independent [1] [3].

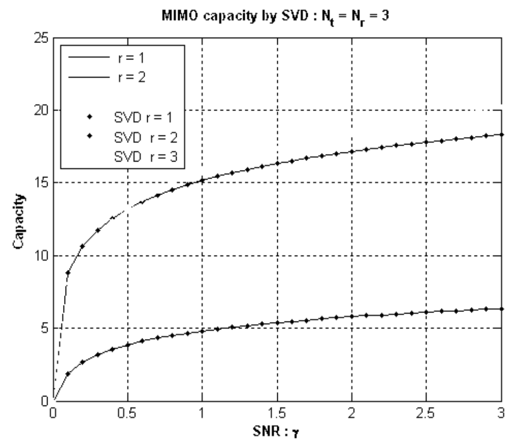

Fig. 4 Capacity by SVD factorization with different value of r

When λ_i are independently and identically distributed $\lambda_i = \lambda$, the capacity is expressed as:

$$C_{SVD} = N_T log_2\left(1+\frac{\gamma\lambda}{N_T}\right) \quad \text{si} \quad N_R \geq N_T \quad (15)$$

$$C_{SVD} = N_R log_2\left(1+\frac{\gamma\lambda}{N_T}\right) \quad \text{si} \quad N_T > N_R \quad (16)$$

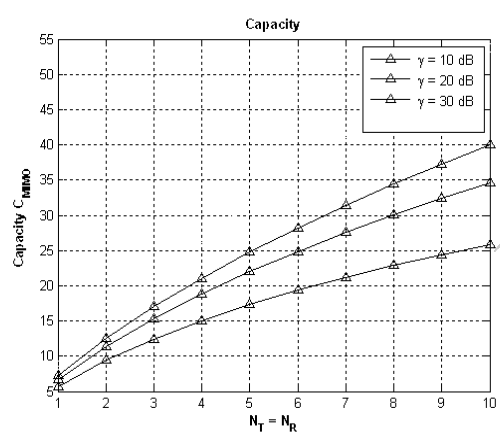

Fig. 5 MIMO capacity by SVD with different value of λ

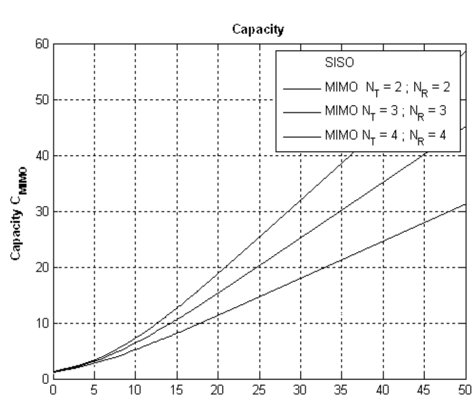

Fig. 6 MIMO capacity by SVD with different value of N_T and N_R.

3.4 MIMO capacity with Water-filling

When the channel statement information is available at the transmitter, intelligent power allocation is adopted. The total power of the link is expressed as :

$$\sum_{p=1}^{N_T} P_p = P_T \quad (17)$$

The optimal distribution of the power on every p channel is defined by [1] [3] :

$$P_p = \left(\mu - \frac{\sigma_b^2}{\lambda_p}\right)^+ \quad (18)$$

Where $(a)^+ = \max(a, 0)$ and is a constant that satisfies the constraint of the total power. The capacity is defined then by [1] [3]:

$$C_{WF} = \sum_{p=1}^{m} log_2\left[\left(\frac{\lambda_p\mu}{\sigma_b^2}\right)^+\right] \quad (19)$$

3.5 Outage probability

The outage probability is the probability for which the capacity is lower than a threshold capacity R which is fixed. Under this capacity the transmission is not possible. Outage probability is expressed [4]:

$$P_{out} = \Pr(C \leq R) \quad (20)$$

For SISO channel, outage probability is expressed as:

$$P_{out} = Pr(log_2(1+\gamma|h|^2) \leq R) \quad (21)$$

If |h| is a random variables distributed with Rayleigh, then it is expressed as [5] :

$$|h| = \sqrt{\mathcal{Re}\{h\}^2 + \mathcal{Im}\{h\}^2} \quad (22)$$

Where $\mathcal{Re}\{h\}$ and $\mathcal{Im}\{h\}$ are Gaussian $\mathcal{N}(0,1)$. Well, $|h|^2 = \mathcal{Re}\{h\}^2 + \mathcal{Im}\{h\}^2$ are random variables distributed according to a chi-square distribution with 2 degrees of freedom [4] :

$$f_k(x) = \frac{1}{2^{(k/2)}\Gamma(k/2)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}} \quad (23)$$

$$f_2(x) = \frac{1}{2\Gamma(1)} e^{-\frac{x}{2}} \quad (24)$$

Where Γ is gamma function.
With (21), $|h|^2$ is expressed as :

$$|h|^2 \leq \frac{2^R - 1}{\gamma} \quad (25)$$

$$P_{out} = \int_0^{|h|^2{}_{th}} f_2(|h|^2) d|h|^2 \quad (26)$$

$$P_{out} = -\int_0^{|h|^2{}_{th}} -\frac{1}{2\Gamma(1)} e^{-\frac{|h|^2}{2}} d|h|^2 \quad (27)$$

$$P_{out} = 1 - e^{-\frac{|h|^2}{2}} \quad (28)$$

The outage probability of Rayleigh SISO channel is defined by :

$$P_{out\,SISO\ Rayleig\ h}(R) = 1 - e^{-\frac{2^R-1}{2\gamma}} \quad (29)$$

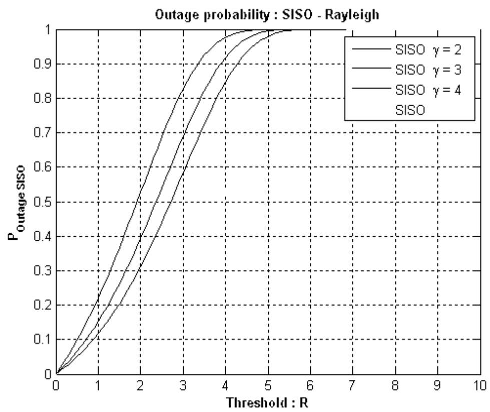

Fig. 7 Outage probability of Rayleigh SISO channel

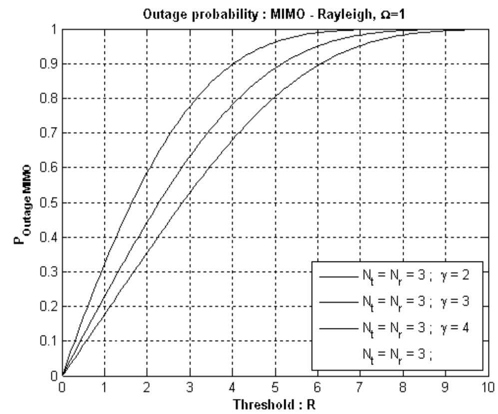

Fig. 8 Outage probability of Rayleigh MIMO Channel

For a MIMO channel, the outage probability is defined by :

$$P_{out} = Pr\left(log_2\left(det\left\{I_{N_R} + \frac{\gamma}{N_T} HH^H\right\}\right) \le R\right) \quad (30)$$

With singular value decomposition of H, the outage probability is defined by :

$$P_{out\_SVD} = \Pr\left(N_T \log_2\left(1 + \frac{\gamma\lambda}{N_T}\right) \le R\right) \quad (31)$$

If, λ is distributed with Rayleigh random variable, the density probability is expressed as:

$$f(x) = \frac{2x}{\Omega} e^{\left(-\frac{x^2}{\Omega}\right)} \quad (32)$$

Where, $\Omega = E[x^2] = \overline{x^2}$

With, $N_T \log_2\left(1 + \frac{\gamma\lambda}{N_T}\right) \le R$, λ is expressed as:

$$\lambda \le \frac{N_T\left(2^{\frac{R}{N_T}} - 1\right)}{\gamma} \quad (33)$$

Then,

$$P_{out\_SVD} = \Pr(C_{SVD} \le R) \quad (34)$$

$$P_{out\_SVD} = \int_0^{\lambda_{th}} f(\lambda) d\lambda \quad (35)$$

$$P_{out\_SVD} = -\int_0^{\lambda_{th}} -\frac{2\lambda}{\Omega} e^{\left(-\frac{\lambda^2}{\Omega}\right)} d\lambda \quad (36)$$

$$C_{out\_SVD} = 1 - e^{\left(-\frac{\lambda_{th}^2}{\Omega}\right)} \quad (37)$$

Where, $\lambda_{th} = \frac{N_T\left(2^{\frac{R}{N_T}} - 1\right)}{\gamma}$.

3. Ergodic capacity of channel MIMO

Ergodic capacity refers to the maximum rate that can be achieved during a long observation communication to exploit all channel information. For SISO channel, ergodic capacity is defined by :

$$\bar{C}_{SISO} = E[log_2(1 + \gamma|h|^2)] \quad (38)$$

For MIMO channel, ergodic capacity is defined by :

$$\bar{C}_{MIMO} = E\left\{\max_{p(x):E[X^H X] \le P_T} I(X;Y)\right\} \quad (39)$$

$$\bar{C}_{MIMO} = E\left\{log_2\left(det\left\{I_{N_R} + \frac{\gamma}{N_T} HH^H\right\}\right)\right\} \quad (40)$$

4. Conclusions

The In wireless radio, quality of transmission increase linearly, with the number of antenna used. Capacity of channel radio, depend of the technique of diversity. In MIMO channel, the numbers of link which transport information depend of the rank of the channel matrix. In the receiver side, there are many techniques to combine the different versions of the signal in an optimal manner exist also.

References

[1] A. A. Name, and B. B. Name, "Mimo systems, theory and applications", InTech, Mar. 2011.

[2] J. Casse, and T. Horel, "Maximal theoretical debit of the MIMO communication", 2009.

[3] A. Goldsmith, S. A. Jafar, N. Jindal, S. Vishwanath, "Fundamental Capacity of MIMO Channels", Nov. 2002.

[4] J. Pardonche, "Systems transmission cordless multi-emitters, multi-receptors for applications transportation. Survey of the models of propagation channel", Thesis, University of Sciences et Technologies of Lille, 2004.

[5] M. Terre, "Propagation", National conservatory of the Arts and Professions, 2006.

Ando N. Andriamanalina was born in Antananarivo, Madagascar on 1984. He received his M.S. degrees in Information Theory Student in 2012 at University of Antananarivo (Madagascar).He work as a Teacher assistant and a Ph.D. student at High School Polytechnic of Antananarivo. His currents interests include Wireless transmission, digital transmission.

Andry A. Randriamitansoa was born in Antananarivo, Madagascar on 1984. He received respectively his M.S degree Ph.D. in Information Theory Student and information engineering in 2010 and 2013 at University of Antananarivo (Madagascar).He work as a professor at High School Polytechnic of Antananarivo. His currents interests include automatism, digital servitude.

Tahina E. Rakotondraina was born in Antsirabe, Madagascar on 1984. He received respectively his M.S degree Ph.D. in Information Theory Student and information engineering in 2010 and 2013 at University of Antananarivo (Madagascar).He work as a professor at High School Polytechnic of Antananarivo. His currents interests include Cryptography, multimedia, information Hiding, VOIP.