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# Capacity and Performance Analysis of MIMO-STBC in Rayleigh Fading Channels

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***Abstract*— In wireless communications, spectrum is a scarce resource and hence imposes a high cost on the high data rate transmission. Fortunately, the emergence of multiple antenna system has opened another very resourceful dimension i.e. space for information transmission. Multi-antenna systems are expected to play very important role in future multimedia wireless communication systems. Such systems are predicted to provide tremendous improvement in spectrum utilization. Here, the orthogonal space-time block codes are considered for the capacity and error probability analysis of MIMO systems. The numerical and simulation results obtained using MATLAB are presented for the multi-antenna system channel capacity and bit-error rate in Rayleigh fading channels.**

***Index Terms*— Space-time codes, transmit diversity, Rayleigh fading channels, channel capacity, MIMO.**

## I. INTRODUCTION

Wireless communications has made a tremendous impact on the life style of a human being. As compared to fixed wireless systems, today's wireless networks provide high-speed mobility for voice as well as data traffic. The time-varying nature of wireless channels, such as fading, multipath makes it difficult for wireless system designers to satisfy the ever-increasing expectations of mobile users in terms of data rate and Quality of Service (QoS). Continuous exponential growth of Internet, cellular mobile and Multimedia services in near past has been the driving forces for the increased demand of data rates in communication networks.

Receiver diversity is used in present cellular mobile systems to gain certain benefits like improving quality and range of uplink. Error control coding can be combined with transmit diversity to achieve improved performance of multiple antenna transmission systems and thus leads to coding gain advantage in addition to diversity benefit but at the cost of bandwidth expansion due to code redundancy. A joint design of error control coding, modulation and transmit diversity as a single block needs to use Space-Time codes, then it is possible to achieve coding gain as well as diversity benefit without bandwidth expansion. The combination of receive diversity with Space-Time codes can further enhance the performance of multi-antenna system by minimizing multipath fading effect and help achieve the capacity of MIMO systems.

The work carried out by [2] and [3] on MIMO capacity limits shows that, capacity of MIMO channels increase approximately linearly with increased number of antennas. In 1998, Tarokh et al in [1] introduced the fundamentals of Space-Time coding utilizing multiple transmit antennas and optionally multiple receive antennas. Alamouti in [4] has proposed a simple 2x2 system achieving full diversity.

## II. MIMO SYSTEM MODEL

We consider a MIMO system as shown in Fig.1 with array of $N_T$ transmit antennas and $N_R$ receiving antennas. The transmitted signals in each symbol period are represented by an $N_T$ x1 column matrix $x$.

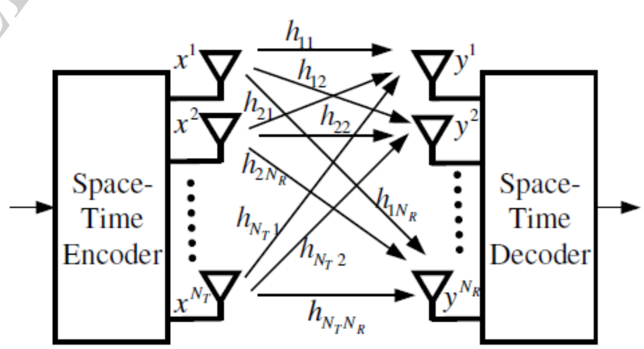


Fig.1: MIMO system model

The wireless channel between transmitter and receiver is described by $N_R$ x $N_T$ complex matrix, denoted by $\boldsymbol{H}$. The $ij^{th}$ entry of matrix $\boldsymbol{H}$ denoted by $h_{ij}$ represents the channel fading co-efficients from $i^{th}$ transmit antenna to $j^{th}$ receive antenna. Rayleigh distribution is the most representative of Non-line of sight (N-LOS) wireless radio propagation and hence the MIMO channel capacity has been investigated for Rayleigh fading channel model.

The received signals denoted by $\boldsymbol{y}$ are represented by $N_R$ x1 column matrix. Similarly, the noise at the receiver is represented by $N_R$ x1 column matrix, denoted by $n$. The received vector can be represented as

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$$\boldsymbol{y} = \boldsymbol{H}x + n \qquad (1)$$

## III. CAPACITY ANALYSIS OF MIMO

The Shannon's capacity formula for capacity per channel use b/s/Hz is given by $C = \log_2(1+\gamma H^2)$ where $\gamma$ is received SNR and $H^2$ is channel transfer characteristic. As the Rayleigh channel matrix $\boldsymbol{H}$ has entries, which are independent and identically distributed (i.i.d.) complex zero mean Gaussian random variables with unit variance.

The capacities for Single-Input Single-Output (SISO) i.e. no diversity, Single- Input Multiple-Output (SIMO) i.e. receive diversity, Multi-Input Single-Output (MISO) i.e. transmit diversity, and MIMO i.e. combined transmit-receive diversity for slow Rayleigh fading channel are given by [2].

SISO system (Single antenna link):

$$C = \log_2(1+\gamma \chi_2{}^2) \qquad (2)$$

Where $\chi_2{}^2$ is a chi-squared random variable with 2 degrees of freedom.

SIMO system (Receive Diversity):

$$C = \log_2(1+\gamma \chi_{2NR}{}^2) \qquad (3)$$

MISO system (Transmit diversity):

$$C = \log_2(1+\frac{\gamma}{N_T} \chi_{2NT}{}^2) \qquad (4)$$

MIMO system: For the case of $N_T \geq N_R$, lower bound on the capacity in terms of chi-squared random variable is given by

$$C = \sum_{i=NT-(NR-1)}^{NT} \log_2(1+\frac{\gamma}{NT} \chi_{2i}{}^2) \qquad (5)$$

For a special case of $N_T = N_R$ =N, the lower bound in (5) reduces to

$$C = \sum_{i=1}^{NT} \log_2(1+\frac{\gamma}{NT} \chi_{2i}{}^2) \qquad (6)$$

## IV. THE ORTHOGONAL SPACE-TIME BLOCK ENCODER

The orthogonal space-time block encoder consists of two functional units: a mapper and the block encoder itself.

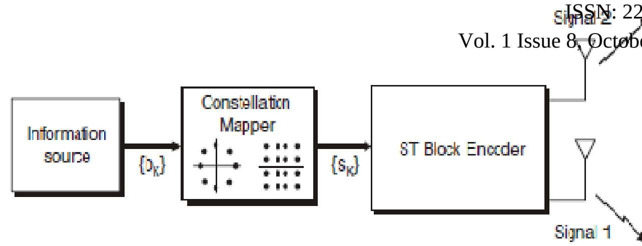


Fig.2: Space-Time Block Encoder.

The mapper takes the incoming binary data stream $\{b_k\}$, and generates a new sequences of blocks, with each block made up of multiple symbols that are complex. The mapper may be in the form of an M-ary PSK or M-ary QAM mapper.

All the symbols in a particular column of the transmission matrix are pulse shaped and then modulated into a form suitable for simultaneous transmission over the channel by the transmit antennas.

The block encoder converts each block of complex symbols produced by the mapper into a $l-by-N_T$ transmission matrix, where $l$, $N_T$ represents the temporal and spatial dimension respectively of the transmission matrix.

## V. CAPACITY ANALYSIS OF MIMO-STBC OVER RAYLEIGH FADING CHANNELS.



Considering that the receiver knows the channel whereas the transmitter does not the channel, the general expression for the channel capacity of a random MIMO channel is given by in [2] as

$$C = E\left[B \log_2 \det(I + \frac{E_S}{N_T N_0} Q)\right] \text{b/s/Hz} \qquad (7)$$

The capacity of equivalent STBC channel with code rate R will then be given by

$$\overline{C}_{STBC} = E\left[BR \log_2 \det(I_r + \frac{E_S}{RN_T N_0} Q)\right] \qquad (8)$$

$$= E\left[BR \log_2(1+\gamma_S)\right]$$

with the bound for channel capacity at the output of STBC system expressed as

$$\overline{C}_{STBC} \leq E\left[BR \log_2(1+\overline{\gamma}_{STBC})\right] \qquad (9)$$

Where $\gamma_S$ is the effective instantaneous SNR per symbol at the receiver given by [5].

$$\gamma_S = \frac{E_S}{N_T R N_0} \|H\|_F{}^2 \quad \text{and}$$

$$\|H\|_F{}^2 = \sum_{i=1}^{NT} \sum_{j=1}^{NR} \|h_{ij}\|^2 \qquad (10)$$

and $\bar{\gamma}_{STBC}$ is average SNR per symbol in STBC channel.

## VI. ERROR PROBABILITY ANALYSIS OVER RAYLEIGH FADING CHANNELS.

The well-known Bit-error probability (BEP) equations as given in [6], for Binary Phase Shift Keying (BPSK) and QPSK over an AWGN channel are given as

$$P_{Mb}(\gamma_S) = Q(\sqrt{2\gamma_S}) \qquad P_{Mb}(\gamma_S) = Q(\sqrt{\gamma_S}) \qquad (11)$$

The complex exact equation for Symbol Error Probability (SEP) of M-ary PSK in AWGN channel can be approximated under the assumptions of large values of SNR and large values of M. The approximated SEP expression can be given as

$$P_M(\gamma_S) \approx 2Q(\sqrt{2\gamma_S}\,\sin\frac{\pi}{M}) \qquad (12)$$

and the approximated BEP is given as

$$P_{Mb} \approx \frac{1}{\log_2 M} P_M \qquad (13)$$

Defining the error probability of M-ary signal constellation with STBC in AWGN channel as $P_{STBC,M}$, the error probability with Rayleigh fading channel can be obtained by averaging $P_M(\gamma_S)$ over the PDF of $\gamma_S$ and can be given as in [5]

$$P_{STBC,M} = \int_0^{\infty} P_M(\gamma_S) P_{Rayleigh}(\gamma_S)\, d\gamma_S \qquad (14)$$

The BER performance between Alamouti schemes and MRC (maximal ratio combiner) i.e. one transmitter and two receiver case is also shown here.

From the post on MRC, the BER of BPSK modulation in Rayleigh channel with one transmitter, two receiver case is

$$P_e(MRC) = P_{MRC}{}^2[1 + 2(1 - P_{MRC})] \qquad (15)$$

$$\text{where } P_{MRC} = \frac{1}{2} - \frac{1}{2}\left[1 + \frac{1}{\frac{E_b}{N_0}}\right]^{-\frac{1}{2}} \qquad (16)$$

Similarly, with Alamouti scheme two transmitters and one receiver case, the BER is

$$P_e(STBC) = P_{STBC}{}^2[1 + 2(1 - P_{STBC})] \qquad (17)$$

$$\text{where } P_{STBC} = \frac{1}{2} - \frac{1}{2}\left[1 + \frac{2}{\frac{E_b}{N_0}}\right]^{-\frac{1}{2}} \qquad (18)$$

## VII. NUMERICAL AND SIMULATION RESULTS.

The numerical and simulation results are presented to illustrate and verify the information theoretic capacity of MIMO systems and to observe the effect of several STBCs on MIMO channel capacity.

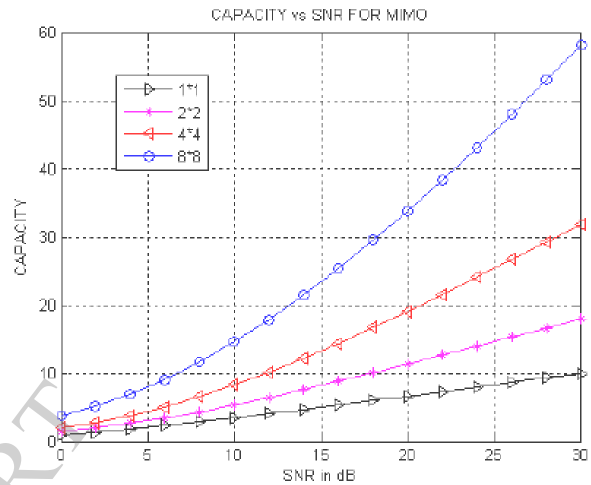


Fig.3: MIMO Ergodic Capacity.

Fig.3 shows the graph for Ergodic capacity plotted against SNR for different systems. The graph shows that increasing the number of antennas increases the Ergodic capacity.

The capacity using different STBCs over a Rayleigh fading channel is given in Fig.4.

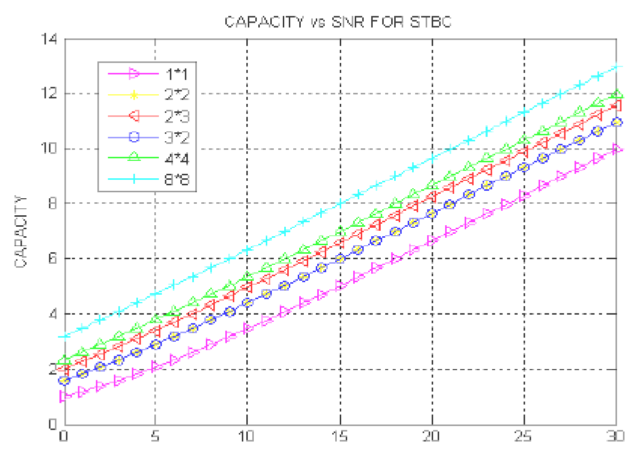


Fig.4: MIMO-STBC Capacity.

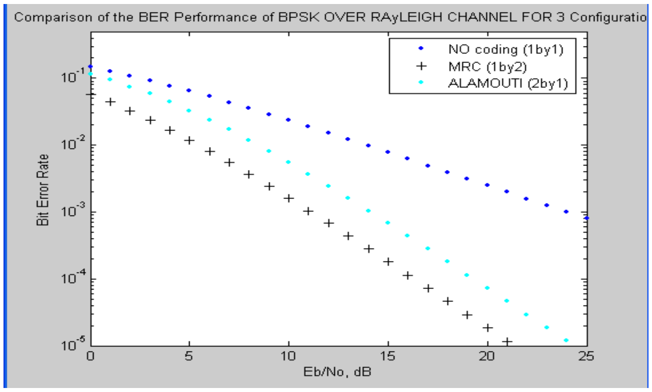


Fig.5: Comparison of BER Performance.

Fig.5 shows that the BER performance of Alamouti STBC [2 by 1] has around 3dB poorer performance as compared to MRC [1 by 2].

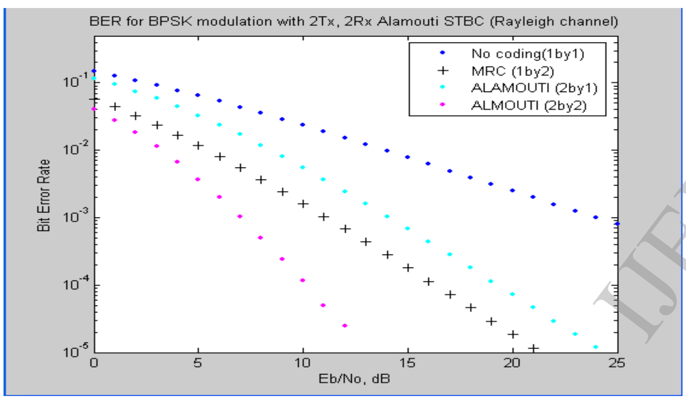


Fig.6: Comparison of BER Performance with Alamouti STBC [2 by 2].

Fig.6 shows that the BER performance of Alamouti STBC [2 by 2] is much better than MRC case.

## VIII. CONCLUSION.

In this paper, the capacity and BER of MIMO systems in Rayleigh fading channels has been examined. It has been seen that the use of multiple antennas increases the capacity although significant improvement can be achieved using equal or higher number of receive antennas compared to transmit antennas.

Similarly, the performance of Alamouti code [2 by 1] is worse by about 3dB compared to MRC. This is because in space-diversity-on transmit scheme using Alamouti code, the transmit power in each of the two antennas is one-half of the transmit power in the space-diversity-on receive scheme using MRC. But the BER performance of MIMO-STBC [2 by 2] scheme is better than MRC case due to higher diversity order.

## IX. REFERENCES.

1. V. Tarokh, N. Seshadri, and A.R. Calderbank, "Space-time codes for high data rate wireless communication: Performance analysis and code construction," IEEE Trans. Inform. Theory, vol. 44, No.2, Mar.1998, pp.744-765.
2. G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", Wireless Personal Communications, vol. 6, 1998, pp. 311-335.
3. E. Telatar, "Capacity of multi-antenna Gaussian channels", European Transactions on Telecommunications, vol. 10, no. 6, Nov./Dec. 1999, pp. 585-595.
4. S.M. Alamouti, "A simple transmit diversity technique for wireless communications", IEEE Journal Selected. Areas in Communications, vol. 16, No. 8, Oct.1998, pp. 1451-1458.
5. Hao Zhang and T. Aaron Gulliver, "Capacity and Error Probability analysis for Orthogonal Space-Time Block Codes over fading channels", IEEE Transactions on Wireless Communications, vol. 4, N0. 2, March 2005, pp. 808-819.
6. J.G. Proakis, "Digital Communications", 4th Edition, McGraw Hill Inc.