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Buoyant Force Determination for Different Wooden Shapes in Water Using Analytical Method

Determining the buoyant force acting on wooden shapes like cuboid, sphere, cylinder and cone in water using Analytical method and metacentric height with approximate time of oscillation and list angle

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***Abstract*—We determine the buoyant force and metacentric height in water acting on different wooden pieces, namely, a cube, cylinder, sphere and cone using an analytical method system. We also calculate the time of oscillation and list angle. We then compare the results of each system to obtain the final table of comparison.**
***Keywords*— buoyancy, wood, cube, cone, cylinder, sphere, metacenter, time of oscillation, list angle**

I. INTRODUCTION

Buoyancy is defined as the tendency for an immersed body to be lifted up in the fluid due to an upward force acting opposite to the action of gravity. The force tending to lift up the body under such conditions is known as buoyant force or upthrust.

II. Theory

The equilibrium of floating bodies is of the following types:

1. Stable Equilibrium: When a body is given a small angular displacement (i.e. slightly tilted), by some external force, and then it returns back to its original position due to the internal forces (weight and upthrust), such an equilibrium is called a stable equilibrium.
2. Unstable Equilibrium: If the body does not return to its original position from the slightly tilted angular position and heels farther away, when given a small angular displacement, such an equilibrium is called an unstable equilibrium.
3. Neutral Equilibrium: If a body, when given a small angular displacement, occupies a new position and remains at rest in this new position, it is said to possess a neutral equilibrium.

Metacenter and Metacentric Height:

Metacenter: When a body floating in a liquid is given a small angular displacement, it starts oscillating about a point (M). This point about which the body starts oscillating is called the metacenter. The metacenter may also be defined as a point of intersection of the axis of body passing through center of gravity (G) and original center of buoyancy (B) and a vertical line passing through the center of buoyancy (B1) of the tilted position of the body. This position of the metacenter, M remains pror the small angle of tilt θ.

Metacentric Height: The distance between the center of gravity of a floating body and the metacenter (i.e. distance GM as shown in Fig 4.6 (b) is called metacentric height.

- For stable equilibrium, the position of metacenter M remains higher than the center of gravity of the body G.
- For unstable equilibrium, the position of metacenter M remains lower than G.
- For neutral equilibrium, the position of metacenter coincides with G.

Determination of Metacentric Height:

- Analytical Method:

  BM = I/V

  where

  I = area moment of inertia at the water line about the axis O

  V = volume of the water displaced

  GM = BG +/- BM

- Experimental Method:

  GM = (W1.z.l)/(W.d) = (W1.z)/(W.tan(θ))

  where

  l = length of plumb bob

  d = displacement of plumb bob

III. UNITS

- All units are in the SI system.
- specific weight of water = 9.81 kN/m^3
- specific weight of wood = 6.4 kN/m^3
- damping coeffiecnt of water = 0.2 N-s/m

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IV. ASSUMPTIONS:

1. Archimedes principle: Bouyant force is equal to the weight of the fluid displaced by the body.
2. Pascal's Law: The intensity of pressure at any point on a liquid at rest is the same in all directions which is independent of / regardless of the inclination surface of the element of the liquid.
3. Pressure of a fluid on a surface will always act normal to the surface.
4. Hydrostatic law: It states that rate of increase of pressure of a fluid in the vertically downward direction is equal to the weight density of the fluid at that point.
5. The fluid (water) is incompressible and is in STP conditions (ideal).
6. The only forces acting on the wooden body and the fluid are gravity and pressure forces.
7. Velocity or Kinetic Energy of the system is zero.
8. System is steady(does not change with time), continuous (mass is conserved) and fluid flow is streamlined.

V. EQUATIONS

A. Buoyant Force on Rectangular Wooden Block*:*

Width of rectangular wooden block = a
Depth of rectangular wooden block = a
Length of rectangular wooden block = b
Volume of rectangular wooden block = b*a^2
Specific weight of wood = 6.4 kN/m^3
For equilibrium the weight of water displaced =
Weight of rectangular wooden block = 6.4*b*a^2kN
= 6.4*b*a^2 kN
Volume of water displaced
= Weight of water displaced / Weight density of water
= 6.4*a^2*b/9.81
= 0.6524*a^2*b m^3

Position of center of buoyancy:

We know that

Volume of rectangular wooden block in water = Volume of water displaced

or

a*a*h = 0.6524*a^2*b m^3

(where h = depth of wooden block in water)

Therefore,

h = 0.6524*b m

Hence,
Center of buoyancy OB from bottom
= 0.6524*b/2 m
= 0.3262*b m (from the base of the rectangular wooden cube)
Center of gravity OG from base
= b/2 m
BG = OG - OB = 0.174*b m
Also
BM = I/V
where
I = area moment of inertia of a rectangular plan about axis YOY'
= a*a^3/12
= a^4/12 m^4
V = Volume of water displaced or Volume of wood in water
= 0.6524*a^2*b m^3
BM = I/V
= a^2/(12*0.6524*b)
= a^2/(7.8288*b) m
We know that the metacentric height,
GM = BM -BG
= a^2/(7.8288*b) - 0.174*b m
-ve sign means the metacenter (M) is below the center of gravity (G). Thus the rectangular wooden block is in unstable equilibrium.
Time of oscillation:

T = 2*pi*√(GM/g)

T = 2*√[a^2/(7.8288*b) - 0.174*b] seconds

Or

Weight of rectangular wooden block = Buoyant force

c.v = Mg

0.02. w.r_e = 6400*b*a^2

w = (6400*b*a^2)/(0.02* r_e)

where r_e = effective radius of container

T = 2*pi/ w

T = 2*pi* r_e /(320*10^3*b*a^2)

List angle θ
tan θ = h/GM
= 0.6524*b / a^2/(7.8288*b) - 0.174*b rad
Or
cos θ = $r_{e\,/\,BM}$ (for small values of θ)
= $r_{e\,/}$ [a^2/(7.8288*b)] rad
Torque t = Fb. BG. sin θ
= 6400*a^3* 0.174*b*sin(θ) N-m

B. Buoyant Force on Cubic Wooden Block:

Width of cubic wooden block = a
Depth of cubic wooden block = a
Length of cubic wooden block = a
Volume of cubic wooden block = a^3
Specific weight of wood = 6.4 kN/m^3
For equilibrium the weight of water displaced =
Weight of cubic wooden block = 6.4*a^3 kN
= 6.4*a^3kN
Volume of water displaced
= Weight of water displaced / Weight density of water
= 6.4*a^3/9.81

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= 0.6524*a^3 m^3

Position of center of buoyancy:
We know that
Volume of cubic wooden block in water = Volume of water displaced
or
a*a*h = 0.6524*a^3
(where h = depth of wooden block in water)
Therefore,
h = 0.6524*a m
Hence,
Center of buoyancy OB from bottom
= 0.6524*a/2
= 0.3262*a m (from the base of the rectangular wooden cube)
Center of gravity OG from base
= a/2
= 0.5*a m
BG = OG - OB = 0.1376*a m
Also
BM = I/V
where
I = area moment of inertia of a cubic plan about axis YOY'
= a*a^3/12
= a^4/12 m^4
V = Volume of water displaced or Volume of wood in water
= 0.6524*a^3 m^3
BM = I/V
= a/(12*0.6524)
= a/7.8288 m
We know that the metacentric height,
GM = BM -BG
= a/7.8288 - 0.1376*a
= -0.00987a m
-ve sign means the metacenter (M) is below the center of gravity (G). Thus the cubic wooden block is in unstable equilibrium.
Time of oscillation:
T = 2*pi*√(GM/g)
T = 2*√(-0.00987a) seconds
Or
Weight of cubic wooden block = Buoyant force
c.v = Mg
0.02. w.r_e = 6400*a^3
w = (6400*a^3)/(0.02* r_e)
where r_e = effective radius of container
T = 2*pi/ w
T = 2*pi* r_e /(320*10^3*a^3)
List angle θ
tan θ = h/GM
= 0.6524*a / -0.00987a rad
Or
cos θ = $r_{e\,/\,BM}$ (for small values of θ)
= $r_{e\,/}$ (a/7.8288) rad
Torque t = Fb. BG. sin θ
= 6400*a^3*0.1376*a* sin(θ) N-m

C. Buoyant Force on Cylindrical Wooden Block:

Width of cylindrical wooden block = a
Depth of cylindrical wooden block = a
Length of cylindrical wooden block = p
Volume of cylindrical wooden block = π*a^2*p/4 m^3
Specific weight of wood = 6.4 kN/m^3
For equilibrium the weight of water displaced =
Weight of cylindrical wooden block = 6.4*π*a^2*p/4 kN
= 5.0275*a^2*p kN
Volume of water displaced
= Weight of water displaced / Weight density of water
= 5.0275*a^2*p/9.81
= 0.5125*a^2*p m^3

Position of center of buoyancy:
We know that
Volume of cylindrical wooden block in water = Volume of water displaced
or
pi*a^2*h/4 = 0.5125*a^2*p
(where h = depth of wooden block in water)
Therefore,
h = 0.6525*p m
Hence,
Center of buoyancy OB from bottom
= 0.6525*p/2 m
= 0.3263*p m (from the base of the circular wooden block)
Center of gravity OG from base
= p/2
= 0.5*p m
BG = OG - OB = 0.1737*p m
Also
BM = I/V
where
I = area moment of inertia of a circular plan about axis YOY'
= pi/64*a^4 m^4
V = Volume of water displaced or Volume of wood in water
= 0.5125*a^2*p m^3
BM = I/V
= (pi/64)*a^4/(0.5125*a^2*p)
= 0.0958*a^2/p m
We know that the metacentric height,
GM = BM -BG
= 0.0958*a^2/p - 0.1737*p m
-ve sign means the metacenter (M) is below the center of gravity (G). Thus the cylindrical wooden block is in unstable equilibrium.
Time of oscillation:
T = 2*pi*√(GM/g)
T = 2*√[0.0958*a^2/p - 0.1737*p] seconds
or
Weight of cylindrical wooden block = buoyant force
c.v = Mg

0.02. w.r_e = 5027.5*a^2*p

w = (5027.5*p*a^2)/(0.02* r_e)

where r_e = effective radius of container

T = 2*pi/ w

T = 2*pi* r_e /(251.375*10^3*b*a^2)

List angle θ
tan θ = h/GM
= 0.6525*p / 0.0958*a^2/p - 0.1737*p rad
Or
cos θ = $r_{e / BM}$ (For small values of θ)
= $r_{e /}$ [0.0958*a^2/p] rad
Torque t = Fb. BG. sin θ
= 6400*a^3*0.1737*p * sin(θ) N-m

D. Buoyant Force on Spherical Wooden Block:

Width of spherical wooden block = a
Depth of spherical wooden block = a
Length of spherical wooden block = a
Volume of spherical wooden block = (4/3)*pi*a^3/2^3 m^3
Specific weight of wood = 6.4 kN/m^3
For equilibrium the weight of water displaced =
Weight of spherical wooden block
Weight of spherical wooden block = 6.4*(4/3)*pi*a^3/2^3 kN
= 3.349*a^3 kN

Volume of water displaced
= Weight of water displaced / Weight density of water
= 3.349*a^3/9.81
= 0.3414*a^3 m^3

Position of center of buoyancy:
We know that
Volume of spherical wooden block in water = Volume of water displaced
or
⅔*(π/8)*a^3 + pi/4*a^2*h = 0.3414*a^3
(where h = depth of wooden block in water from the C.G)
Therefore,
pi/4*a^2*h = 0.0796*a^3 m
h =0.1014*a m
Hence,
Center of buoyancy OB from bottom
= 0.1014*a + 0.5*a
= 0.6014*a/2
= 0.3007*a m (from the base of the spherical wooden block)
Center of gravity OG from base
= a/2
= 0.5*a m
BG = OG - OB = 0.1993*a m
Also
BM = I/V
where
I = area moment of inertia of a circular plan about axis YOY'
= ⅔*pi*a^4/2^4 m^4
= 0.1309*a^4 m^4
V = Volume of water displaced or Volume of wood in water
= 0.3414*a^3 m^3

BM = I/V
= (0.1309*a^4)/(0.3414*a^3)
= 0.3834*a m
We know that the metacentric height,
GM = BM -BG
= 0.3834*a - 0.1993*a
= 0.1841*a m
+ve sign means the metacenter (M) is above the center of gravity (G). Thus the spherical wooden block is in stable equilibrium.
Time of oscillation:

T = 2*pi*√(GM/g)

T = 2*√(0.1841*a) seconds

or

Weight of cylindrical wooden block = buoyant force

c.v = Mg

0.02. w.r_e = 3349*a^3

w = (3349*a^3)/(0.02* r_e)

where r_e = effective radius of container

T = 2*pi/ w

T = 2*pi* r_e /(167.450*10^3*a^3)

List angle θ
tan θ = h/GM
= 0.6014*a/0.1841*a rad
Or
cos θ = $r_{e / BM}$ (For small values of θ)
= r_e /[0.3834*a]rad
Torque t = Fb. BG. sin θ
= 6400*a^3*0.1993*a * sin(θ) N-m

E. Buoyant Force on Conical Wooden Block:

Radius of conical wooden block = R
Apex angle of conical wooden block = 2a
Height of conical wooden block = H
Volume of conical wooden block = ⅓*pi*R^2*H m^3
Specific weight of wood = 6.4 kN/m^3
Weight of conical wooden block
= 6.4*⅓*pi*R^2*H kN
= ⅓*pi*H^3*tan(a)^2*6.4 kN
Since,
tan(a) = R/H
R= H tan(a)
r = h tan(a)
For equilibrium the weight of water displaced =
Weight of conical wooden block = ⅓*pi*H^3*tan(a)^2*6.4 kN
Volume of water displaced
= Weight of water displaced / Weight density of water
= ⅓*pi*H^3^tan(a)^2*6.4/9.81 m^3

Position of center of buoyancy:
We know that
Volume of conical wooden block in water = Volume of water displaced
or

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⅓*pi*r^2*h= ⅓*pi*H^3*tan(a)^2*6.4/9.81 m^3;
⅓*pi*h^3*tan(a)^2=⅓*pi*H^3*tan(a)^2*6.4/9.81 m^3;
Simplifying,
h^3=H^3*6.4/9.81 m^3
(where h = height of wooden block in water from the apex of cone O)
Therefore,
h = 0.8673H m
Hence,
Center of buoyancy OB from apex
= ⅔*h
=0.5782 H m

Center of gravity OG from apex
= ⅔*H m
= 0.6667 H m
BG = OG - OB = (0.6667 - 0.5782)*H m
= 0.0885*H m
Also
BM = I/V
where
I = area moment of inertia of a circular plan of cone about axis YOY'
= pi*r^4/4 m^4
= pi*h^4*tan(a)^4/4 m^4
V = Volume of water displaced or Volume of wood in water
= ⅓*pi*H^3^tan(a)^2*6.4/9.81 m^3
BM = I/V
= 3*h^4/H^3*tan(a)^2*9.81/(4*6.4)
= 0.6505*H*tan(a)^2 m
We know that the metacentric height,
GM = BM -BG
= 0.6505*H*tan(a)^2 - 0.0885*H
= (0.6505*tan(a)^2 - 0.0885)*H m
-ve sign means the metacenter (M) is below the center of gravity (G). Thus the conical wooden block is in unstable equilibrium.
Time of oscillation
T = 2*pi*√(GM/g)
T = 2*√[(0.6505*tan(a)^2 - 0.0885)*H] seconds
or
Weight of cylindrical wooden block = buoyant force
c.v = Mg
0.02. w.r_e = 6400*⅓*pi*R^2*H
w = (6698.67*R^2*H)/(0.02* r_e)
where r_e = effective radius of container
T = 2*pi/ w
T = 2*pi* r_e /(334.93*10^3*R^2*H)
List angle θ
tan θ = h/GM
= 0.8673H/ (0.6505*tan(a)^2 - 0.0885)*H rad
Or
cos θ = $r_{e\,/\,BM}$ (For small values of θ)
= r_e /[0.6505*H*tan(a)^2] rad
Torque t = Fb. BG. sin θ
= 6400*a^3*0.0885*H * sin(θ) N-m

VI. SUMMARY TABLE

Solid Shape	Cuboid	Cube	Cylinder	Sphere	Cone
Centre of Gravity (OG) m	b/2	0.5*a	0.5*p	0.5*a	0.6667 H
List angle tan θ rad	0.6524*b / a^2/(7.8288*b) - 0.174*b	0.6524*a / -0.00987a	0.6525*p / 0.0958*a^2/p - 0.1737*p	0.6014*a/ 0.1841*a	0.8673H/ (0.6505*tan(a)^2 - 0.0885)*H
Height of the wooden block submerged in water (h) m	0.6524*b	0.6524*a	0.6525*p	0.6014*a	0.8673 H
Center of Buoyancy (OB) m	0.3262*b	0.3262*a	0.3263*p	0.3007*a	0.5782 H
Buoyant Force (Fb) kN	6.4*b*a^2	6.4*a^3	5.0275*a^2*p	3.349*a^3 kN	⅓*pi*H^3*tan(a)^2*6.4
Metacentric Height (GM) m	a^2/(7.8288*b) - 0.174*b	-0.00987a	0.0958*a^2/p - 0.1737*p	0.1841*a	(0.6505*tan(a)^2 - 0.0885)*H
Time of Oscillation (T) sec	a^2/(7.8288*b)	2*√(-0.00987a)	2*√[0.0958*a^2/p - 0.1737*p]	2*√(0.1841*a)	2*√[(0.6505*tan(a)^2 - 0.0885)*H]
Torque (t) N-m	6400*a^3 * 0.174*b* sin(θ)	6400*a^3*0.1376*a* sin(θ)	6400*a^3*0.1737*p * sin(θ)	6400*a^3*0.1993*a * sin(θ)	6400*a^3*0.0885*H * sin(θ)

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VII. FIGURES

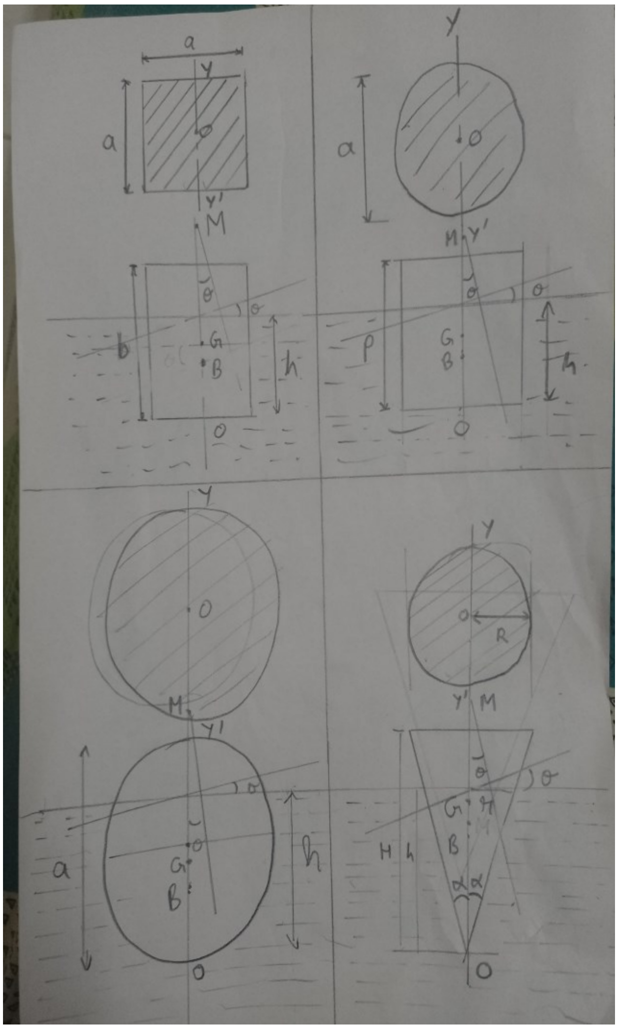

Fig 1: Shows the metacenter, list angle , C.G and center of buoyancy for various shapes of solids

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IX. REFERENCES

1. R.K.Rajput, "A Textbook of Fluid Mechanics" S. Chand publications, pp. 125-153, Reprint 2003.
2. R.S. Khurmi , Theory of Machines, S. Chand Publications, pp. 66-85, 3rd Edition, 2003.
3. PSG, Design Data Handbook, Kalaikathir Achchagam Coimbatore, pp. 6.1-6.32, 2003 Edition.