Buckling Analysis Of Straight Helical Compression Springs Made Of ASTM A229 Gr-II, ASTM A 313 Materials (Type 304 & 316).

Rajkumar V. Patil[1], Dr. P. Ravinder Reddy[2] and Dr. P. Laxminarayana[3]

[1]*Department of Mechanical Engineering, Rajiv Gandhi Institute of Technology,Mumbai 400053, India.*

[2]*Department of Mechanical Engineering, Chaitanya Bharati Institute of Technology, Hyderabad 500075, India*

[3]*Dept. of Mechanical Engg., University College of Engineering, Osmania University,Hyderabad 500007, India.*

Abstract

The coil compression springs will have a tendency to buckle when the deflection (for a given free length) becomes too large and thereby spring can no longer provide the intended force. Though the buckling is mainly depends upon their geometrical properties rather than their material properties, an attempt has been made to confirm experimentally the results obtained previously by different researchers and to carry out analyses with springs made of different materials for their suitability in various applications. With this analysis it will be possible to provide valuable comparisons on the critical relative compression and buckling loads between springs made of commonly used materials.

Keywords: helical spring, buckling, critical relative compression, spring index, squareness, Parallelism, spring stiffness, helix angle, slenderness ratio.

Nomenclature

L_f	Free length or unloaded length of spring
D_m	Mean diameter of the coil spring
α_o	Uncompressed helix angle
p	pitch of the coil spring
k	stiffness of the spring
E	Modulus of elasticity
G	Modulus of rigidity or shear modulus
ξ	Critical relative compression
δ_{cr}	critical deflection
n	Total number of coils in spring
n'	Active number of coils in spring
d	wire diameter of the spring
e_1	Deviation in squareness
e_2	Deviation in parallelism
P	Static axial compressive load
Ψ	Deflection factor
υ	Poisson's ratio
δ	axial deflection of spring

1. Introduction

Buckling of spring refers to its deformations in non-axial (lateral) direction under compression. Compression coil springs will buckle when the free length of the spring is larger and the end conditions are not proper to evenly distribute the load all along the circumference of the coil. The coil compression springs will have a tendency to buckle when the deflection (for a given free length) becomes too large and thereby spring can no longer provide the intended force. Once buckling starts, the off-axis deformation typically continues rapidly until the spring fails. As a result, it is important to design compression springs such that their likeliness to buckle is minimized.

Research to date, shows that the buckling of springs mainly depends on the ratio of the initial spring length (L_f) to the coil diameter D_m and on the method of attaching the spring ends. However, the detail study indicates that, the buckling of springs is also depends on the following factors:

1) Spring coil ends - ground ends, parallel ends, non-parallel ends.
2) End coil fixity (end configuration) – fixed and / or free ends.
3) Off sets between end coil centers.
4) Arrangement of springs- equal span linearly and circumferentially.
5) Helix angle (α)
6) pitch (p)
7) Expansion of coil diameter.
8) Spring index (D/d).
9) Variable radius of curvature of each turn
10) Spring Stiffness (k)
11) The factors, depending on spring wire material, which affect buckling of springs are;
    a) Modulus of elasticity 'E' N/mm^2
    b) Modulus of rigidity 'G' N/mm^2

Though the buckling is mainly depends upon their geometrical properties rather than their material properties, an attempt has been made in this paper firstly, to confirm experimentally the results obtained previously by different researchers and to carry out analyses with springs made of different materials for their suitability in various applications. With this analysis it will be possible to provide valuable comparisons on the critical relative compression and buckling loads between springs made of commonly used different materials.

Theoretical calculation with respect to the elastic stability of helical compression springs of circular wire section by J.A.Haringx[5] shows that spring will buckle when the critical relative compression $\xi \geq 5.24$ (or 2.62 in case of both spring ends being hinged or constrained parallel i.e., only free to move in a lateral direction without any rotation). A.M.Wahl[9] , who summarized the earlier work on this subject by Haringx[5], has given the formula(1) for the critical buckling deflection of a compressive spring with **fixed ends** as

$$\frac{\delta_{cr}}{L_f} = 0.812[\,1 \pm \sqrt{1 - 6.87\left(\frac{2D_m}{L_f}\right)^2} \qquad (1)$$

Theory related to buckling behaviours of helical springs presented by D.Pearson[2] implies that the buckling occurs when $L_f / D_m \approx 3.86$ for both ends fixed. This neither agrees with the elementary theory by Haringx[5] nor with the model given by L.E.Becker and W.L.Cleghorn[7]. Buckling curve given by J.E.Mottershead[6] has no limiting value for L_f/D_m other than at full compression and his graph implies that springs can buckle up to $L_f/D_m \approx 4$. Hence his result show poor agreement with those by Haringx and L.E.Becker & W.L. Cleghorn. To confirm all of the above results obtained in various findings, it is proposed to carry out the experiments to verify them testing the springs made of different materials such as Spring Steel Wire, Grade –II, ASTM A229 (Equiv. as per IS 4454 of 1981). *Refer* [9],[8] (C-0.72 %, Mn-0.69%, Si-0.21%, P-0.018%, S-0.019%. for a size of 2mm). Stainless Steel Wires (SS 304, i.e., ASTM A 313, Type 304) *Refer* [9],[3] (C-0.08% max, Mn-2% max, Si-1% max,Cr-18 – 20%, Ni- 8-12%, P-0.045max,S-0.030%max. for a size of 2 mm). Stainless Steel Wires (SS 316, i.e., ASTM A 313, Type 316) *Refer* [9],[3] (C-0.08%max, Cr-16-18%, Ni-10-14%, Mn-2%, Si-1%,P-0.045% max, S-0.030% max, Mo-2-3%. for a size of 2 mm). *Refer* [9],[3].

2.TEST-RIG FOR EXPERIMENTATION

Figure 1. Test rig for experimentation

To analyze the behavior of helical compression spring, the test-rig (fig.1) has been developed and fabricated in the institute laboratory. This spring testing machine is capable of taking load of 300 N.

Test Rig Specifications:
Max. Height – 315 mm
Max. Diameter - 150 mm

Springs Specifications:
Outer diameter(D_o) of each coil : 20 mm
Wire diameter(d) of each coil : 2 mm
Free length (L_f) : ranging 95 – 170 mm (set of seven springs of each of the above three materials).
No.of coils(n') : ranging 15 – 25
Coil ends : Squared and ground.
Helix angle : ranging $5^0 – 12^0$ and
Spring index, $D_m / d = 9$

Some of the tests are carried out in industries, where the test-rigs (fig. 2 & 3) have been used: Capacity : 3000 N

Figure 2. Testing for buckling.


Figure 3. Checking for surface cracks by exposing to UV lights.

The rest-rigs (fig.2) are having two parallel plates in between which the springs are compressed to test for their buckling. The axial load applied on the spring can be directly read from the digital display and corresponding spring deflection be noted from another similar digital indicator. The critical buckling load may be noted at the stage where it remains more or less constant and the corresponding excessive lateral deflection could be noticed. Thereafter, the springs are further tested for detection of surface cracks (fig.3) if any by exposing them to ultra violet light.

The pitch (p) and the helix angle (α) of all the twenty one springs are measured in the laboratory on the profile projector as shown in figure 4.

Specifications of Profile Projector(fig4)**:**
Magnification-10x
Field View – 25 mm
Cross travel stage size – 125 x 125 mm Table travel upto 50 x 50 mm,

Figure 4. Profile-Porjector

A set of seven springs made of each of the above specified materials have been tested and the test results of all twenty one springs have been given in table1 through table3.

Table 1. Critical relative compression of the spring with L_f/D_m ratio and critical load.

Material : ASTM A229, Gr-II; OD= 20mm; WD=2 mm ; E=207 kN/mm2, G=79.3 kN/mm2 ; E/G = 2.61			
		Critical relative compression ($\xi = \delta_{cr} / L_f$)	
L_f/D_m Ratio	Critical Load (N)	Experimental approach	Theoretical approach
5.42222	8.3x10	0.563524	0.60546
5.75	7x10	0.454106	0.479036
6.7277	4.4x10	0.305532	0.309805
7.31666	3.4x10	0.235383	0.245871
7.89444	2.7x10	0.190001	0.205145
8.88888	2.2x10	0.1375	0.15644
9.68333	1.7x10	0.126219	0.129453

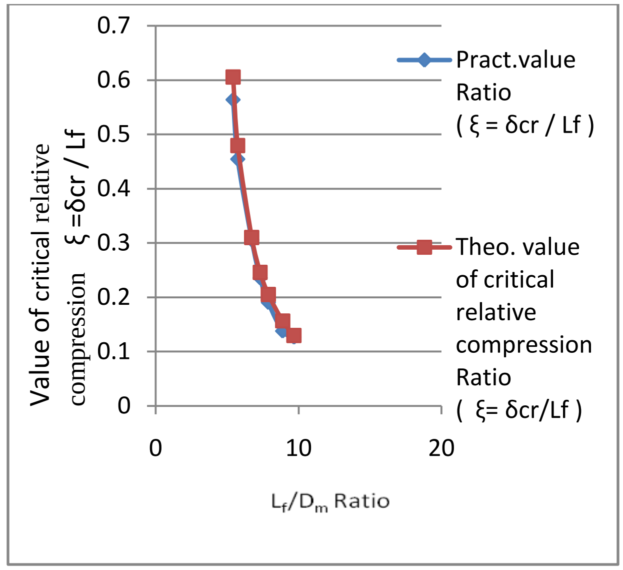

Fig 5. Critical relative compression of the spring versus L_f/D_m Ratio

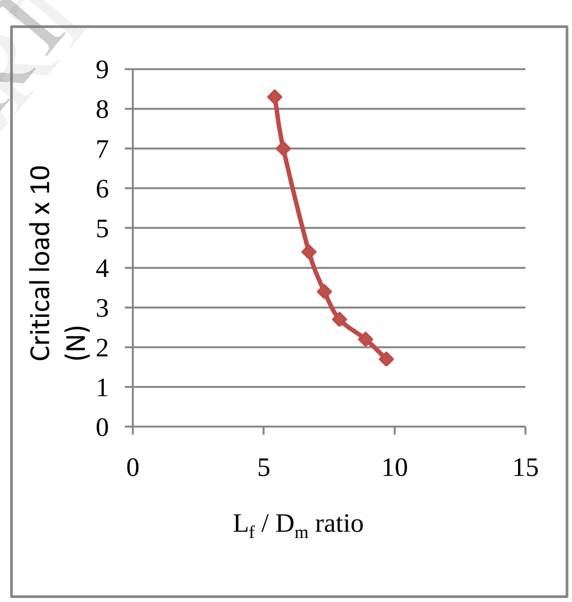

Fig 6. Critical load versus L_f/D_m Ratio

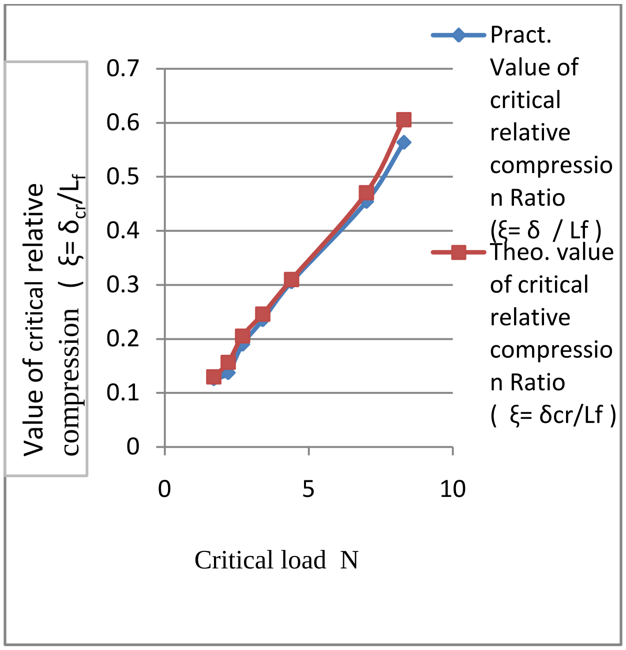

Fig 7. Critical relative compression of the spring versus critical load

The test results indicates that, the buckling load will decrease as L_f/D_m ratio increases non-linearly. For an average critical deflection of 0.2874 (28.74%) for the springs having slenderness ratio (L_f/D_m) between 5.422 to 9.683, an average buckling load has been found to be 42.4 N. From figure 6 & 7, one can decide the operating load for the required deflection to avoid the buckling of springs.

Table 2. Critical relative compression of the spring with L_f/D_m ratio and Critical load.

Material : ASTM A 313, Type 304 (SS-304); OD= 20mm; WD=2 mm ;E=187.5 kN/mm^2, G=70.3 kN/mm^2 ; E/G = 2.667			
L_f / D_m Ratio	Critical Load (N)	Critical relative compression (ξ= δ_{cr} / L_f)	
		Experimental approach	Theoretical approach
5.427	7.45x10	0.5783	0.60238
5.7	6.23x10	0.438596	0.49386
6.8277	3.7x10	0.284783	0.292134
7.3722	2.7x10	0.211002	0.244751
8.15	2.45x10	0.184049	0.19052
9.03333	1.9x10	0.135301	0.15091
9.5277	1.7x10	0.128279	0.1371688

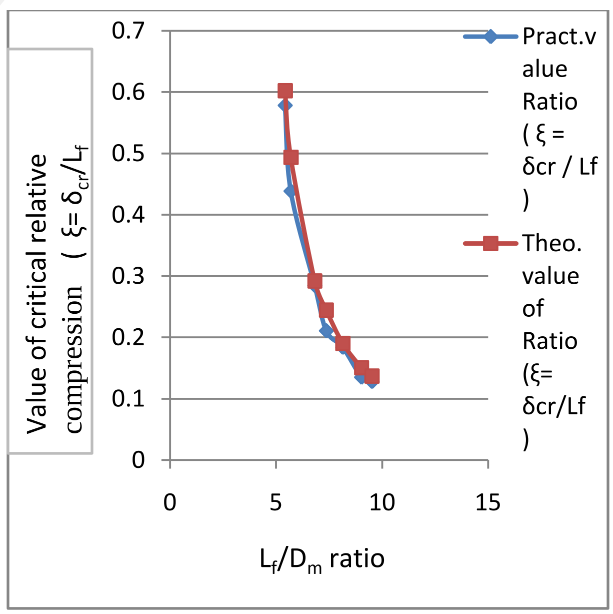

Figure 8. Critical relative compression versus L_f/D_m ratio

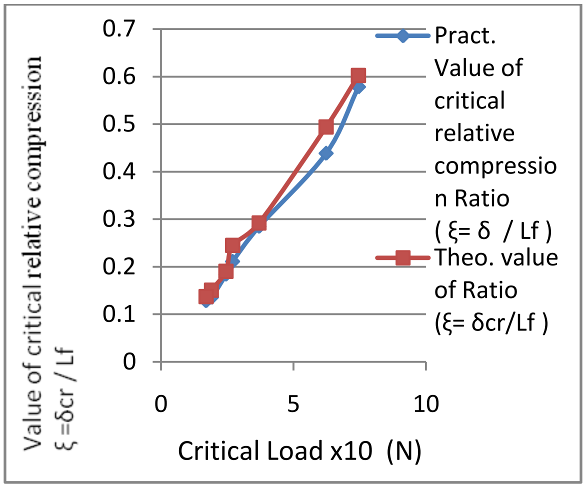

Fig 9. Critical relative compression of the spring versus critical load

Table 3. Critical relative compression of the spring with L_f/D_m ratio and Critical load.

Material : ASTM A 313, Type 316 (SS-316); OD= 20mm; WD=2 mm ; E=187.5 kN/mm^2, G=70.3 kN/mm^2 ; E/G = 2.667			
L_f/D_m Ratio	Critical Load (N)	Critical relative compression ($\xi=\delta_{cr} / L_f$) Experimental approach	Theoretical approach
5.27777	7x10	No Buckling	No Buckling
5.88888	5.8x10	0.443396	0.442672
6.68888	3.6x10	0.315614	0.31307
7.52222	2.6x10	0.217872	0.233083
7.86666	2.5x10	0.201271	0.20951
9.01111	1.625x10	0.135635	0.153499
9.55555	1.42x10	0.122093	0.13476

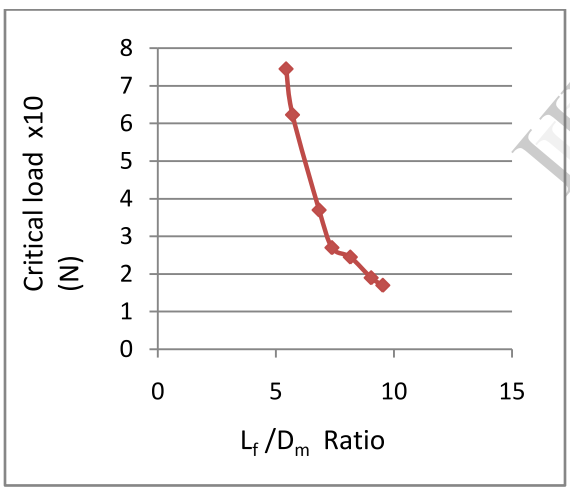

Figure 10. Critical load versus L_f/D_m Ratio

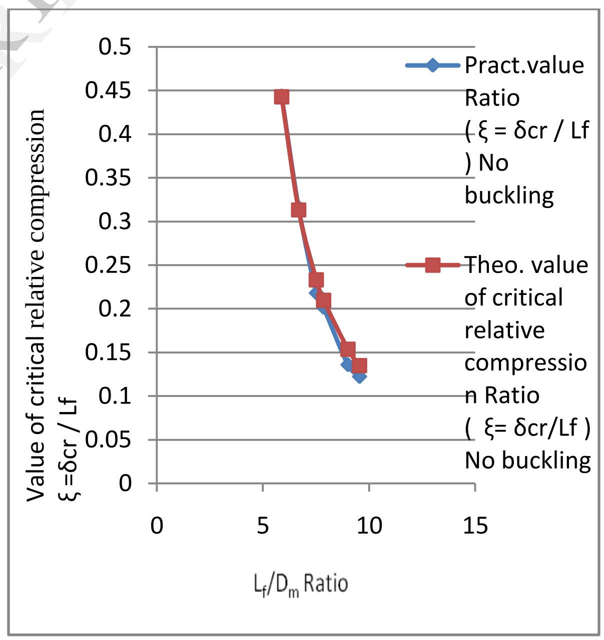

Figure 11. . Critical relative compression of the spring versus L_f/D_m Ratio

The test values indicate that, the buckling load will decrease as L_f/D_m ratio increases non-linearly. For an average critical deflection of 0.28 (28%) for the springs having slenderness ratio (L_f/D_m) between 5.427 to 9.5277, an average buckling load has been found to be 37.32 N. From the figure 9 & 10, one can decide the operating load for the required deflection to avoid the buckling of springs.

Table4. Comparison between various springs for their critical relative compression and buckling loads

Material	L_f/D_m Ratio	Critical or Buckling Load(N)	Practical value of percentage of critical deflection	Theoretical value of percentage of critical deflection
Spring Steel Wire, ASTM A229 Grade –II	5.42222	8.3x10	0.563524	0.60546
	5.75	7x10	0.454106	0.479036
	6.7277	4.4x10	0.305532	0.309805
	7.31666	3.4x10	0.235383	0.245871
	7.89444	2.7x10	0.190001	0.205145
	8.88888	2.2x10	0.1375	0.15644
	9.683333	1.7x10	0.126219	0.129453
ASTM A 313, Type 304 (SS-304)	5.427	7.45x10	0.5783	0.60238
	5.7	6.23x10	0.438596	0.49386
	6.8277	3.7x10	0.284783	0.292134
	7.3722	2.7x10	0.211002	0.244751
	8.15	2.45x10	0.184049	0.19052
	9.03333	1.9x10	0.135301	0.15091
	9.5277	1.7x10	0.128279	0.1371688
ASTM A 313, Type 316 (SS-316)	5.27777	7x10	No Buckling	No Buckling
	5.88888	5.8x10	0.443396	0.442672
	6.68888	3.6x10	0.315614	0.31307
	7.52222	2.6x10	0.217872	0.233083
	7.86666	2.5x10	0.201271	0.20951
	9.01111	1.625x10	0.135635	0.153499
	9.55555	1.42x10	0.122093	0.13476

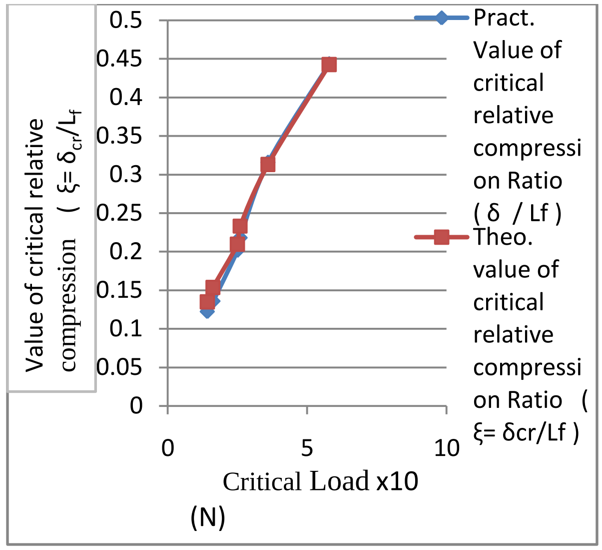

Figure 12. . Critical relative compression of the spring versus critical load

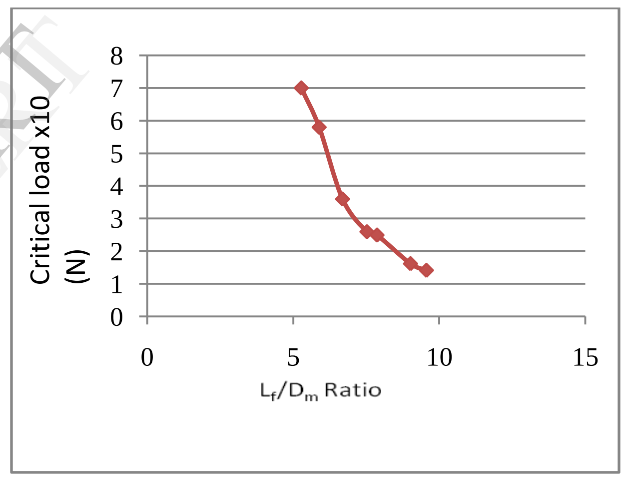

Figure 13. Critical load versus L_f/D_m Ratio

Similarly, these test results shows that, the buckling load will decrease as L_f/D_m ratio increases non-linearly. For an average critical deflection of 0.2803 (28.03%) for the springs having slenderness ratio (L_f/D_m) between 5.277 to 9.5555, an average buckling load has been found to be 35.06 N. Looking at the figure 12 & 13. it will help in deciding the operating load for the required deflection to avoid the buckling of springs.

3. Results and Discussion:

The test results are very close to the hypothetical values as indicated by equation (1).

This equation(1) was derived for a materials having E/G ratio as 2.6. However, it won't give the correct values of critical compression ratio (ξ) for materials having little higher values of E/G. and the condition of buckling L_f /D_m = 5.24 for symmetrical deflection for material having E/G=2.667 will get shifted to little higher side of L_f /D_m ratio for fixed ends conditions. From the test results it is very clear that for nearly same average critical deflection, the percentage difference in buckling load between the springs under test has been found to be :

a) Between springs made of ASTM A 313, Type 304 (SS-304) & ASTM A 313, Type 316 (SS-316) = 6.5 %
b) Between springs made of ASTM A229, Gr-II & ASTM A 313, Type 304 (SS-304) = 13.5%
c) Between springs made of ASTM A229, Gr-II & ASTM A 313, Type 316 (SS-316) = 20.5%

The close deviation between practical and theoretical results is mainly due to

1) Exceeding the limiting values in

(a) Tolerance on squareness of unloaded springs (1975)

(b) Deviation in parallelism of squared and ground ends(1975).

The limiting value of tolerance on squareness and parallelism of ground faces shall be(1975)

Deviation in squareness, $e_1 = 0.03 \times L_f$.
(1.7^0).

Deviation in parallelism, $e_2 = 0.02 \times D_m$.
(1.15^0).

But the measured average value of these two deviations in the above twenty one springs lies around 2.1^0 and 1.5^0 respectively. If a compression spring having some off sets between end coil centers is compressed between two parallel plates, it is found that in general the resultant load is displaced from the spring axis by a small amount, e. The effect of this eccentric loading is to increase the stress on one side and decrease it on the other side of its axis and thereby resulting into the buckling of spring elements.This eccentric loading will cause certain percentage difference between the theoretical and practical values of deflections.

(2) End coil fixity (end configuration) – fixed and / or free ends.

In the experiment, the ends of the springs being compressed between parallel plates are taken as fixed ends. The test results shows somewhat lower buckling load than would be obtained using the theoretical calculations. The reason for this is that the spring ends are not perfectly fixed as assumed in the theory, since some flexibility is generally present.

(3) pitch and pitch angle of the spring.

A more exact analysis by Ancker and Goodier(1958) for springs deflection taking pitch angle into account yields the formula,

$$\delta = \left(\frac{8PD_m^3}{Gd^4}\right)\psi \qquad (2)$$

Where deflection factor,

$$\psi = 1 - \frac{3}{16\,c^2} + \frac{(3+\upsilon)\tan^2\propto_o}{2(1+\upsilon)}$$

and

$$Poisson's\ ratio,\ \upsilon = 0.3$$

From this, it is clear that for spring indexes(c) greater than about 4 and for pitch angles ($\propto_o$) less than 5^0, the error is less than about 1 ½ per cent. For small indexes and small pitch angles, deflections are slightly less than those figured from the usual formula; however, for large-index springs, rather large deflections are possible without excessive stress. In the present investigation, the values of pitch angle of the various springs is ranging from 5^0 to 9^0. This factor also would cause certain difference in theoretical and practical relative critical compression (ξ).

(4) Variation of material characteristics of the springs when they are used in different arrangement in parallel combination of them would also results into difference in practical and theoretical values.

(5) Expansion of coil diameter.

Since the spring deflection, other things being equal, is proportional to the cube of the coil radius, it follows that the spring becomes more flexible as it is compressed. If the spring deflection per turn becomes large, the effect due to change in coil radius is more pronounced in such springs(1963). This change will also cause difference between theoretical and practical values of deflections. Change in coil diameter is given by(1963)

$$\frac{\Delta D}{D} = 0.05 \left(\frac{p^2 - d^2}{D^2}\right) \qquad (3)$$

Where, ΔD – Change in coil diameter.
D - Initial mean coil diameter.
p – pitch
d – wire diameter

Thus, the reason could be many more in addition to the above factors.

4. Conclusions

From the above test results it confirms that the practical values deflections and buckling loads are in better agreement with that of the hypothetical values indicated by equation (1). However, it implies that for almost same average critical deflection and range of L_f/ D_m ratio, the average percentage difference in buckling loads between springs made of ASTM A229, Gr-II and ASTM A 313 has been found to be 13.5 % to 20.5% . Although the practical and theoretical tests data are in good agreement, the buckling of above springs has occurred before reaching their theoretical critical deflection. Comparative statement of the test results would help to understand the relative buckling loads and propose a suitable springs according to their suitability in various applications. From this experimentation, based on the requirement of deflections and operating loads, appropriate springs can be selected to transmit effectively the maximum load without any buckling of springs.

Acknowledgment

The authors would like to thank Modern Engineerng & Spring Co (MESCO) Industries and especially Mr.Nilesh Shah for his support to this experimental work and M.Coil Spring Manufacturing Co. for providing necessary test springs samples.

References

[1] C.J.Ancker and J.N.Goodier, Theory of pitch and curvature corrections for the helica springs-I(Tension) J.Appl.Mech., December, p.471, 1958.

[2] D.Pearson., The transfer matrix method for the vibration of compressed helical springs .J. Mech. Sci., 24, 163-171, 1982.

[3] Harold, Carlson, Spring desiner's handbook, Marcel Dekker,Inc., New York 10016, pp 71-74, 1978.

[4] Indian Standard, IS:7906(Part II).1975 (Reaffirmed 2004), Edition1.1 (1990-05), pp 3,10. 1975.

[5] J.A.Haringx., Elastic stability of helical springs at a compression larger than original length.Applied Sci. Res. A1,417-434, 1948.

[6] J.E.Mottershead, The large displacements and dynamic stability of springs using helicalfiniteelements,Int.J.Mech.Sc.vol.24 ,No.9, pp 547-558, 1982.

[7] L.E. Becker and W.L. Cleghorn., On the buckling of helical compressionsprings., J.Mech.Sci.Vol.34,No.4,pp 275-282, 1992.

[8] Robert E.Joerres., Standard handbook of machine design, part-2, McGraw-Hill (digitalengineeringlibrary.com), pp 6.8 – 6.20. 2004.

[9] Wahl A.M., Mechanical springs, 2nd edition,McGraw-Hill, New York, 1963.