Buckling Analysis of Plates with Rectangular Cutouts

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Abstract - Mechanical analyses were performed on rectangular plates with central rectangular cutouts. The finite-element structural analysis method was used to study the effects of plate-support conditions, plate aspect ratio, hole geometry, and hole size on the mechanical buckling strengths of the aluminum 6061–T6 plates. The compressive-buckling strengths of the plates could be increased considerably only under certain boundary conditions and aspect ratios. The plate-buckling mode can be symmetrical or antisymmetrical, depending on the plate boundary conditions, aspect ratio, and the hole size. The experimental values are in good agreement with the finite element solution. For a particular aspect ratio the critical buckling load values are less because of the cutout orientation (60°) and applied boundary condition (SFSF).

Key words: Buckling, FEA, cutouts, critical buckling load, boundary conditions.

I. INTRODUCTION

The plates are being extensively used in civil, marine, aeronautical and mechanical engineering applications. Different parts of aero planes are idealized by introducing substitute structures in the form of plates with cutouts to reduce the weight. Complex alignment problems in bridge design arising from functional, aesthetic or structural requirement are often solved by the use of plates with cutouts. Other applications of plates with different cutouts can be found in ship hulls, parallelogram slabs in buildings, etc.

The plate with holes or cutouts is mainly considered in the above said fields just to reduce the weight and to give the same effect as that of parent metals. This analysis mainly concentrated on the plates with rectangular cutouts with varying aspect ratios and effect of orientation of the cutouts too. In many industries this problem was considered as one of the major problem in designing the main components which will plays important role in sharing loads in the structures.

II. MATERIALS AND METHODS

A. Introduction

The methods used for determining the critical buckling loads of isotropic plates with rectangular cutouts under uniaxial compression are (i) buckling experiment in Computerized Universal testing machine (UTM) and (ii) finite element analysis using MSC/NASTRAN. The details of the test specimens used in the experimental methods are also presented.

B. Test specimens used in the experimental studies

Isotropic plate specimens (with circular holes) made of Aluminum 6061-T6 was used. Each specimen had one rectangular cutout with varying aspect ratio between 1, 1.5, 2 and 2.5. The cutouts are located at the center of each plate specimen. The material was supplied by D S Srinivasa Shetty and Company, Shivamogga. The material properties of the isotropic plates made of Aluminum 6061-T6 is: E = 69 GPa, µ = 0.33 and ρ = 2700 kg/m³ and these data were supplied by the manufacturer. The values provided by the manufacturer were verified by conducting experiments as per ASTM standards. The experimental values obtained were quite close to those supplied by the manufacturer and hence the values given by the manufacturer were adopted. The isotropic assumption was verified by conducting experiments. The thickness of all the isotropic skew plate specimens is 2 mm. The values of a and b for various values of aspect ratio for plate and rectangular hole are given in Table 1.

<table>
<thead>
<tr>
<th>a/b</th>
<th>a</th>
<th>b</th>
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</thead>
<tbody>
<tr>
<td>1.0</td>
<td>100mm</td>
<td>100mm</td>
</tr>
<tr>
<td>1.5</td>
<td>150mm</td>
<td>100mm</td>
</tr>
<tr>
<td>2.0</td>
<td>200mm</td>
<td>100mm</td>
</tr>
<tr>
<td>2.5</td>
<td>250mm</td>
<td>100mm</td>
</tr>
</tbody>
</table>

C. Mounting of Strain gauge:

Step 1: Selection of material
Step 2: Surface preparation
Step 3: Marking
Step 4: Cleaning of prepared surface
Step 5: Placing the strain Gauge
Step 6: Pasting the adhesive material to specimen and strain gauge
Step 7: Soldering the strip wires to the mounted strain gauge

D. Experimental setup and procedure

The fixtures for holding the test specimens are shown in Figures 1. The test specimen was inserted between the end plates of the fixture so that no slippage of the test specimen
occurs. The tests were conducted in a 1000 kN, computerized universal testing machine after positioning properly the test specimen using universal vice as shown in Figure 2.

Figure 1: Fixtures for holding skew plate specimen

Figure 2: Experimental Test Setup

The measuring instrumentation consists of strain gages. The strain gages were placed at each surface of the test specimen at midspan. ASTM E1237 - 93(2009) were followed for installing bonded resistance strain gages. The computerized universal testing machine was used to measure the in-plane deflection. The testing was carried out with one loaded edge restrained completely and the other loaded edge restrained except translationally in the direction of loading.

E. Experimental determination of critical buckling load

The different procedures used to evaluate the critical buckling load of plates are described in and depicted in Figure 3.

![Methods used to determine critical buckling load experimentally](image)

The procedures use applied load versus deflection, applied load versus end shortening and applied load versus strain plots. In the present study, five different methods are used which are designated as Method I, Method II etc. Method I employs a plot of applied load (P) versus out-of-plane deflection (W) at midspan. Method II employs a plot of applied load (P) versus end shortening (Δ) in the direction of applied load. Method III employs a plot of applied load (P) versus square of out-of-plane deflection (W²).

The Method IV utilizes the fact that the surface strain on one side of the specimen becomes tensile when the specimen has buckled. In this method, the applied load is plotted against the algebraic mean strain, \( \varepsilon_A = (\varepsilon_1 + \varepsilon_2)/2 \), where \( \varepsilon_1 \) and \( \varepsilon_2 \) are strains at the two surfaces of the specimen at midspan in the direction of loading.

The Method V employs a plot applied load versus strain difference \( \varepsilon_D = (\varepsilon_1 - \varepsilon_2) \). In this experimental work method II and Method IV have been used to get the experimental critical buckling load for the plates with rectangular cutout specimens.

F. Finite element analysis

In linear static analysis, a structure is assumed to be in a state of stable equilibrium. As the applied load is removed, the structure is assumed to return to its original, undeformed position. Under certain combinations of loadings, however, the structure continues to deform without an increase in the magnitude of loading. In this case the structure has become unstable; it has buckled. For elastic, or linear, buckling analysis, it is assumed that there is no yielding of the structure and that the direction of applied forces does not change.

Elastic buckling incorporates the effect of the differential stiffness, which includes higher order strain displacement relationships that are functions of the geometry, element type, and applied loads. From a physical standpoint, the differential stiffness represents a linear approximation of softening (reducing) the stiffness matrix for a compressive axial load and stiffening (increasing) the stiffness matrix for a tensile axial load.

In buckling analysis, the equations are solved for the eigenvalues that are scale factors that multiply the applied load in order to produce the critical buckling load. In general, only the lowest buckling load is of interest, since the structure will fail before reaching any of the higher order buckling loads. Therefore, usually only the lowest eigenvalue needs to be computed.
The buckling eigenvalue problem reduces to:

\[ [K] + \lambda [K_d] = 0 \quad (1) \]

Where \( K \) is the system stiffness matrix, \( K_d \) is the differential stiffness matrix (generated automatically by MSC/NASTRAN, based on the geometry, properties, and applied load), and \( \lambda \) are the eigenvalues to be computed. Once the eigenvalues are found the critical buckling load is calculated by using the equation:

\[ P_{cr} = \lambda P \quad (2) \]

Where, \( P_{cr} \) are the critical buckling loads and \( P \) are the applied loads.

In the present work, finite element method is employed to obtain the critical buckling load using MSC/NASTRAN software. The buckling analyses of isotropic skew plates are performed using CQUAD4 elements. The CQUAD4 element is a four-node plate element having six degrees of freedom/node (translational \( (u, v, w) \) and rotational \( (\theta_x, \theta_y, \theta_z) \)).

Figure 4 shows the finite element meshes of plate with global. \( u \) and \( v \) are the displacement components in the global \( x \) and \( y \) directions respectively.

III. RESULTS AND DISCUSSION

A. Isotropic plates with rectangular cutouts

Isotropic plates with rectangular cutouts were tested under uniaxial compression, varying the aspect ratios from 1.0 to 2.5. The experimental values of the critical buckling load were determined in accordance with the Methods II and V. Figure 5 shows a typical plot from which the experimental critical buckling load has been determined according to the Method II for a plate with aspect ratio 1.0 and rectangular hole size 1 and orientation ranging \( 0^\circ \) to \( 60^\circ \). Figure 6 shows a typical plot from which the experimental critical buckling load has been determined according to the Method V for a plate with aspect ratio 1.0 and rectangular hole size 1 and orientation ranging \( 0^\circ \) to \( 60^\circ \). Figure 7 shows a typical buckled shape of isotropic plate specimen with rectangular cutouts.

Classical linear buckling analysis was performed and the finite element solution for the critical buckling load determined. The values of the critical buckling load obtained are tabulated in Table 2 through 5.

<table>
<thead>
<tr>
<th>Angle of Orientation</th>
<th>Critical Buckling Load (kN)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>FEM</td>
</tr>
<tr>
<td>0°</td>
<td>19.350</td>
</tr>
<tr>
<td>30°</td>
<td>19.642</td>
</tr>
<tr>
<td>45°</td>
<td>19.354</td>
</tr>
<tr>
<td>60°</td>
<td>19.353</td>
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</tbody>
</table>
Table 3: Critical buckling load for plate with aspect ratio 1.5, cutout aspect ratio = 1.5

<table>
<thead>
<tr>
<th>Angle of Orientation</th>
<th>FEM</th>
<th>Method II (P v/s δ)</th>
<th>Method V (P v/s ε)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>8.551</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td>30°</td>
<td>8.551</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td>45°</td>
<td>8.550</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td>60°</td>
<td>8.550</td>
<td>8.36</td>
<td>8.36</td>
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</tbody>
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Table 4: Critical buckling load for plate with aspect ratio 2, cutout aspect ratio = 2

<table>
<thead>
<tr>
<th>Angle of Orientation</th>
<th>FEM</th>
<th>Method II (P v/s δ)</th>
<th>Method V (P v/s ε)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>4.780</td>
<td>3.5</td>
<td>3.5</td>
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<tr>
<td>30°</td>
<td>4.780</td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>45°</td>
<td>4.780</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>60°</td>
<td>4.779</td>
<td>3.5</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 5: Critical buckling load for plate with aspect ratio 2.5, cutout aspect ratio = 2.5

<table>
<thead>
<tr>
<th>Angle of Orientation</th>
<th>FEM</th>
<th>Method II (P v/s δ)</th>
<th>Method V (P v/s ε)</th>
</tr>
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<tbody>
<tr>
<td>0°</td>
<td>3.042</td>
<td>2.5</td>
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<tr>
<td>30°</td>
<td>3.042</td>
<td>2.95</td>
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<tr>
<td>45°</td>
<td>3.042</td>
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<tr>
<td>60°</td>
<td>3.041</td>
<td>2.75</td>
<td>2.75</td>
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From Table 2 and Figure 5, the following observations are made:

- Method II and V yield the highest experimental value for critical buckling load.
- The experimental values are in good agreement with the finite element solution.
- For a particular aspect ratio, the critical buckling load is observed to decreases with orientation of rectangular cutout.

Figure 8: Applied load versus in plane deflection (Method II) for isotropic plates (Aspect Ratio = 1.5)

Figure 9: Applied load versus in plane deflection (Method II) for isotropic plates (Aspect Ratio = 2)

Figure 10: Applied load versus in plane deflection (Method II) for isotropic plates (Aspect Ratio = 2.5)

Figure 11: Applied load versus Average Strain (Method V) for isotropic plates (Aspect Ratio = 1.5)

Figure 12: Applied load versus Average Strain (Method V) for isotropic plates (Aspect Ratio = 2)
From figures 8 through 13 following observation have been made:

- Experimental results are in good agreement with FEM results.
- For a particular aspect ratio the critical buckling load values are less because of the cutout orientation (60°) and applied boundary condition (SFSF).
- The experimental results are varying in nature because of the simply supported – free – simply supported – free boundary condition.

CONCLUSIONS

- The critical buckling load of a plate reduces when holes are introduced.
- Method II and V yield the highest experimental value for critical buckling load.
- The experimental values are in good agreement with the finite element solution.
- For a particular aspect ratio the critical buckling load values are less because of the cutout orientation (60°) and applied boundary condition (SFSF).
- This experience was a great opportunity to plunge into the area of experimental and numerical research and has considerably enhanced the confidence and research abilities of the author and has been unique.

REFERENCES


