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Blind Carrier Frequency Offset Estimation Based on Gravitational Search Algorithm for Interleaved OFDMA Uplink Systems

Ann-Chen Chang
Department of Information Technology
Ling Tung University
Taichung, Taiwan 408, R.O.C.

***Abstract*—This study deals with carrier frequency offset (CFO) estimation for interleaved orthogonal frequency division multiple access (OFDMA) uplink systems. Firstly, the gravitational search algorithm (GSA) with the center-symmetric (CS) trimmed correlation matrix and the multiple signal classification (MUSIC) criterion is presented for the purpose of efficient estimation. It has been shown that the estimate accuracy of the searching-based estimator strictly depends on the number of search grids used during the peaks searching process, which is time consuming and the required number of search grids is not clear to determine. However, the searching grid size is no need to know previously for the proposed GSA-based approach. Meanwhile, the advantage of inherent interleaved OFDMA signal structure also is exploited to conquer the problems of local optimization and the effect of ambiguous peaks for the proposed estimators. Finally, several simulation results are provided for illustration and comparison.**

***Keywords*— *Carrier Frequency Offset; Interleaved OFDMA; Gravitational Search Algorithm; Multiple Signal Classification*.**

I. INTRODUCTION

Emerging as a promising technology for next-generation wireless communications, the orthogonal frequency division multiple access (OFDMA) has drawn a lot of attention due to its high bandwidth efficiency and robustness to narrowband interference. OFDMA inherits from orthogonal frequency division multiplexing the weakness of being sensitive to inaccurate frequency references [1]. Inaccurate carrier frequency offset (CFO) will disrupt the orthogonality among subcarriers and lead to inter-carrier interference as well as multiple access interference (MAI), which degrades the system performance. In OFDMA uplink systems, every user transmits its signal to base station (BS) through different channel, thus the received signals at the BS are the mixture of all users. Due to the different propagation delays and channel attenuations, each user has different time delay and CFO at the receiver [2]. The estimation of time delays and CFOs all become multiple-parameter estimation problems. So this requires an accurate CFOs estimation and compensation method which becomes a more crucial task in multiuser environments in OFDMA uplink.

For the interleaved OFDMA uplink systems, a blind CFO estimator [2] based on the estimation of signal parameters via rotational invariance technique (ESPRIT) exploits the inherent signal structure without pilot symbols. For the searching-based minimum variance distortionless response (MVDR) [1] and multiple signal classification (MUSIC) [3] estimators, the searching complexity and estimating accuracy strictly depend on the number of search grids used during the search, which is time consuming and the required number of search grids is not clear to determine. Unfortunately, the neighboring peaks cannot be distinguished for the MVDR and MUSIC estimators under relatively low signal-to-noise ratio (SNR) and large active users [4]. This implies that one user's original peak location is pulled into an adjacent user's range, severely distorting the original peak location [4]. It is well known that the polynomial root-version estimator has been proposed to improve the searching-based estimation [1, 5]. The resolution threshold performance of root-MVDR [1] and root-MUSIC [4] estimators is better than that of MVDR and MUSIC estimators, respectively, but for the noises dominant situation the appearance of both methods' threshold performance is opposite.

Recently, a novel heuristic search algorithm, called gravitational search algorithm (GSA), has been proposed motivated by the law of gravity and mass interactions [6]. It is characterized as a simple concept that is both easy to implement and computationally efficient. In GSA, the individuals, called agents, are a collection of masses which interact with each other based on the Newtonian gravity and the laws of motion. All agents move to a new place, the direction and distance are determined by their velocities. The agents share information using the gravitational force to guide the search toward the best location in the search space. By changing the velocities over time, the agents are likely to move toward the global optima. In this study, the feasibility of applying GSA to the MUSIC criterion is investigated for accurate CFO estimation of interleaved uplink OFDMA. First, in conjunction with the centro-symmetric (CS) trimmed correlation matrix [7] and the MUSIC criterion, the estimate performance of the centro-symmetric MUSIC (CSMUSIC) estimator can be improved under the low SNR case. Therefore, with a self-searched GSA and CSMUSIC scheme, the proposed CSMUSIC-GSA estimator does not require conventional spectral search, and can improve the CFO estimate accuracy. At each iteration, every agents can choice an appropriate acceleration along every dimension of search space according to its own situation. Finally, the proposed GSA-based estimators develop the CFO estimation by taking advantage of inherent structure of interleaved OFDMA signals. By dividing the whole possible CFO range into several smaller search ranges, the maximum value of fitness function is searched in each smaller range to get each user's CFO. Therefore, the proposed estimators can conquer the effect of ambiguous peaks and local optimization.

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II. BACKGROUND

A. Signal Model

Consider the interleaved uplink of an OFDMA system with N subcarriers in which K active users simultaneously communicate with the BS through an independent multipath channel. The N subcarriers are interleaved into Q $(Q>K)$ subchannels, each of which has $P=N/Q$ subcarriers. And, each subcarrier is exclusively used by only one user. Subchannel q_k contains the subcarriers with index $\{q_k, Q+q_k, \cdots, (P-1)Q+q_k\}$ and the value of $q_k=k-1$ with $k=1,2,\cdots,Q$. After removing the cyclic prefix (CP), the received signal of one OFDMA block at BS can be expressed as $y(n)=\sum_{k=1}^{K} r_k(n)+z(n)$, $n=0,\ 1,\cdots,N-1$, where $z(n)$ is the additive white Gaussian noise with zero mean and variance σ_z^2 . The discrete channel impulse between the kth user and BS is characterized by a L_k order finite impulse response filter as $\mathbf{h}_k=[h_k(0),\ h_k(1),\ \cdots,\ h_k(L_k-1)]^T$. The channel frequency response of the kth user is $\bar{H}_k(v)=\sum_{l=0}^{L_k-1} h_k(l)\exp(-j2\pi lv/N)$ for $v=0,\ 1,\cdots,N-1$ [2]. Let $\theta_k=(q_k+\varepsilon_k)/Q$ denote the effective CFO of the kth user, $\varepsilon_k\in(-0.5,\ 0.5)$ denote the kth user's CFO normalized by the subcarrier spacing $2\pi/N$, then the received signal sample from the kth user is given by

$$r_k(n)=\sum_{p=0}^{P-1} X_k(p)H_k(p)\exp\{j\frac{2\pi np}{P}\}\exp\{j\frac{2\pi n\theta_k}{P}\} \qquad (1)$$

where $X_k(p)$ is a set of P data streams of the kth user, $H_k(p)$ represents the sample from $\bar{H}_k(v)$ at $v=pQ+q_k$. The structure of (1) has a special periodic feature with every samples, $r(n+\xi P)=\sum_{k=1}^{K}\exp(j2\pi\xi\theta_k)r_k(n)$, where ξ $(0\le\xi\le Q-1)$ is an integer. In one OFDMA block, $\{y(n)\}_{n=0}^{N-1}$ can thus be arranged into a $Q\times P$ matrix

$$\mathbf{Y}=\begin{bmatrix} y(0) & y(1) & \cdots & y(P-1) \\ y(P) & y(P+1) & \cdots & y(2P-1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-P) & y(N-P+1) & \cdots & y(N-1) \end{bmatrix} \qquad (2)$$

Therefore, Equation (2) in one block can be expressed as $\mathbf{Y}=\mathbf{A}(\theta)\mathbf{S}+\mathbf{Z}$, where $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\ \cdots\ \mathbf{a}(\theta_M)]$ with $\mathbf{a}(\theta_k)=[1,\ e^{j2\pi\theta_k},\ \cdots,\ e^{j2\pi(Q-1)\theta_k}]^T$, $\mathbf{Z}$ is a $Q\times P$ noise matrix, $\mathbf{S}=\mathbf{D}\otimes(\mathbf{BW})$ with $\mathbf{W}$ is a $P\times P$ inverse discrete Fourier transform (IDFT) matrix, and $\otimes$ indicates an element-by-element product. $\mathbf{B}=[\mathbf{b}_1\ \mathbf{b}_2\ \cdots\ \mathbf{b}_K]^T$ with $\mathbf{b}_k=[X_k(0)H_k(0),\ X_k(1)H_k(1),\ \cdots,\ X_k(P\text{-}1)H_k(P\text{-}1)]^T$ and $\mathbf{D}=[\mathbf{d}_1\ \mathbf{d}_2\ \cdots\ \mathbf{d}_K]^T$ with $\mathbf{d}_k=[1,\ e^{j2\pi\theta_k/P},\ \cdots,\ e^{j2\pi(P-1)\theta_k/P}]^T$, where $(\bullet)^T$ denotes the transpose operation. Then, the ensemble correlation matrix of (2) is $\mathbf{R}=E\{\mathbf{YY}^H\}=\mathbf{AR}_s\mathbf{A}^H+\sigma^2\mathbf{I}$, where $E\{\bullet\}$ and $(\bullet)^H$ denote the expectation and Hermitian operations, respectively. $\mathbf{R}_s=E\{\mathbf{SS}^H\}$ is the correlation matrix of $\mathbf{S}$ and $\mathbf{I}$ is a $Q\times Q$ identity matrix. For finite received signal's samples, $\mathbf{R}$ is replaced by the estimated sample average $\hat{\mathbf{R}}=(1/FP)\sum_{\tau=1}^{F}\mathbf{Y}(\tau)\mathbf{Y}(\tau)^H$ and F is the total blocks of observation. The cost function of the searching-based MVDR estimator [1] with the search grid μ are defined as

$$S_{MVDR}(\theta)=\underset{\theta}{Max}\ [\mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)]^{-1} \qquad (3)$$

where $\mathbf{a}(\theta)$ is the CFO scanning vector.

B. Searching-based MUSIC estimators

The MUSIC is a noise subspace-based estimator for high resolution CFO estimation. Assume that the number of active users K is known. Then, the eigenvalue decomposition (EVD) of $\mathbf{R}$ is $\mathbf{R}=\sum_{i=1}^{Q}\lambda_i\mathbf{e}_i\mathbf{e}_i^H=\mathbf{E}_s\mathbf{\Sigma}_s\mathbf{E}_s^H+\mathbf{E}_z\mathbf{\Sigma}_z\mathbf{E}_z^H$, where $\lambda_1\ge\lambda_2\ge\cdots\ge\lambda_K\ge\lambda_{K+1}=\cdots=\lambda_Q=\sigma_z^2$ are the eigenvalues of $\mathbf{R}$ in the descending order. $\mathbf{e}_i$ are the corresponding orthonormal eigenvector associated with λ_i for $i=1,\ 2,\ \cdots,\ Q$. Moreover, $\mathbf{E}=[\mathbf{E}_s\ \mathbf{E}_z]$, both $\mathbf{E}_s=[\mathbf{e}_1\ \mathbf{e}_2\ \cdots\ \mathbf{e}_K]$ and $\mathbf{E}_z=[\mathbf{e}_{K+1}\ \mathbf{e}_{K+2}\ \cdots\ \mathbf{e}_Q]$ are orthogonal and span the signal subspace and noise subspace corresponding to $\mathbf{R}$, respectively. $\mathbf{\Sigma}_s=diag\{\lambda_1,\ \lambda_2,\cdots,\lambda_K\}$ and $\mathbf{\Sigma}_z=diag\{\lambda_{K+1},\ \lambda_{K+2},\ \cdots,\ \lambda_Q\}$. Furthermore, $\mathbf{E}_s$ spans the same signal subspace as that spanned by $\{\mathbf{a}(\theta_k)\}_{k=1}^{K}$. Thus, we have $\mathbf{E}_z^H\mathbf{a}(\theta_k)=\mathbf{0}$ and $\mathbf{a}^H(\theta_k)\mathbf{E}_z=\mathbf{0}$ for $k=1,\ 2,\cdots,K$. The cost function of the MUSIC estimator with the searching grid μ defined as [3] is given by

$$S_{MUSIC}(\theta)=\underset{\theta}{Max}\ [|\,\mathbf{a}^H(\theta)\mathbf{E}_z\mathbf{E}_z^H\mathbf{a}(\theta)\,|]^{-1} \qquad (4)$$

The whole possible range $((-0.5)/Q,\ (Q-0.5)/Q)$ is divided into Q smaller search ranges, and then the search rang $((q_k-0.5)/Q,\ (q_k+0.5)/Q)$ is set for the kth user when q_k belongs to the user k. The maximum value of cost function is calculated in each smaller range to get $\hat{\theta}_k$ respectively, rather than finding the K peaks in the whole range. Through K searches, $\{\hat{\theta}_k\}_{k=1}^{K}$ can be obtained. Hence, $\hat{\varepsilon}_k=Q\hat{\theta}_k-q_k$ for $k=1,\ 2,\ \cdots,\ K$.

To improve the CFO estimating accuracy, it can be further dealing $\hat{\mathbf{R}}$ with a CS trim to mitigate the finite sample effect. Let $\tilde{\mathbf{R}}$ denote a sample estimate of $\mathbf{R}$. If we assume that $\tilde{\mathbf{R}}$ is a CS matrix [7], i.e., $\tilde{\mathbf{R}}=\tilde{\mathbf{I}}_Q\tilde{\mathbf{R}}^*\tilde{\mathbf{I}}_Q$, where $\tilde{\mathbf{I}}_Q$ denotes the "exchange" matrix and $(\bullet)^*$ is the conjugate operation. A sample correlation matrix with the above property can be easily obtained by way of performing $\tilde{\mathbf{R}} = 0.5(\hat{\mathbf{R}} + \tilde{\mathbf{I}}_Q \hat{\mathbf{R}}^* \tilde{\mathbf{I}}_Q)$. Therefore, since $\mathbf{R}$ is a Toeplitz structure, $\tilde{\mathbf{R}}$ can be expected to be a better estimate of $\mathbf{R}$ than $\hat{\mathbf{R}}$ is. Here, $\tilde{\mathbf{R}}$ is to replace $\hat{\mathbf{R}}$ for performing EVD. Then, the improved noise subspace $\tilde{\mathbf{E}}_z$ can be obtained. Therefore, the CS trim is applied on the MUSIC estimator, and then the resulting improved estimator is termed CS-MUSIC estimator.

The estimate performances of the searching-based CFO estimators are governed by the scanning grid size. Smaller grid size can improve estimate accuracy, but the required computational load also has relatively larger. For increasing estimate performance and searching efficient, this study presents the GSA-based optimization to replace the spectral searching approach. Therefore, the proposed GSA-based estimators with the MUSIC or MVDR criterion can increase the estimate accuracy and have robust capability in both of the low SNR scenarios.

III. GSA-BASED CFO ESTIMATION

GSA is inspired from Newton's theory and can be considered as a collection of agents (candidate solutions) whose have masses proportional to their value of fitness function. During generations, all masses attract each other by the gravity forces between them. A heavier mass has the bigger attraction force. Therefore, the heavier masses which are probably close to the global optimum attract the other masses proportional to their distances. It starts with randomly placing all agents in search space. Suppose a system with N_P agents and ε_i^k represent the position of ith agent, where k denotes the kth dimension of agent (the kth active user). Then during all iterations, the gravitational force from agent j on agent i at a specific iteration time t is defined as follow [6]:

$$F_{ij}^k = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t)+\kappa}(\varepsilon_j^k(t) - \varepsilon_i^k(t)) \tag{5}$$

where $i \neq j = 1, 2, \cdots, N_P$ and $k = 1, 2, \cdots, K$. M_{aj} is the active gravitational mass related to agent j , M_{pi} is the passive gravitational mass related to agent i , $G(t)$ is gravitational constant at time t , κ is a small constant, and $R_{ij}(t)$ is the Euclidian distance between two agents i and j . The $G(t)$ is calculated as

$$G(t) = G_0 \exp\{-\beta t / N_s\} \tag{6}$$

where G_0 and β are initial value of gravitational constant and descending coefficient, respectively, N_s is maximum number of iterations. The total force that acts on agent i at iteration time t is calculated as

$$F_i^k(t) = \sum_{j=1, j\neq i}^{N_P} rand_j F_{ij}^k(t) \tag{7}$$

where $rand_j$ is a random number in $[0,\ 1]$. According to the law of motion, the acceleration of an agent is proportional to the result force and inverse of its mass, so the acceleration of all agents should be calculated as

$$acc_i^k(t+1) = F_i^k(t) / M_{ii}(t) \tag{8}$$

where M_{ii} is the mass of agent i . The updating formula of gravitational and inertial masses of agent i follows as bellow:

$$m_i = \frac{Fitness_i^k(t) - worst^k(t)}{best^k(t) - worst^k(t)} \tag{9}$$

where $Fitness_i^k(t)$ is the fitness value of the agent i at time t . Then, $worst^k(t)$ and $best^k(t)$ are defined as follows:

$$\begin{cases} best^k(t) = \max\limits_{j\in\{1,\ldots,N_P\}} Fitness_j^k(t) \\ worst^k(t) = \min\limits_{j\in\{1,\ldots,N_P\}} Fitness_j^k(t) \end{cases} \tag{10}$$

For simplify, let $M_{ai} = M_{pi} = M_{ii} = M_i$, and $M_i(t) = m_i(t) / \sum_{j=1}^{N_P} m_j(t)$. The velocity and position of the ith agent for the kth user are calculated as

$$vel_i^k(t+1) = rand_i \times vel_i^k(t) + acc_i^k(t+1) \tag{11}$$

$$\varepsilon_i^k(t+1) = \varepsilon_i^k(t) + vel_i^k(t+1) \tag{12}$$

where $rand_i$ is a random number in the interval $[0,\ 1]$.

At first all agents are initialized with random values and each agent $\varepsilon_i^k(0)$ is a candidate solution which represents the desired impinging angle. After initialization, velocities for all agents are defined using (11). Meanwhile the gravitational constant, total forces, and accelerations are calculated as (6), (7), and (8), respectively. The positions of agents are calculated using (12). If the movement of agent exceeds the searching space, the position of agent will be re-generalized randomly. Also, the fitness function of the desired kth user obeys the CSMUSIC criterion as bellow

$$Fitness_i^k(t) = [\mathbf{a}^H(\varepsilon_i^k(t))\tilde{\mathbf{E}}_z\tilde{\mathbf{E}}_z^H\mathbf{a}(\varepsilon_i^k(t))]^{-1} \tag{13}$$

Finally, the CSMUSIC-GSA estimator will be stopped by meeting an end criterion and return the best solution. Abbreviations and Acronyms

IV. SIMULATION RESULTS

For illustrating the performance of the proposed estimators, several simulation results are conducted. For comparison, the results of the ESPRIT [2], MUSIC [4], root-MUSIC [4], MVDR [1], root-MVDR [1], MVDR-GSA, and MUSIC-GSA estimators are also provided. It is noted that the fitness functions of MVDR-GSA, MUSIC-GSA, and CSMUSIC-GSA estimators are (3), (4), and (13), respectively.

The parameters in the OFDMA uplink system are $K=8$, $N=1024$, $Q=32$, and $P=32$. All OFDMA signals were generated with binary phase-shift-keying (BPSK) modulation and the average received signal power from all users is the same. Each user transmits signal to BS through independent multipath channel. The channel taps $h_k(l)$ are modeled as statistically independent Gaussian random variables with zero mean and an exponentially decaying power profile, $E[h_k(l)^2]=\alpha_l e^{(-l/5)}$, $0\le l\le L_k-1$, where α_l is a normalized factor used to set the channel power to unity and $L_k=10$ [2]. As indices of evaluation, the mean square error (MSE) and the input SNR of the kth user were defined as $\mathrm{MSE}=(1/M\Pi)\sum_{\rho=1}^{\Pi}\sum_{k=1}^{K}(\hat{\varepsilon}_k^{\rho}-\varepsilon_k^{\rho})^2$ and $\mathrm{SNR}=20\log E[r_k(n)^2]/\sigma_z^2$, where Π is the number of Monte Carlo runs. For all simulations, the CFOs are $\{-0.4041,-0.3355,-0.0407,-0.1175,$ $0.1254, 0.2193, 0.3612, 0.4091\}$. For searching-based estimators, the proper searching grids are set to $\mu=10^{-4}$ in the scenario range $[0\text{ dB}, 30\text{ dB}]$ [4]. In GSA, the gravitational constant $G(t)$ is set using (6), where G_0=100, β=20, and a small constant $\kappa=5\times10^{-3}$. All GSA start with random initializations and are terminated if the maximum iteration (the total age of system) N_s is reached. Each of the simulation results presented is after one OFDMA block processed and is the average of $\Pi=10^3$ runs with independent noise samples for each run.

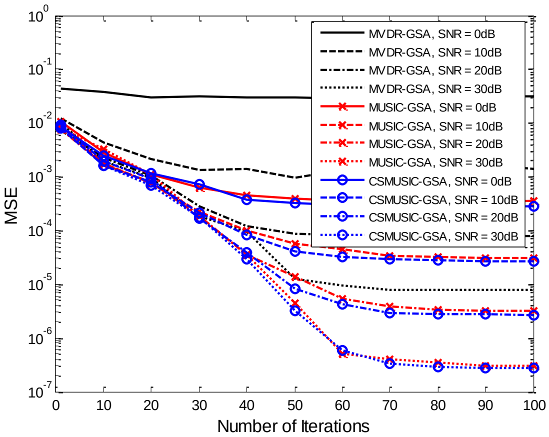

Fig. 1. MSE versus the number of iterations.

We investigate the effects of parameters $\{N_p, N_s\}$ on the MSE performance with SNR as parameters. Fig. 1 depicts MSE versus the number of iterations N_s for GSA-based estimators under the number of agents (particles) $N_p=12$, whereas Fig. 2 depicts MSE versus the number of agents N_p for GSA-based estimators under the number of iterations $N_s=80$. It also demonstrates that the performance of the GSA-based estimators heavily depend on the selected the number of iterations and the number of agents. In the scenario range $[0\text{ dB}, 30\text{ dB}]$, the proper choices of the number of particles and the number of iterations for GSA-based estimators are $N_p=12$ and $N_s=80$, respectively. It also demonstrates that both of large agent and iteration numbers have better estimation resolution. The total required complex multiplications (CM) of the estimators contain the CM of computing fitness (objective) function and the CM of search (iteration) procedure. For computing the fitness function, the CM of the GSA-based and searching-based estimators are equal. However, the proposed advantage is the reduction of computational complexity by reducing the number of the search while maintaining comparable performance. According to [4], a proper choice is $\mu=10^{-4}$ in the scenario range $[0\text{ dB}, 30\text{ dB}]$. Assume that all the searching grid of searching-based estimators are $\mu=10^{-4}$, then the number of searching $F_1=10001$. But, the calculating fitness function number of the CSMUSIC-GSA is only $N_pN_s=960$. However, the required N_p and N_s of the GSA-based estimators are less than those of the searching-based estimators for all SNR cases, respectively. It is clearly that the proposed GSA-based estimators have high searching efficiency.

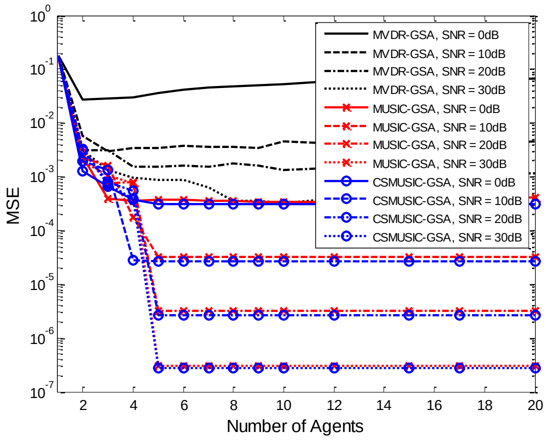

Fig. 2. MSE versus the number of agents.

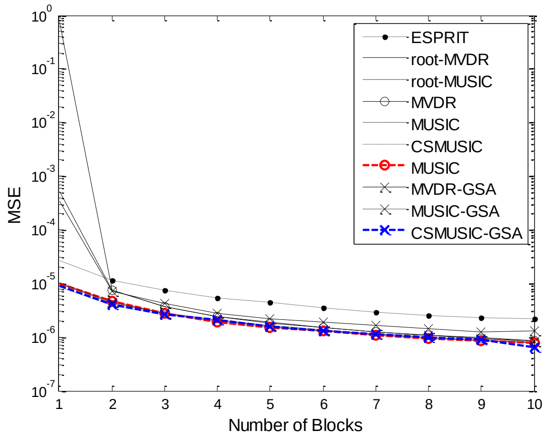

Fig. 3. MSE versus the number of blocks.

Fig. 3 shows the results of MSE versus the number of blocks under SNR=15dB. Clearly, increasing the number of blocks induces the performance improvement for all estimation methods. It shows that all GSA-based approaches have the same convergence speed. Fig. 4 presents MSE of the CFO estimation versus SNR. By using the correlation matrix with a CS trim to mitigate the finite sample effect, the CSMUSIC estimator has better performance improvement especially under the low SNR case. However, the statistical behavior of the root-MVDR estimator for a single data block appears difficult to establish. Observe that the accuracy of the CFO estimation of the searching-based estimators is governed by the search grid size whereas all GSA-based estimators do not suffer from this limitation. Again, these figures are presented to verify the efficiency of the proposed estimators.

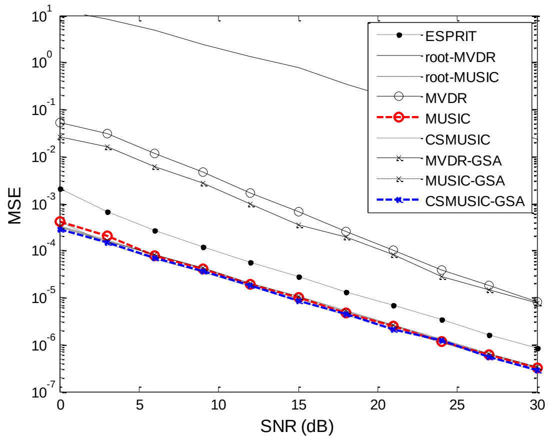

Fig. 4. MSE versus SNR.

V. CONCLUSIONS

This study has presented GSA-based searching CFO estimation methods with the MVDR/MUSIC criterion, named as the MVDR-GSA, MUSIC-GSA, and CSMUSIC-GSA estimators, to achieve the CFO estimate accuracy under a single data block. By dividing the whole possible CFO range into Q smaller search ranges, the maximum value of fitness function is calculated in each smaller range to get each user's CFO. Therefore, the convergence of GSA to the global maximum is confirmed.

ACKNOWLEDGMENT

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