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Bit Error Rate Performance of TAPSK using Block Coded Modulation

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***Abstract*—In this paper, we calculate minimum non coherent distances of block-coded TAPSK (twisted amplitude and phase shift keying) and QAM (quadrature-amplitude modulation), both using hamming distance. According to the derived distances, non coherent block-coded TAPSK (NBC-TAPSK) and non coherent block-coded QAM (NBC-QAM) are presented. If we change the radius of NBC-TAPSK then it performs best among allnon coherent schemes and NBC-QAM performs worse due to its small minimum non coherent distance. However, if we consider block length is not short, NBC-QAM has the best error performance because the code words with small non coherent distances are rare.Here we also change the value of r and see the performance of BER and also see the effect of Rayleigh channel on BER.**

Index Terms*—Non coherent detection, BCH Codes, channels Block coded modulation, multilevel coding.

I. INTRODUCTION

THE ADDITIVE white Gaussian noise (AWGN) channel which introduces an unknown carrier phase rotation has been investigated in many works. This channel offers a useful abstraction of the flat fading channel,when the effects of the phase rotation need to best studied independently of the amplitude variations. A simple model that is commonly used is one where the unknown carrier phase is constant over a block of Nsymbols, and independent from block to block. This model is correct for frequency hopping systems. For this non coherent channel with large N,pilot symbols used for the carrier phase estimation with codes designed for coherent decoding perform well.However, for small N, block codesdesigned for non coherent decoding outperform this training-based non coherent codes.The minimum non coherent distances of codes are obtained by brute-force searching for all codeword-pairs.

For the transmitted baseband codeword**x** =($x1,x2, \cdots , xN$), the received baseband blocky = ($y1$, $y2$, ⋯ , yN) is given by y = x exp{$j\theta$} + n .signal point in the signal constellation of 8PSK, is labeled by (a, b, c) where a, b, and $c \in \{0, 1\}$. Let ($a1$, $b1$, $c1$), ($a2$, $b2$, $c2$), ⋯ , (aN, bN, cN) bea block of transmitted signals. If ca = ($a1$, $a2$, ⋯ ,aN), cb = ($b1$, $b2$, ⋯ , bN) and cc = ($c1$, $c2$, ⋯ , cN) are code words of binary block codes Ca, Cb and Cc, are also called components codes. the minimum non coherent Hamming distance of $Ci$is defined by $dncH,i$ = min{di,min,N −di,max}where d,min and di,maxdenote the minimum and maximum values of Hamming distance between any two code words corresponding to different data bits in C.

II. NONCOHERENT BLOCKMODULATION USING LINEAR COMPONENT CODES

A.TAPSKFor TAPSK with labeling in Fig. 1, the bit in level $a$decidesSymbol energy. The radiuses of the inner and outer circles are denoted by r_0 and r_1, respectively. The values of r_1 and r_0 $(r_0 \le 1 \le r_1)$ satisfy r=2 when a=0 has thesame probability as a =1, $r_0^2 + r_1^2 = 2$.With the proof given in Appendix A, we have the following theoremDefine$f(d)$ by $f(d)$=

$$\frac{r_1^2 - r_0^2}{2} d - \sqrt{(r_0^2(N-d) + r_0 r_1 \cos\Phi)^2 + (r_0 r_1 d \sin\Phi)^2}$$

Block coded generalized-8TAPSK Cwhose component codes are all linear, the minimum squared non coherent distance is $d_{nc}^2 = \min\{d_{nc,a}^2, d_{nc,b}^2, d_{nc,c}^2\}$,where

$d_{nc,a}^2 = \min\{f(d_{a,\min}), f(d_{a,\max})\}$, $d_{nc,b}^2 = r_0^2(N - \sqrt{(N - d_{ncH,b})^2}$) $d_{nc,c}^2 = 2r_0^2 d_{ncH,c}$ For block-coded generalized-16TAPSK, by a similar derivation $d_{nc}^2 = \min\{d_{nc,a}^2, d_{nc,b}^2, d_{nc,c}^2, d_{nc,d}^2\}$ Where, $d_{nc,a}^2 =$ $\min\{f(d_{a,\min}), f(d_{a,\max})\}$,

$$d_{nc,b}^2 = r_0^2(N - \sqrt{(N - \frac{2-\sqrt{2}}{2} d_{ncH,b})^2 + \frac{d_{ncH,b}^2}{2}}),$$

$$d_{nc,c}^2 = r_0^2(N - \sqrt{(N - d_{ncH,c})^2 + d^2_{ncH,c}})$$ and

$d_{nc,d}^2 = 2r_0^2 d_{ncH,d}$,for NBC-8TAPSK. Table I compares NBC-8TAPSK with NBC-8PSK in terms of $d$for N =15, 31, 63, and$N$ →8. In this paper, only (15,11,1) code, (31,26, 1) code and (63,57,1) BCH codes are used as componentcodes. The values of (da,min,db,min,dc,min).are shown in the column of "code", and the values of $r$which maximize the same rate and N, NBC-8TAPSK always has larger d^2nc than NBC-8PSK.Figure 2 presents the results for N =4. For the pilotoptimized 16QAM, the amplitude of the pilot signal is 1.225. NBC-16QAM has better BER than 16QAM(H) and 16QAM(L), but they all do not decrease exponentially.

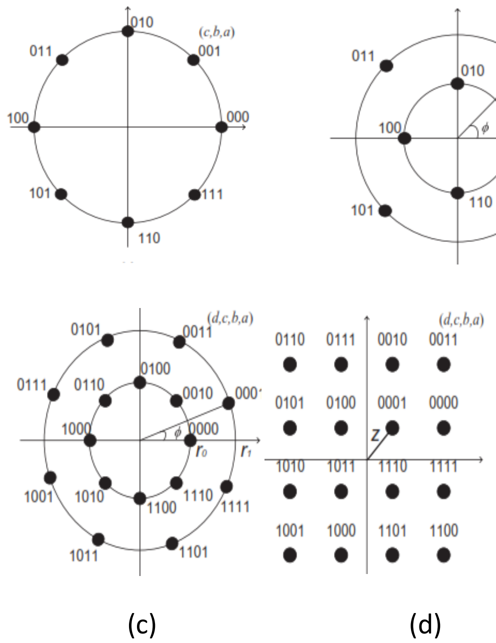
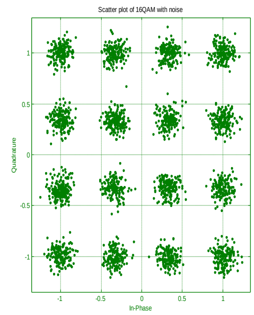


Fig. 1. Constellations with bit labeling for (a) 8PSK (b) 8TAPSK (ϕ = π/4)(c) 16TAPSK (ϕ =π/8) (d) 16QAM

When r =1, i.e. TAPSK becomes MPSK, we have (N)=0and $f(d)=f(N - d)\forall d$. Consequently, d of block-coded MPSK is equal to (d). Therefore, forBlockcoded MPSK, Ca should be a binary block code withlarge d. We proposed NBC-MPSK in [5] by setting $d_{a,\max}$, $d_{ncH,a} = N - d_{a,\min}$ such that $dncH,a$= da,min at the priceof sacrificing one data bit. But as $r$increases, (N) also increases. For block-coded8TAPSK where $r$is large enough, (N)=$(r_1 - r_0)^2 N/2$ can be larger than f(da,min). Ifr>1.61238, (N) is always larger than f(da,min) for any value of da,min(d= N/2).In such case, since $d^2_{nc,a}$ = f(da,min), Ca,mincould be anormal code with large da,min and thus the one-bit loss isunnecessary.

B. 16QAM

The distance between the smallest-energy point and the origin in the 16QAM constellation is denoted by$z$.



From this diagram we can calculate minimum non coherent distance $d^2_{nc\min}$ .If we define $d_{\min H} = 4$, then

$d_{0\min} = (d_{\min H} / \delta_0^2)$, $d_{1,\min} = (d_{\min H} / \delta_1^2)$,

$d_{2,\min} = (d_{\min H} / \delta_2^2)$ and $d_{3,\min} = (d_{\min H} / \delta_3^2)$

For blockcoded 16QAM whose component codes are all linear, the minimum squared non coherent distance

$$d^2_{nc} = \min\{d^2_{nc,a}, d^2_{nc,b}, d^2_{nc,c}, d^2_{nc,d}\}$$

Spectral efficiency	code	d^2_{nc}		
		N=15	N=31	N=63
4.34	8PSK	0.212	0.351	0.361
2.24	8TAPSK(H)	0.401	0.356	0.371
2.43	8TAPSK(L)	0.352	0.412	0.423
3.23	16QAM	0.450	0.453	0.464
2.56	16TAPSK	0.554	0.621	0.632

Table I compares NBC-8TAPSK with NBC-8PSK in terms of d^2_{nc} for N =15, 31, 63,and N →8.In this paper, only (15,11,1) code, (31,27,1) code and (63,57,1) BCH CODES are used as component codes. The values of (da,min,db,min dc,min)For the same rate and N, NBC-8TAPSK always has larger dncthan NBC-8PSK.

COMPARISON OF THEORETICAL BEST VALUES AND SIMULATION BEST VALUES OF rFOR NBC-16TAPSK.

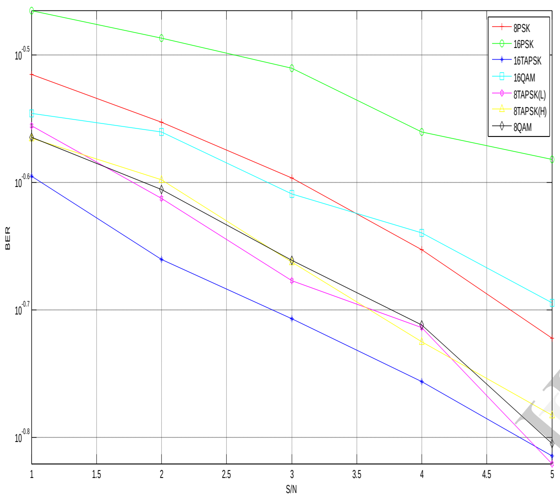
Spectral efficiency	N=15		N=31		N=63	
	Theo.	Simu.	Theo	Simu	Theo	simu
2.23	0.51	0.52	0.46	0.48	0.64	0.67
3.24	0.46	0.49	0.56	0.58	0.56	0.63
3.67	0.43	0.45	0.59	0.61	0.66	0.67
4.34	0.34	0.36	0.61	0.63	0.67	0.69

For NBC-16TAPSK, Table II compares the best values of$r$for simulations with the theoretical best values of $r$thatmaximize dnc. The values of (da,min,db,min,dc,min)are shown in the column of "data rate" In the multistagedecoding, a decoding error in level $a$probably causes errorpropagation, so slightly larger $r$which results in better BER in level $a$would have the best overall BER. Let N_a and N_b denote the numbers of the nearest-neighbor codewords for C_a and C_b respectively, shown in Table II also. We find that ifis less than or approximately equal to 1, the best $r$forsimulations is close to (slightly larger than) the best $r$fordnc.ButifNis not small, the BER in level $a$is increaseddue to the large number of the nearest-neighbor codewords,so the best $r$for simulations is larger than the best $r$for d_{nc}.

NBC-16TAPSK is better than NBC16QAM at high SNRs which agrees with the minimum noncoherent distance analysis. For NBC-16QAM, the gap betweennoncoherent decoding and ideal coherent decoding is quite wide.

III. SIMULATION RESULTS AND DISCUSSIONS

At high SNRs, the pilot-optimized 16QAM outperforms NBC-16QAM, and NBC-16TAPSK is the best among all noncoherent schemes. The results for N =15 are shown in Fig. 3 in which the amplitude of the pilot signal is 1.673.We find that the average number of codewords with smallnoncoherent distances is too tiny to affect the curves above BER of 10^{-6} for all noncoherent 16QAM schemes.

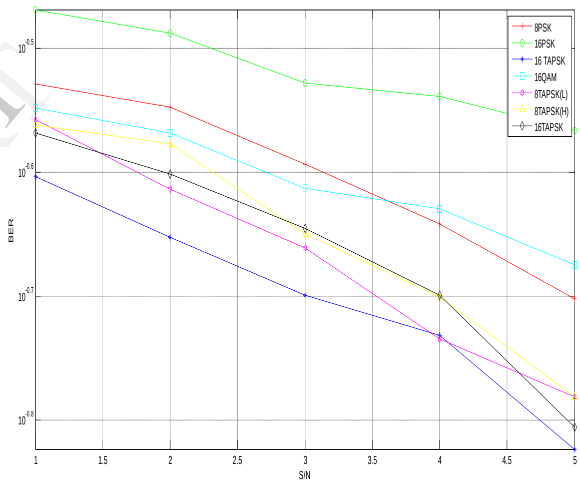


(BER Vs S/Na t(r=o.35))

At the receivers, the channel-quantization decoding algorithmin [6, Sec. III] is used. This algorithm uses the estimate of $\theta$from the family T={0,2π/MQ, ⋯ ,2$\pi(Q$-1)$/MQ$},M =4for NBC-8TAPSK and NBC-16QAM, M =8forNBC-8PSK and NBC-16TAPSK. In all simulations, we set$Q$ =6,but if $C'b$and $C'c$are un-coded bits(db,min= dc,min=1), the labeling of bits $b$and $c$shouldbe Gray labeling of QPSK for theminimization of bit errorrate (BER). The labeling in Fig. 1(c) For NBCTAPSKand nonlinear NBC-TAPSK, we look for the value of$r$ that needs the lowest SNR at the BER of $10^{-6}$by simulation results, and use it in simulations. In Fig. 2 and Fig. 3, we consider noncoherent blockcodes using sixteen signal points with data rate (4N -4)/Nbits/symbol, including NBC-16TAPSK and NBC-16QAMwhose (da,min,db,min, dc,min,dd,min) is (2, 1, 1, 1), and the differentially-encoded16QAM scheme in [9] denoted by 16QAM(H). We modify the scheme in [9] by choosing the low energy codewordsinstead of the high-energy codewords, denoted by 16QAM(L),as suggested by [7]. The results of ideal coherent decoding for NBC-16TAPSK and NBC 16QAM areexplained in [7] is also compared .

Figure 2 presents the results for N =31. For the pilotoptimized 16QAM, the amplitude of the pilot signal is 1.225. NBC-16QAM has better BER than 16QAM(L), but they all do not decrease exponentially because the average number of codewords with small noncoherent distances is little, but not little enough. For ideal coherent decoding,NBC-16TAPSK is worse than NBC-16QAM. But for noncoherent decoding, NBC-16TAPSK is better than NBC16QAM at high SNRs which agrees with the minimum noncoherent distance analysis. For NBC-16QAM, the gap between noncoherent decoding and ideal coherent decoding is quite wide given as references.

But here we take fixed minimum hamming distance, then find $\delta_a^2, \delta_b^2, \delta_c^2$, and then minimum required $d^2_{\min}$ .for evaluating system performance, we compute BER versus E/Nb graph for the AWGN channel or Rayleigh channel. For encoding we use the BCH encoder, then transmitted the signals by thisencoding, at the receiver we use same type of decoder and see the error which place we have to correct.

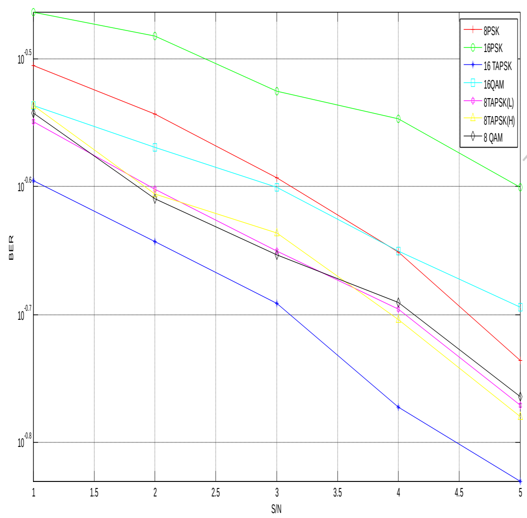


(BER Vs S/Na t(r=o.40))

At high SNRs, the pilot-optimized 16QAM outperforms NBC-16QAM, and NBC-16TAPSK is the best among all noncoherent schemes. The results for N =15 are shown in Fig. 3 in which the amplitude of the pilot signal is 1.673.We find that the average number of codewords with smallnoncoherent distances is too tiny to affect the curves above BER of $10^{-6}$for all noncoherent 16QAM schemes. NBC16QAMoutperforms NBC-16TAPSK and the pilot-optimized16QAM, and its gap between noncoherent decoding and idealcoherent decoding is less than 1dB.

Various non coherent block codes using eight or sixteen signal points with data rate (3N -3)/Nbits/symbol for N =16 are compared in Fig. 4. NBC-16TAPSK and NBC-16QAM both use (da,min,db,min,dc,min,dd,min)=(8, 4, 1, 1),and NBC-8TAPSK using $C(H)$ (denoted by NBC-8TAPSK(H) and NBC-8TAPSK(denoted by NBC-8TAPSK(L)) andboth use(da,min,db,min,dc,min))=(1, 1, 1). NBC-8TAPSK using (O)has almost the same BER as NBC-8TAPSK andthus is not shown in the figure-2. The used values of $r$are1.94, 1.95 and 1.6 for NBC 8TAPSK(H), NBC-8TAPSK(L)and NBC-8TAPSK, respectively. We find that NBC-8PSK isthe worst, and NBC-8TAPSK has better BER than NBC8TAPSK(L)and NBC-8TAPSK(H). At high SNRs, NBC16TAPSKoutperforms NBC-8TAPSK. This is reasonablesince its dnc, 0.6277, is larger than $d$ncof NBC-8TAPSK,0.6030. After all, NBC-16QAM whose $dnc$is only 0.1649 isthe best. It provides about 1.6dB gain over NBC-16TAPSK ata BERof 10^{-6}.

Quite different from NBC-MPSK and NBC-TAPSK, the average number of nearest neighbors of NBC-16QAM is very small. It is complicated to compute the average number of nearest neighbors of NBC-16QAM, so we take an example to illustrate this point as follows. Suppose that the transmitted has component codeword in level a c_a= 0. Consider another component codewordca.Help of scatter plot shown in above figure-4.then we compute BER for 16 QAM , For N=31, the minimum non coherent distance of energy constraint 16-MAPSK is larger than that of energy constraint 16-QAM. Therefore, it is reasonable that the performance of energy constraint 16-MAPSK is better than energy constraint 16-QAM.



(BER V*s* S/Na t(r=o.55))

Quite different from NBC-MPSK and NBC-TAPSK, the average number of nearest neighbors of NBC-16QAM is very small. It is complicated to compute the average number of nearest neighbors of NBC-16QAM, so we take an example to illustrate this point as follows. Suppose that the transmitted codeword, denoted by x, has a component codeword in level$a$ ca= 0. Consider another component codewordcin level$a$and the Hamming distance between caand c'a is dmin,Assume that dnc= dnc,. For thiscase, we compute the number of nearest neighbors caused by C'a for NBC-16TAPSK and NBC-16QAM as follow.

IV. CONCLUSION

In this paper, the minimum non-coherent distances of block-codedTAPSK and 16QAM using linear component codes arecalculated. The minimum non-coherent distance of block-codedQAM with more signal points can be derived similarly. Wefind that the minimum noncoherent distance of block-codedMPSK derived is a special case of the derived minimumnoncoherent distance of block-coded TAPSK. According tothe derived distances, we propose NBC-TAPSK and NBC-QAM.The comparison of minimum noncoherent distancesshows the superiority of NBC-TAPSK over NBC-QAM athigh data rates. We compare various non-coherent block codesbased on the simulation results. By changing the value of radius in TAPSK we get optimum value of radius (r) in which TAPSK has better BER performance among all digital modulation techniques and QAM has worse error performance due to its smallminimum noncoherent distance.

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