Bipolar Interval Valued Fuzzy Contra Generalized Semiprecontinuous Mappings

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***Abstract*: In this paper, bipolar interval valued fuzzy contra generalized semi-precontinuous mapping is defined and introduced. Using this definitions, some theorems are introduced.**

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INTRODUCTION:

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [16] in the year 1965, the subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper C.L.Chang [2] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Dontchev [3] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [12]. Tapas kumar mondal and S.K.Samantha [11] have introduced the topology of interval valued fuzzy sets. Jeyabalan.R and Arjunan [5, 6] have introduced interval valued fuzzy generalized semi pre continuous mapping. After that interval valued fuzzy generalized semi pre continuous mapping has been generalized into interval valued intuitionistic fuzzy generalized semi pre continuous mapping by S.Vinoth and K.Arjunan[14, 15]. The interval valued fuzzy set has been extended into the bipolar interval valued fuzzy topological spaces. P.Kongeswarn et. al [7, 8] have defined and introduced the bipolar interval valued fuzzy generalized semipreclosed sets and bipolar interval valued fuzzy generalized semiprecontinuous mapping. After that R.Selvam et.al [15] have defined and introduced the bipolar interval valued multi fuzzy generalized semipreclosed sets. In this paper, we introduce bipolar interval valued fuzzy contra generalized semi-precontinuous mapping and some properties are investigated.

1.PRELIMINARIES:

Definition 1.1[16]. Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X→[0, 1].
Definition 1.2[16]. Let X be any nonempty set. A mapping A : X→D[0, 1] is called an interval valued fuzzy subset (briefly, IVFS) of X, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1].
Definition 1.3[9]. A bipolar valued fuzzy set A in X is defined as an object of the form A = {⟨ x, M(x), N(x) ⟩/ x∈X}, where M : X→ [0, 1] and N : X→ [−1, 0]. The positive membership degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.
Example 1.4. A = { ⟨ a, 0.8, −0.6 ⟩, ⟨ b, 0.6, −0.7 ⟩, ⟨ c, 0.3, −0.9 ⟩ } is a bipolar valued fuzzy subset of X = { a, b, c }.
Definition 1.5[7]. A bipolar interval valued fuzzy set A in X is defined as an object of the form A = {⟨ x, M(x), N(x) ⟩/ x∈X}, where M : X→D[0, 1] and N : X→D[−1, 0]. The positive membership interval degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar interval valued fuzzy set A and the negative membership interval degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar interval valued fuzzy set A.
Example 1.6. A = { ⟨ a, [0.6, 0.9], [−0.6, −0.4] ⟩, ⟨ b, [0.8, 0.9], [−0.7, −0.5] ⟩, ⟨ c, [0.5, 0.8], [−0.8, −0.6] ⟩} is a bipolar interval valued fuzzy subset of X ={ a, b, c }.

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Definition 1.7[7]. Let A = ⟨ M, N ⟩ and B = ⟨ O, P ⟩ be any two bipolar interval valued fuzzy subsets of a set X. We define the following relations and operations:
(i) $A \subseteq B$ if and only if $M(x) \le O(x)$ and $N(x) \ge P(x)$ for all x in X.
(ii) A = B if and only if M(x) = O(x) and N(x) = P(x) for all x in X.
(iii) $(A)^c = \{ \langle x, (M)^c(x), (N)^c(x) \rangle / x \in X \}$.
(iv) $A \cap B = \{ \langle x, rmin\{ M(x), O(x) \}, rmax\{ N(x), P(x) \} \rangle / x \in X \}$.
(v) $A \cup B = \{ \langle x, rmax\{ M(x), O(x)\}, rmin\{ N(x), P(x) \} \rangle / x \in X \}$.
Remark 1.8. $\bar{0} = \{ \langle x, [0, 0], [0, 0] \rangle : x \in X\}$ and $\bar{1} = \{ \langle x, [1, 1], [-1, -1] \rangle : x \in X \}$.
Definition 1.9[7]. Let X be a set and $\Im$ be a family of bipolar interval valued fuzzy subsets of X. The family $\Im$ is called a bipolar interval valued fuzzy topology (BIVFT) on X if $\Im$ satisfies the following axioms

(i) $\bar{0}, \bar{1} \in \Im$ (ii) If $\{ A_i ; i \in I \} \subseteq \Im$, then $\bigcup_{i=1}^{\infty} A_i \in \Im$

(iii) If $A_1, A_2, A_3, \ldots A_n \in \Im$, then $\bigcap_{i=1}^{n} A_i \in \Im$.

The pair (X, $\Im$) is called a bipolar interval valued fuzzy topological space (BIVFTS). The members of $\Im$ are called bipolar interval valued fuzzy open sets (BIVFOS) in X. An bipolar interval valued fuzzy subset A in X is said to be bipolar interval valued fuzzy closed set (BIVFCS) in X if and only if $(A)^c$ is a BIVFOS in X.
Definition 1.10[7]. Let (X, $\Im$) be a BIVFTS and A be a BIVFS in X. Then the bipolar interval valued fuzzy interior and bipolar interval valued fuzzy closure are defined by $bivfint(A) = \cup\{G : G$ is a BIVFOS in X and $G \subseteq A\}$, $bivfcl(A) = \cap\{K : K$ is a BIVFCS in X and $A \subseteq K\}$. For any BIVFS A in (X, $\Im$), we have $bivfcl(A^c) = (bivfint(A))^c$ and $bivfint(A^c) = (bivfcl(A))^c$.
Definition 1.11[7]. A BIVFS A of a BIVFTS (X, $\Im$) is said to be a
(i) bipolar interval valued fuzzy regular closed set (BIVFRCS for short) if $A = bivfcl(bivfint(A))$
(ii) bipolar interval valued fuzzy semiclosed set (BIVFSCS) if $bivfint(bivfcl(A)) \subseteq A$
(iii) bipolar interval valued fuzzy preclosed set (BIVFPCS) if $bivfcl(bivfint(A)) \subseteq A$
(iv) bipolar interval valued fuzzy α closed set (BIVF**α**CS for short) if $bivfcl\left(bivfint(bivfcl(A))\right) \subseteq A$
(v) bipolar interval valued fuzzy β closed set (BIVF**β**CS for short) if $bivfint\left(bivfcl(bivfint(A))\right) \subseteq A$.
Definition 1.12[7]. A BIVFS A of a BIVFTS (X, $\Im$) is said to be a
(i) bipolar interval valued fuzzy generalized closed set (BIVFGCS for short) if $bivfcl(A) \subseteq U$, whenever $A \subseteq U$ and U is a BIVFOS
(ii) bipolar interval valued fuzzy regular generalized closed set (BIVFRGCS for short) if $bivfcl(A) \subseteq U$, whenever $A \subseteq U$ and U is a BIVFROS.
Definition 1.13[7]. A BIVFS A of a BIVFTS (X, $\Im$) is said to be a
(i) bipolar interval valued fuzzy semipreclosed set (BIVFSPCS for short) if there exists a BIVFPCS B such that $bivfint(B) \subseteq A \subseteq B$
(ii) bipolar interval valued fuzzy semipreopen set (BIVFSPOS for short) if there exists a BIVFPOS B such that $B \subseteq A \subseteq bivfcl(B)$.
Definition 1.14[7]. Two BIVFSs A and B are said to be not q-coincident if and only if $A \subseteq B^c$.
Definition 1.15[7]. Let A be a BIVFS in a BIVFTS (X, $\Im$). Then the bipolar interval valued fuzzy semipre interior of A ($bivfspint(A)$ for short) and the bipolar interval valued fuzzy semipre closure of A ($bivfspcl(A)$ for short) are defined by $bivfspint(A) = \cup\{G : G$ is a BIVFSPOS in X and $G \subseteq A\}$, $bivfspcl(A) = \cap\{K : K$ is a BIVFSPCS in X and $A \subseteq K\}$. For any BIVFS A in $(X, \Im)$, we have $bivfspcl(A^c) = (bivfspint(A))^c$ and $bivfspint(A^c) = (bivfpcl(A))^c$.
Definition 1.16[7]. A BIVFS A in BIVFTS (X, $\Im$) is said to be a bipolar interval valued fuzzy generalized semipreclosed set (BIVFGSPCS for short) if $bivfspcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a BIVFOS in (X,$\Im$).
Example 1.17. Let $X = \{a, b\}$ and $\Im = \{\bar{0}, G, \bar{1}\}$ is a BIVFT on X, where G = { ⟨ a, [0.5, 0.5], [–0.4, –0.4] ⟩, ⟨ b, [0.4, 0.4], [–0.3, –0.3] ⟩}. And the BIVFS A = { ⟨ a, [0.3, 0.3], [–0.2, –0.2] ⟩, ⟨ b, [0.2, 0.2], [–0.1, –0.1] ⟩ } is a BIVFGSPCS in (X,$\Im$).
Definition 1.18[7]. The complement A^c of a BIVFGSPCS A in a BIVFTS (X, $\Im$) is called a bipolar interval valued fuzzy generalized semi-preopen set (BIVFGSPOS) in X.
Definition 1.19[7]. A BIVFTS (X, $\Im$) is called a bipolar interval valued fuzzy semi-pre $T_{1/2}$ space ($BIVFSPT_{1/2}$), if every BIVFGSPCS is a BIVFSPCS in X .
Definition 1.20[8]. Let (X, $\Im$) and (Y, σ) be BIVFTSs. Then a map h: X→Y is called a (i) bipolar interval valued fuzzy continuous (BIVF continuous) mapping if $h^{-1}(B)$ is BIVFOS in X for all BIVFOS B in Y.
(ii) a bipolar interval valued fuzzy closed mapping (BIVFC mapping) if h(A) is a BIVFCS in Y for each BIVFCS A in X.

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(iii) bipolar interval valued fuzzy semi-closed mapping (BIVFSC mapping) if $h(A)$ is a BIVFSCS in Y for each BIVFCS A in X.

(iv) bipolar interval valued fuzzy preclosed mapping (BIVFPC mapping) if $h(A)$ is a BIVFPCS in Y for each BIVFCS A in X.

(v) bipolar interval valued fuzzy semi-open mapping (BIVFSO mapping) if $h(A)$ is a BIVFSOS in Y for each BIVFOS A in X.

(vi) bipolar interval valued fuzzy generalized semi-preopen mapping (BIVFGSPO mapping) if $h(A)$ is a BIVFGSPOS in Y for each BIVFOS A in X.

(vii) bipolar interval valued fuzzy generalized semi-preclosed mapping (BIVFGSPC mapping) if $h(A)$ is a BIVFGSPCS in Y for each BIVFCS A in X.

Theorem1.21[8]. Every BIVFC mapping is a BIVFCoGSP continuous mapping but not conversely.

2. SOME PROPERTIES

Definition 2.1. A mapping f : X → Y is said to be a bipolar interval valued fuzzy contra generalized semiprecontinuous mappings (BIVFCoGSPCM) if $f^{-1}(A)$ is a BIVFGSPCS in X for every BIVFOS A in Y.

Example 2.2. Let X={a, b}, Y ={ u, v} and G_1 = {⟨ a, [0.2, 0.2], [–0.3, –0.3] ⟩, ⟨ b, [0.4, 0.4], [–0.5, –0.5] ⟩ }, G_2 = { ⟨ u, [0.3, 0.3], [–0.5, –0.5] ⟩, ⟨ v, [0.5, 0.5], [–0.8, –0.8] ⟩ }. Then τ = { 0_X, G_1, 1_X } and σ = { 0_Y, G_2, 1_Y } are BIVFT on X and Y respectively. Define a mapping f : (X, τ)→ (Y, σ) by f(a) = u and f(b) = v. Then f is a BIVFCoGSPCM.

Theorem 2.3. Every BIVFC mapping is a BIVFCoGSPCM but not conversely.

Proof. Let A ⊆ Y be a BIVFOS. Then $f^{-1}(A)$ is a BIVFCS in Y, by hypothesis. Hence $f^{-1}(A)$ is a BIVFGSPCS in X. Therefore f is a BIVFCoGSPCM.

Example 2.4. Let X ={a, b}, Y = {u, v} and G_1 ={⟨ a, [0.6, 0.6], [–0.8, –0.8] ⟩, ⟨ b, [0.2, 0.2], [–0.5, –0.5] ⟩ }, G_2 = { ⟨ u, [0.3, 0.3], [–0.2, –0.2] ⟩, ⟨ v, [0.6, 0.6], [–0.7, –0.7] ⟩ }. Then τ = { 0_X, G_1, 1_X } and σ = { 0_Y, G_2, 1_Y } are BIVFT on X and Y respectively. Define a mapping f : (X, τ)→ (Y, σ) by f(a) = u and f(b) = v. Then f is a BIVFCoGSPCM but not a BIVFC mapping, since G_2 is a BIVFOS in Y, but $f^{-1}(G_2)$ = { ⟨ a, [0.3, 0.3], [–0.2, –0.2] ⟩, ⟨ b, [0.6, 0.6], [–0.7, –0.7] ⟩ } is not a BIVFCS in X, because bivfcl($f^{-1}(G_2)$) = $G_1^c \neq f^{-1}(G_2)$.

Theorem 2.5. Every BIVFCα continuous mapping is a BIVFCoGSPCM but not conversely.

Proof. Let A ⊆ Y a BIVFOS. Then $f^{-1}(A)$ is a BIVFαCS in X, by hypothesis. Hence $f^{-1}(A)$ is a BIVFGSPCS in X. Therefore f is a BIVFCoGSPCM.

Example 2.6. Let X = {a, b}, Y= {u, v} and G_1 = {⟨a, [0.1, 0.1], [–0.3, –0.3] ⟩, ⟨ b, [0.5, 0.5], [–0.8, –0.8] ⟩ }, G_2 = { ⟨ u, [0.4, 0.4], [–0.7, –0.7] ⟩, ⟨ v, [0.3, 0.3], [–0.5, –0.5] ⟩ }. Then τ = { 0_X, G_1, 1_X } and σ = { 0_Y, G_2, 1_Y } are BIVFT on X and Y respectively. Define a mapping f : (X, τ)→ (Y, σ) by f(a) = u and f(b) = v. Then f is a BIVFCoGSPCM but not a BIVFCα continuous mapping, since G_2 is a BIVFOS in Y, but $f^{-1}(G_2)$ = {⟨ a, [0.4, 0.4], [–0.7, –0.7] ⟩, ⟨ b, [0.3, 0.3], [–0.5, –0.5] ⟩ } is not a BIVFαCS in X, because bivfcl(bivfint (bivfcl($f^{-1}(G_2)$))) = bivfcl(bivfint (G_1^c) = bivfcl(G_1) = $G_1^c \not\subseteq f^{-1}(G_2)$.

Theorem 2.7. Every BIVFCP continuous mapping is a BIVFCoGSPCM but not conversely.

Proof. Let A ⊆ Y be a BIVFOS. Then $f^{-1}(A)$ is a BIVFPCS in X, by hypothesis. Hence $f^{-1}(A)$ is a BIVFGSPCS in X. Therefore f is a BIVFCoGSPCM.

Example 2.8. Let X = {a, b}, Y= {u, v} and G_1 ={⟨ a, [0.3, 0.3], [–0.5, –0.5] ⟩, ⟨ b, [0.6, 0.6], [–0.8, –0.8] ⟩ }, G_2 = { ⟨ u, [0.6, 0.6], [–0.7, –0.7] ⟩, ⟨ v, [0.5, 0.5], [–0.8, –0.8] ⟩ }. Then τ = { 0_X, G_1, 1_X } and σ = { 0_Y, G_2, 1_Y } are BIVFT on X and Y respectively. Define a mapping f : (X, τ)→ (Y, σ) by f(a) = u and f(b) = v. Then f is a BIVFCoGSPCM but not a BIVFCP continuous mapping, since G_2 is a BIVIFOS in Y but it is not a BIVFPCS in X, because bivfcl (bivfint ($f^{-1}(G_2)$)) = bivfcl(G_1) = $1_X \not\subseteq f^{-1}(G_2^C)$.

Theorem 2.9. Let f : X→ Y be a mapping. Then the following statements are equivalent:

(i) f is a BIVFCoGSPCM,

(ii) $f^{-1}(A)$ is a BIVFGSPOS in X for every BIVFCS A in Y.

Proof. (i) ⇒ (ii) Let A be a BIVFCS in Y. Then A^c is a BIVFOS in Y. By hypothesis, $f^{-1}(A^c)$ is a BIVFGSPCS in X. That is ($f^{-1}(A)$ $)^c$ is a BIVFGSPCS in X. Hence $f^{-1}(A)$ is a BIVFGSPOS in X.

(ii) ⇒ (i) Let A be a BIVFOS in Y. Then A^c is a BIVFCS in Y. By hypothesis, $f^{-1}(A^c)$ is a BIVFGSPOS in X. Hence $f^{-1}(A)$ is a BIVFGSPCS in X. Thus f is a BIVFCoGSPCM.

Theorem 2.10. Let f : X →Y is a bijective mapping. Suppose that one of the following properties hold:

(i) f^{-1}(bivfcl(B)) ⊆ bivfint(bivfspcl ($f^{-1}(B)$)) for each BIVFS B in Y,

(ii) bivfcl (bivfspint ($f^{-1}(B)$)) ⊆ f^{-1}(bivfint(B)) for each BIVFS B in Y,

(iii) f (bivfcl (bivfspint(A))) ⊆ bivfint(f(A)) for each BIVFS A in X,

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(iv) f (bivfcl(A)) ⊆ bivfint (f(A)) for each BIVFSPOS A in X.
Then f is a BIVFCoGSPCM.

Proof. (i) ⇒ (ii) is obvious by taking the complement in (i).
(ii) ⇒ (iii) Let A ⊆ X. Put B = f(A) in Y. This implies A = f^{-1}(f(A)) = f^{-1}(B) in X. Now bivfcl(bivfspint(A)) = bivfcl(bivfspint(f^{-1}(B))) ⊆ f^{-1}(bivfint(B)) by hypothesis. Therefore f(bivfcl(bivfspint(A))) ⊆ f(f^{-1}(bivfint(B))) = bivfint(B) = bivfint(f(A)). (iii) ⇒ (iv) Let A ⊆ X be a BIVFSPOS. Then bivfspint(A) = A. By hypothesis,f(bivfcl (bivfspint(A))) ⊆ bivfint(f(A)).Thereforef(bivfcl(A)) = f(bivfcl (bivfspint(A))) ⊆ bivfint(f(A)). Suppose (iv) holds: Let A be a BIVFOS in Y. Then f^{-1}(A) is a BIVFS in X and bivfspint(f^{-1}(A)) is a BIVFSPOS in X. Hence by hypothesis, f(bivfcl(bivfspint(f^{-1}(A)))) ⊆ bivfint(f(bivfspint(f^{-1}(A)))) ⊆ bivfint (f (f^{-1}(A))) = bivfint(A) ⊆ A. Therefore bivfcl (bivfspint (f^{-1}(A))) = f^{-1}(f (bivfcl (bivfspint (f^{-1}(A))))) ⊆ f^{-1}(A). Now bivfcl(bivfint (f^{-1}(A))) ⊆ bivfcl (bivfspint (f^{-1}(A))) ⊆ f^{-1}(A). This implies f^{-1}(A) is a BIVFPCS in X and hence a BIVFGSPCS in X. Thus f is a BIVFCoGSPCM.

Theorem 2.11. Let f : X →Y be a mapping. Suppose that one of the following properties hold:
(i) f(bivfspcl(A)) ⊆ bivfint(f(A)) for each BIVFS A in X,
(ii) bivfspcl(f^{-1}(B)) ⊆ f^{-1}(bivfint(B)) for each BIVFS B in Y,
(iii) f^{-1}(bivfcl(B)) ⊆ bivfspint(f^{-1}(B)) for each BIVFS B in Y.
Then f is a BIVFCoGSPCM.

Proof. (i) ⇒ (ii) Let B ⊆ Y. Then f^{-1}(B) is a BIVFS in X. By hypothesis, f(bivfspcl(f^{-1}(B))) ⊆ bivfint(f(f^{-1}(B))) ⊆ bivfint(B). Now bivfspcl(f^{-1}(B)) ⊆ f^{-1}(f(bivfspcl (f^{-1}(B)))) ⊆ f^{-1}(bivfint(B)).
(ii)⇒ (iii) is obvious by taking complement in (ii). Suppose (iii) holds. Let B be a BIVFCS in Y. Then bivfcl(B) = B and f^{-1}(B) is a BIVFS in X. Now f^{-1}(B) = f^{-1}(bivfcl(B)) ⊆ bivfspint(f^{-1}(B)) ⊆ f^{-1}(B), by hypothesis. This implies f^{-1}(B) is a BIVFSPOS in X and hence a BIVFGSPOS in X. Thus f is a BIVFCoGSPCM.

Theorem 2.12. Let f : X → Y be a bijective mapping. Then f is a BIVFCoGSPCM if bivfcl (f(A)) ⊆ f(bivfspint(A)) for every BIVFS A in X.

Proof. Let A be a BIVFCS in Y. Then bivfcl(A) = A and f^{-1}(A) is a BIVFS in X. By hypothesis bivfcl (f(f^{-1}(A))) ⊆ f(bivfspint (f^{-1}(A))). Since f is onto, f(f^{-1}(A)) = A. Therefore A = bivfcl(A) = bivfcl (f(f^{-1}(A))) ⊆ f(bivfspint(f^{-1}(A))). Now f^{-1}(A) ⊆ f^{-1}(f (bivfspint (f^{-1}(A)))) = bivfspint(f^{-1}(A)) ⊆ f^{-1}(A). Hence f^{-1}(A) is a BIVFSPOS in X and hence a BIVFGSPOS in X. Thus f is a BIVFCoGSPCM.

Theorem 2.13. Let f : X → Y is a BIVFCoGSPCM, where X is a BIVFSP$T_{1/2}$ space, then the following conditions are hold:
(i) bivfspcl(f^{-1}(B)) ⊆ f^{-1}(bivfint (bivfspcl(B))) for every BIVFOS B in Y, (ii) f^{-1}(bivfcl (bivfspint(B))) ⊆ bivfspint(f^{-1}(B)) for every BIVFCS B in Y,**Proof.** (i) Let B ⊆Y be a BIVFOS. By hypothesis f^{-1}(B) is a BIVFGSPCS in X. Since X is a BIVFSP$T_{1/2}$ space, f^{-1}(B) is a BIVFSPCS in X. This implies bivfspcl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(bivfint(B)) ⊆ f^{-1}(bivfint (bivfspcl(B))).
(ii) can be proved easily by taking complement in (i).

Theorem 2.14. (i) If f: X→Y is a BIVFCoGSPCM and g: Y→Z is a BIVF continuous mapping, then g of: X→ Z is a BIVFCoGSPCM.
(ii) If f : X→Y is a BIVFCoGSPCM and g : Y → Z is a BIVFC, then gof : X→ Z is a BIVFGSP continuous mapping.
(iii) If f : X→Y is a BIVFGSP irresolute mapping and g : Y → Z is a BIVFCoGSPCM, then gof : X→ Z is a BIVFCoGSPCM.

Proof. (i) Let A be a BIVFOS in Z. Then g^{-1}(A) is a BIVFOS in Y, since g is a BIVF continuous mapping. As f is a BIVFCoGSPCM, f^{-1}(g^{-1}(A)) is a BIVFGSPCS in X. Therefore gof is a BIVFCoGSPCM.
(ii) Let A be a BIVFOS in Z. Then g^{-1}(A) is a BIVFCS in Y, since g is a BIVFC mapping. As f is a BIVFCoGSPCM, f^{-1}(g^{-1}(A)) is a BIVFGSPOS in X. Therefore gof is a BIVFGSP continuous mapping.
(iii) Let A be a BIVFOS in Z. Then g^{-1}(A) is a BIVFGSPCS in Y, since g is a BIVFCoGSPCM. As f is a BIVFGSP irresolute mapping, f^{-1}(g^{-1}(A)) is a BIVFGSPCS in X. Therefore gof is a BIVFCoGSPCM.

Theorem 2.15. For a mapping f: X→Y, the following are equivalent, where X is a BIVFSP$T_{1/2}$ space:
(i) f is a BIVFCoGSPCM,
(ii) for every BIVFCS A in Y, f^{-1}(A) is a BIVFGSPOS in X,
(iii) for every BIVFOS B in Y, f^{-1}(B) is a BIVFGSPCS in X,
(iv) for any BIVFCS A in Y and for any BIVFP $p_{(\alpha, \beta)}$ ∈D^X, if f($p_{(\alpha, \beta)}$) q A, then $p_{(\alpha, \beta)}$ q bivfspint(f^{-1}(A)),
(v) for any BIVFCS A in Y and for any BIVFP $p_{(\alpha, \beta)}$∈D^X, if f($p_{(\alpha, \beta)}$) q A, then there exists a BIVFGSPOS B such that $p_{(\alpha, \beta)}$ q B and f(B) ⊆ A.

Proof. (i) ⇔ (ii) and (ii) ⇔ (iii) are obvious.
(ii) ⇒ (iv) Let A ⊆ Y be a BIVFCS and let $p_{(\alpha, \beta)}$ ∈D^X. Let f($p_{(\alpha, \beta)}$) q A. Then $p_{(\alpha, \beta)}$ q f^{-1}(A). By hypothesis, f^{-1}(A) is a BIVFGSPOS in X. Since X is a BIVFSP$T_{1/2}$ space, f^{-1}(A) is a BIVFSPOS in X. This implies bivfspint(f^{-1}(A)) = f^{-1}(A). Hence $p_{(\alpha, \beta)}$ q bivfspint(f^{-1}(A)).
(iv) ⇒ (ii) Let A ⊆ Y be a BIVFCS and let $p_{(\alpha, \beta)}$ ∈D^X. Let f ($p_{(\alpha, \beta)}$) q A. Then $p_{(\alpha, \beta)}$ q f^{-1}(A). By hypothesis, $p_{(\alpha, \beta)}$ q bivfspint(f^{-1}(A)). That is f^{-1}(A) ⊆ bivfspint(f^{-1}(A)). But we have bivfspint(f^{-1}(A)) ⊆ f^{-1}(A). Therefore f^{-1}(A) = bivfspint(f^{-1}(A)). Thus f^{-1}(A) is a BIVFSPOS in X and hence a BIVFGSPOS in X.
(iv) ⇒ (v) Let A ⊆ Y be a BIVFCS and let $p_{(\alpha, \beta)}$ ∈D^X. Let f ($p_{(\alpha, \beta)}$) q A. Then $p_{(\alpha, \beta)}$ q f^{-1}(A). By hypothesis $p_{(\alpha, \beta)}$ q bivfspint(f^{-1}(A)). Thus f^{-1}(A) is a BIVFSPOS in X and hence a BIVFGSPOS in X. Let f^{-1}(A) = B. Therefore $p_{(\alpha, \beta)}$ q B and f (B) = f(f^{-1}(A)) ⊆ A.

(v) ⇒ (iv) Let A ⊆ Y be a BIVFCS and let $p_{(\alpha, \beta)} \in D^X$. Let $f(p_{(\alpha, \beta)})$ q A. Then $p_{(\alpha, \beta)}$ q $f^{-1}(A)$. By (v) there exists a BIVFGSPOS B in X such that $p_{(\alpha, \beta)}$ q B and f(B) ⊆ A. Let B = $f^{-1}(A)$. Since X is a $BIVFSPT_{1/2}$ space, $f^{-1}(A)$ is a BIVFSPOS in X. Therefore $p_{(\alpha, \beta)}$ q bivfspint($f^{-1}(A)$).

Theorem 2.16. A mapping f : X → Y is a BIVFCoGSPCM, if f^{-1}(bivfspcl(B)) ⊆ bivfint (f^{-1}(B)) for every BIVFS B in Y.

Proof. Let B ⊆ Y be a BIVFCS. Then bivfcl(B) = B. Since every BIVFCS is a BIVFSPCS, bivfspcl(B) = B. Now by hypothesis, f^{-1}(B) = f^{-1}(bivfspcl(B)) ⊆ bivfint(f^{-1}(B)) ⊆ f^{-1}(B). This implies f^{-1}(B) is a BIVFOS in X. Therefore f is a BIVFC mapping. Then by Theorem 1.21, f is a BIVFCoGSPCM.

Theorem 2.17. A mapping f : X → Y is a BIVFCoGSPCM, where X is a $BIVFSPT_{1/2}$ space if and only if f^{-1}(bivfspcl(B)) ⊆ bivfspint(f^{-1}(bivfcl(B))) for every BIVFS B in Y.

Proof. Necessity. Let B ⊆ Y be a BIVFS. Then bivfcl(B) is a BIVFCS in Y. BY hypothesis f^{-1}(bivfcl(B)) is a BIVFGSPOS in X. Since X is a $BIVFSPT_{1/2}$ space, f^{-1}(bivfcl(B)) is a BIVFSPOS in X. Therefore f^{-1}(bivfspcl(B)) ⊆ f^{-1}(bivfcl(B)) = bivfspint(f^{-1}(bivfcl(B))).

Sufficiency. Let B ⊆ Y be a BIVFCS. Then bivfcl(B) = B. By hypothesis, f^{-1}(bivfspcl(B)) ⊆ bivfspint (f^{-1}(bivfcl(B))) = bivfspint (f^{-1}(B)). But bivfspcl(B) = B. Therefore f^{-1}(B) = f^{-1}(bivfspcl(B)) ⊆ bivfspint (f^{-1}(B)) ⊆ f^{-1}(B). This implies f^{-1}(B) is a BIVFSPOS in X and hence a BIVFGSPOS in X. Hence f is a BIVFCoGSPCM.

Theorem 2.18. A BIVF continuous mapping f : X → Y is a BIVFCoGSPCM if BIVFGSPO(X) = BIVFGSPC(X).

Proof: Let A ⊆ Y be a BIVFOS. By hypothesis, f^{-1}(A) is a BIVFOS in X and hence is a BIVFGSPOS in X. Since BIVFGSPO(X) = BIVFGSPC(X), f^{-1}(A) is a BIVFGSPCS in X. Therefore f is a BIVFGSPCM.

REFERENCES

[1] Azhagappan.M and M.Kamaraj, “Notes on bipolar valued fuzzy rw-closed and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces”, *International Journal of Mathematical Archive,* 7(3), 30-36, 2016.
[2] Chang.C.L., “Fuzzy topological spaces”, *JI. Math. Anal. Appl.,* 24(1968), 182 –190.
[3] Dontchev.J., “On generalizing semipreopen sets”, *Mem. Fac. sci. Kochi. Univ. Ser. A, Math.,*16 (1995), 35 – 48.
[4] Indira.R, Arjunan.K and Palaniappan.N, “Notes on interval valued fuzzy rw-closed, interval valued fuzzy rw-open sets in interval valued fuzzy topological space”, *International Journal of Computational and Applied Mathematics.,*Vol.3, No.1(2013), 23 –38.
[5] Jeyabalan.R and K. Arjunan, “Notes on interval valued fuzzy generalized semipreclosed sets”, *International Journal of Fuzzy Mathematics and Systems,* Volume 3, Number 3 (2013), 215 –224.
[6] Jeyabalan,R and Arjunan.K, “Interval valued fuzzy generalized semi-preclosed mappings”, *IOSR Journal of Mathematics, Vol.8, Issue 5(2013), 40 – 47.*
[7] Kongeswaran.P, K.Arjunan & K.L.Muruganantha Prasad,”Bipolar interval valued fuzzy generalized semipreopen sets”, *Journal of information and computational science,* Vol.9, Iss. 8(2019), 232 –239.
[8] Kongeswaran.P, K.Arjunan & K.L.Muruganantha Prasad,”Bipolar interval valued fuzzy completely generalized semiprecontinuous mappings”, *Infokara,* Vol.8, Iss. 8(2019), 309 –316.
[9] Lee, K.M., “Bipolar-valued fuzzy sets and their operations”, *Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand,* (2000), 307 –312.
[10] Levine.N, “Generalized closed sets in topology”, *Rend. Circ. Math. Palermo,* 19 (1970), 89 –96.
[11] Mondal.T.K., “Topology of interval valued fuzzy sets”, *Indian J. Pure Appl.Math.* 30, No.1 (1999), 23 – 38.
[12] Saraf.R.K and Khanna.K., “Fuzzy generalized semipreclosed sets”, *Jour.Tripura. Math.Soc.,* 3 (2001), 59 – 68.
[13] Selvam.R, K.Arjunan & KR.Balasubramanian,”Bipolar interval valued multi fuzzy generalized semipreclosed sets”, *JASC Journal,* Vol.6, ISS. 5(2019), 2688 –2697.
[14] Vinoth.S & K.Arjunan, “A study on interval valued intuitionistic fuzzy generalized semipreclosed sets”, *International Journal of Fuzzy Mathematics and Systems,* Vol. 5, No. 1 (2015), 47 – 55.
[15] Vinoth.S & K.Arjunan, “Interval valued intuitionistic fuzzy generalized semipreclosed mappings”, *International Journal of Mathematical Archive,* 6(8) (2015), 129 –135.
[16] Zadeh.L.A., “Fuzzy sets”, *Information and control,* Vol.8 (1965), 338 –353.