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Bilinear Predictive Law of the Thermal Conductivity of Tropical Woods with Respect to the Infradensity and the Moisture Content, using Schwartz conditions

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***Abstract*— An experimental study led to linear regressions of the density of tropical woods. These regressions are so forth used to compute the variations of the thermal conductivity with moisture content, by using a series-parallel model in which Wood is considered as an association of wet cells walls (cross and side ones) and air. Linear approximation is done of these variations as well as the ones of the thermal conductivity with respect to basal density (or infradensity). This double linear variation is then used to determine a bilinear law linking the moisture content and the infradensity to the thermal conductivity of tropical wood. The obtained empirical law, subjected to obey the Schwartz condition on partial derivatives, enables the prediction of the thermal conductivity of any tropical wood species, at any desired moisture content, if at least its infradensity is known.**

Keywords— Thermal conductivity; Tropical woods; Infradensity; Moisture content; Porosity; Schwartz conditions

I. INTRODUCTION

Mastering thermal conductivity of materials has constantly left impression on the sensitivity of engineers and architects, all over the years, especially in this century in which man is so attentive to energy related problems, namely for thermal comfort as well as for the material conditioning. In fact, greenhouse gases (GHG) emissions, responsible for the global warming of the planet, is mainly due to the high energy consumption through the use of devices providing a good thermal comfort in the interior of the building, but which strongly emit carbon dioxide (CO_2). This high consumption rate observed is due either to poor insulation of the building or the use of materials with high coefficient value of thermal conductivity [1]. Finally, we also know that the insulation has an important role in the thermal and acoustic applications [2]. Thus, the importance of thermal conductivity is no longer to be shown.

Thermal conductivity can be reached by using transient methods ([3] and [4]). An experimental study, based on a permanent regime technique [5], has been used on woods, and led us to some abacus [6]. Wood, the only renewable raw material to our knowledge, has thus been the subject of several research works whose most important results have been summarized by Siau in [7]. In fact, Siau presents the variations of the thermal conductivity with porosity, but porosity which is a function of moisture content and specific weight of the species; now specific weight varies enormously for the same sample. Infradensity is a characteristic parameter of wood species used in calculations ([8] and [9]). We make an advantageous use of it in order to make thermal conductivity of tropical wood become more accessible. As Gordillo-Delgado F did in [10], the species we use are taken from the bottom, middle and top culm regions of the plants. The purpose of this study is to propose a mathematical model allowing an easier prediction of the thermal conductivity of tropical woods.

Nomenclature

GHG	Greenhouse gases
D_i	Infradensity or basal density (kg.m^{-3})
a	Side of a lumen (m^2) ; ratio between the width of lumen and the total width of the c
R	Thermal resistance
V_a	porosity ($V_a = a^2$)
λ'	Thermal conductivity of the cells walls substance (W.m^{-1}.K^{-1})
λ_a	Thermal conductivity of the air in the lumen (W.m^{-1}.K^{-1})
ρ_ω	Density of liquid water (1000 kg.m^{-3})
λ_T	Transversal thermal conductivity of wood (W.m^{-1}.K^{-1})
$\lambda_{//}$	Parallel or Tangential or Axial thermal conductivity of wood (W.m^{-1}.K^{-1})
Z	Part of normal walls effective for conductive flux
W_1	Fraction of the normal wall, adjacent to the lumen and that could be considered as effective for conduction
G	specific weight
M_0	Dry mass (kg)
H	Moisture content (%)

II. MATERIALS AND METHODS

The study is carried out on five specie (Tali, Bilinga, Sapelli, Sipo and Ayous) covering a wide range of densities of most of the woods used in Cameroon. Experimental values of thermal conductivities are obtained by means of a permanent regime method called ‘‘Méthode des boîtes’’ and whose experimental device is clearly described by NGOHE in [6], [11] and [14]; as for the working techniques, it is based on a semi-empirical model developed by Siau in [7].

A. Theoretical Equation of Thermal Conductivity of Tropical Woods.

A.1. Conduction in the transversal (orthoradial) direction

Siau [7] presents a geometric model of longitudinal wood’s cells (tracheid, fibers, vessel, parenchyma,…) with the following hypotheses (see Fig. 1): the cell is everywhere unitary in dimensions’ viewpoint; tangential and radial (with respect to the flow) cells walls have the same

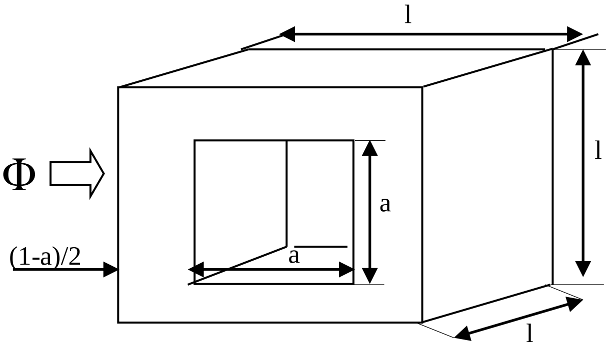

Fig. 1: Geometric model for a wood’s longitudinal cell

thickness , and the proportion of the cells walls thickness with respect to the diameter is the same for all the cells of a given species; the presence of transversal oriented cells, of walls on the edge of the cell, and of pit openings, are neglected, and the lumen is supposed to have a unitary length and a square section of side a.

Siau carries out the mathematical analysis of that model,

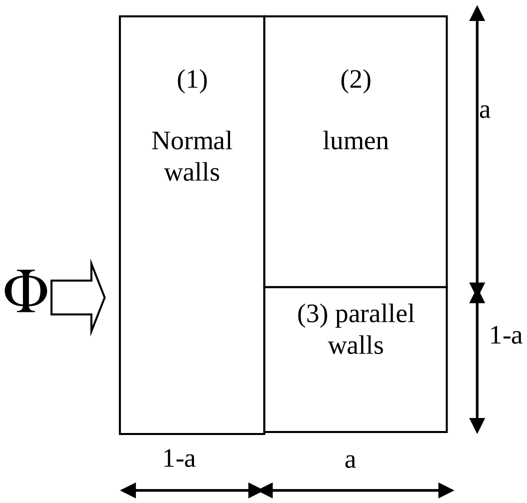

Fig. 2: Physical considerations of Siau (1984) for the geometric model

with the heat flux in the transversal direction, by referring himself to Fig. 2 below.

But this mathematical study can also be carried out using Fig. 3. The only difference in both considerations is the importance given to the conduction through parallel cells. The latter consideration is chosen because it gives more reliable results from the mathematical viewpoint, especially as the Schwartz conditions,

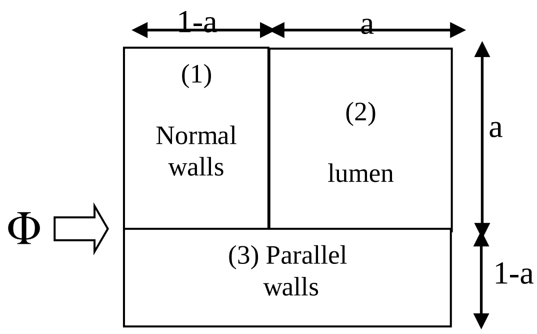

Fig. 3: Adopted configuration for the geometric model

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i.e. the crossed partial derivatives of the found bilinear law, are concerned.

The three parts (Fig. 3) have the following thermal resistances:

$$R_1 = \frac{1-a}{a\lambda'}, R_2 = \frac{1}{\lambda_a} \text{ and } R_3 = \frac{1}{\lambda'(1-a)}$$

The transversal conductivity λ_T of the cell is then given by:

$$\lambda_T = \frac{a(1-a)\lambda'^2 + [(1-a)^2 + a]\lambda_a\lambda'}{a\lambda' + (1-a)\lambda_a} \quad (1)$$

Where *a* stands for the ratio between the width of lumen and the total width of the cell (a^2 = porosity Va according to the hypothesis of the model); λ' and λ_a are respectively the thermal conductivity of the cells walls substance and that of the air in the lumen.

In reality, there is a concentration of thermal flux on lateral walls as shown in Fig. 4, and this makes the flux not uniform on them, as opposed to the hypothesis used to obtain equation (1).

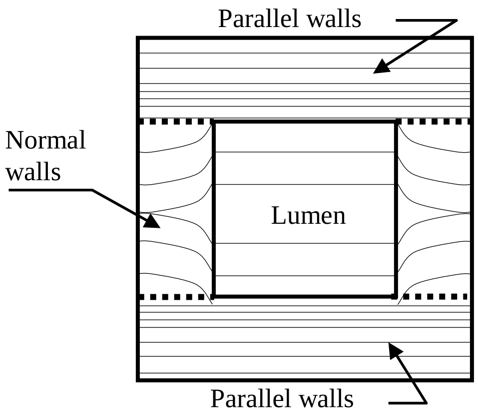

Fig. 4: Flux flow in the cell

Siau [7] then introduces a factor Z in order to bear in mind the fact that, because of non-uniformity, a part of normal walls is not effective for the conductive flux. With this new consideration, our physical model is modified according to Fig. 5 below.

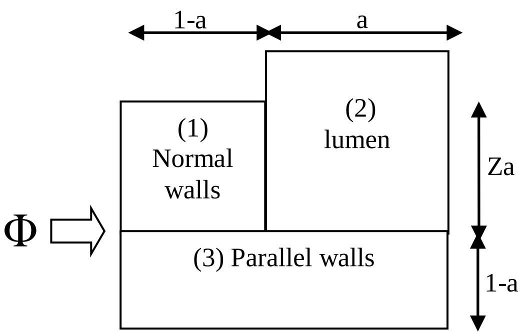

Fig. 5: Effective physical consideration

This confers to the normal wall a thermal resistance given by $R'_1 = \frac{1-a}{\lambda'.Z..a}$; and the transversal thermal conductivity becomes:

$$\lambda_T = \frac{Z.a(1-a)\lambda'^2 + [(1-a)^2 + Z.a]\lambda_a\lambda'}{Z.a\lambda' + (1-a)\lambda_a} \quad (2)$$

The works of Hart [12] and Siau [7] then appear very necessary for the evaluation of the factor Z. In fact, with a null conduction assumed in the lumen, Hart [12] proposes an empirical relationship for the calculation of the fraction W_1 of the normal wall, adjacent to the lumen and that could be considered as effective for conduction as follows:

$$W_1 = 0.48\frac{(1-a)}{a}\left[1 - e^{-2.08\frac{a}{(1-a)}}\right] \quad (3)$$

From the definition of W_1, it's easy to understand that, with a null conduction in the lumen, the width's portion of the normal wall adjacent to the lumen and considered as effective in conduction, divided by the total width (here, a) of the normal wall, is equal to $W_1 a$ [7]. However, in the case of heat flow, there's a significant flux in the lumen, because of the air conductivity. Since flux concentration in the lumen is $\frac{\lambda_a}{\lambda'}$ times that in the lateral walls, the additional fraction of the flux moving through the width of the normal wall is totally conductive, and can be obtained by the following relationship:

$$Z = a.W_1 + a.\frac{\lambda_a}{\lambda'} = a\left[W_1 + \frac{\lambda_a}{\lambda'}\right] \quad (4)$$

Note: According to the physical model there is, in addition to $W_1 a$, no longer any remaining part of the normal wall that is adjacent to the lumen. And consequently, the factor (1-a) of Siau [7] disappears and the Z expression is the one above.

For λ_a and λ' respectively taken, like Siau did in [7], equal to 1.0 10^{-4} and 10.5 10^{-4} cal/cm.°C.s (that is 418 and 4389 10^{-4} W/m.K, knowing that 1 cal/cm.°C.s = 418 W/m.K), equations (3) and (4) allow to get the transversal conductivity λ_T as a function of *a*. For the values of *a* going from 0.01 to 0.99, we obtain a much stronger linear regression ($R^2 = 0.99$) that permits us to conclude that:

$$\lambda_T = 10.61 - 9.53a \quad \text{for } 0.01 \le a \le 0.99 \quad (5)$$

A.2. Conduction in the longitudinal direction

When the geometric model of Fig. 1 is considered in relation to the flux flow in the fibers' direction, it results in the following equation, giving the thermal conductivity in the longitudinal direction:

$$\lambda_{//} = \lambda''(1-a^2) + \lambda_a a^2 \quad (6\text{-a})$$

Or :

$$\lambda_{//} = \lambda''(1-V_a) + \lambda_a V_a \quad (6\text{-b})$$

Va still designates the porosity and *a* the width of the lumen;

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Vol. 6 Issue 02, February-2017

λ'' = 21.0 10^{-4} cal/cm.°C.s and λ_a = 1.0 10^{-4} cal/cm.°C.s respectively stand for the conductivity of the walls cells' substance in the longitudinal direction and that of air in the lumen.

B – Application to the Determination of the Bilinear Law for the Thermal Conductivity of Tropical Woods

B – 1 Conduction in the transversal (orthoradial) direction

In order to relate λ_T to the moisture content H and to the infradensity D_i of wood, we proceed as follows:

By defining the specific weight G of a wood sample of volume V at humidity H as the ratio of its dry mass and the mass of the water it would move by immersion, it appears in [7] that porosity V_a of wood ($V_a = a^2$) can be approached by the following relationship:

$$V_a = 1 - G\left(0.667 + \frac{H}{100}\right) \qquad (7)$$

Where, by definition:

$$G = \frac{M_0}{\rho_w . V} \qquad (8).$$

Now, equation (8) leads to this other one:

$$G = \frac{\rho}{\rho_{H_2O}} . \frac{1}{1 + \frac{H}{100}} \qquad (9)$$

NGOHE [14] gives, for a given species (so a given infradensity), the variations law of the density with respect to the moisture content. With this, it becomes possible for several values of infradensity to establish a table of values for G (using equation 9), then for a (using equation 7), and lastly for the transversal conductivity λ_T (equation 5), with moisture content H and the infradensity D_i. Table 1-a below presents the transversal conductivity coming from Siau's model, while table 1-b below presents its experimental values.

Table 1-a: Transversal conductivity of tropical wood λ_T (10^{-3} W.m^{-1}K^{-1}) as a function of basal density and moisture content (experimental values)

Moisture content H(%)	D_i (infradensity)									
	0,335	0,360	0,343	0,733	0,719	0,670	0,647	0,702	0,704	0,536
0	116,7	141,2	122,9	184,0	184,0	184,6	182,9	180,2	204,8	168,6
12	140,8	157,2	139,3	226,2	230,2	233,0	223,3	237,6	256,2	200,5
15	146,9	161,2	143,5	236,8	241,8	245,1	233,5	251,9	269,0	208,5
20	156,9	167,8	150,3	254,4	261,0	265,2	250,3	275,8	290,4	221,8
40	197,1	194,4	177,7	324,8	338,0	345,8	317,7	371,4	376,0	275,0
60	237,3	221,0	205,1	395,2	415,0	426,4	385,1	467,0	461,6	328,2

Moisture content H(%)	D_i (infradensity)								
	0,555	**0,522**	**0,562**	**0,603**	**0,494**	**0,454**	**0,579**	**0,784**	**0,764**
0	153,3	170,4	154,6	192,7	148,8	86,8	110,2	213,3	206,6
12	183,8	203,0	189,6	216,5	163,7	118,8	146,8	263,3	253,6
15	191,4	211,2	198,4	222,4	167,4	126,9	156,0	275,9	265,4
20	204,1	224,8	213,0	232,3	173,6	140,2	171,2	296,7	285,0
40	254,9	279,2	271,4	271,9	198,4	193,6	232,2	380,1	363,4
60	305,7	333,6	329,8	311,5	223,2	247,0	293,2	463,5	441,8

International Journal of Engineering Research & Technology (IJERT)
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Vol. 6 Issue 02, February-2017

Table 1-b: Transversal conductivity of tropical wood λ_T ($10^{-3}W.m^{-1}K^{-1}$) as a function of basal density and moisture content (Siau's model)

Moisture content H(%)	D_i (infradensity)								
	0,335	0,360	0,343	0,733	0,719	0,670	0,647	0,702	0,704
0	91,6	97,9	90,6	164,4	183,3	172,6	163,7	176,4	177,7
12	102,6	109,7	101,4	187,9	211,4	198,0	188,3	203,9	205,2
15	105,4	112,7	104,2	194,0	218,7	204,6	194,6	211,1	212,4
20	110,1	117,7	108,8	204,3	231,3	215,9	205,5	223,5	224,6
40	129,6	138,5	127,9	248,9	287,6	265,0	252,9	278,5	279,4
60	150,0	160,3	147,9	301,7	362,0	325,5	310,2	349,9	350,2

Moisture content H(%)	D_i (infradensity)									
	0,536	0,555	0,522	0,562	0,603	0,494	0,454	0,579	0,784	0,764
0	144,0	151,2	131,5	148,1	158,6	132,0	124,3	153,0	203,0	203,2
12	162,7	170,0	150,0	167,2	178,9	148,4	139,4	171,9	234,8	233,1
15	167,4	174,8	154,8	172,1	184,1	152,6	143,2	176,7	243,2	241,0
20	175,5	182,8	162,9	180,3	192,8	159,6	149,5	184,7	257,6	254,5
40	209,0	216,4	197,2	214,9	229,6	188,5	175,9	218,3	324,9	315,7
60	246,0	253,2	235,6	253,1	270,7	219,7	203,9	255,1	433,5	404,2

The horizontal exploitation of table 1-b allows us to plot using points the variations of the transversal thermal conductivity of tropical woods as a function of infradensity, for the various values of moisture content (one curve by moisture content), as shown on Fig. 6.

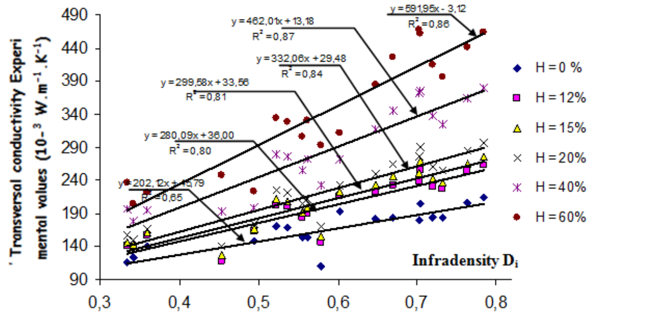

Fig. 6 : Linear correlation of the variations of experimental transversal conductivity λ_T with infradensity D_i (for fixed moisture content)

On the other hand, the vertical exploitation of that table gives rise to the layout of the transversal conductivity as a function of moisture content, for several values of the infradensity (one curve by infradensity), as shown in Fig. 7.

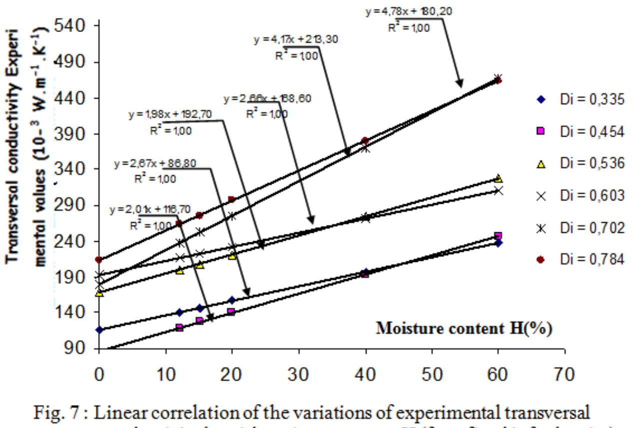

Fig. 7 : Linear correlation of the variations of experimental transversal conductivity λ_T with moisture content H (for a fixed infradensity)

This double linearity then suggests, for transversal thermal conductivity, a bilinear law in the following form:

$$\lambda_T (H,D_i) = \alpha H D_i + \beta D_i + \gamma H + \delta \qquad (12)$$

Now, in order to consider λ_T as a function only of variables H and D_i, one needs to first verify the following Schwartz's condition for the crossed partial derivatives of λ_T:

$$\frac{\partial^2}{\partial H \partial D_i}\lambda_T \approx \frac{\partial^2}{\partial D_i \partial H}\lambda_T \qquad (13)$$

International Journal of Engineering Research & Technology (IJERT)
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Vol. 6 Issue 02, February-2017

Equation (13), with respect to (10) and (11), comes to simply compare $\frac{\partial}{\partial H} a_D$ and $\frac{\partial}{\partial D_i} a_H$. Tables 2 and 3 give the variations $a_D(H)$ and $a_H(D_i)$ which, when plotted as in Fig. 8 and Fig.10 below, enable us to deduce the values 5.6 and 6.2 for those partial derivatives. We then conclude their near equality with a relative uncertainty of about 10%.

Table 2 : Variations of coefficients $a_D(H)$ and $b_D(H)$

Coefficients	Moisture content H(%)					
	0	12	15	20	40	60
$a_D(H)$	202,12	280,09	299,58	332,06	462,02	591,95
$b_D(H)$	45,79	36	33,56	29,48	13,18	-3,12

Table 3 : Variations of coefficients $a_H(D_i)$ and $b_H(D_i)$

Coefficients	D_i (infradensity)					
	0,335	0,454	0,536	0,603	0,702	0,784
$a_H(D_i)$	2,01	2,67	1,98	2,66	4,17	4,78
$b_H(D_i)$	116,7	86,8	192,7	168,6	213,3	180,2

The condition of equaling crossed derivatives is what made us finally adopt the physical model on Fig. 3 rather than that of Siau (Fig. 2) which gives a relative gap greater than 25%.

The procedure for getting empirical values of coefficients α, β, γ and δ of equation (12) is as follows:

Graphics $a_D = a_D(H)$, $b_D = b_D(H)$, $a_H = a_H(D_i)$ and $b_H = b_H(D_i)$ are represented respectively on figures 8, 9, 10 and 11; they also show a strong linear correlation (R^2>0.8) except for $b_H(D_i)$ (which has $R^2 \approx 0{,}6$).

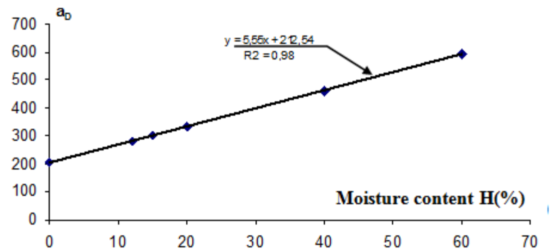


Fig. 8 : Variations of coefficient $a_D(H)$ with the moisture content H

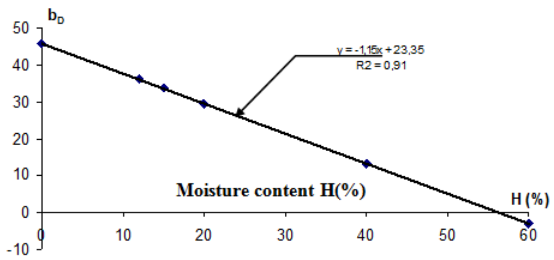


Fig. 9 : Variations of coefficient $b_D(H)$ with the moisture content H

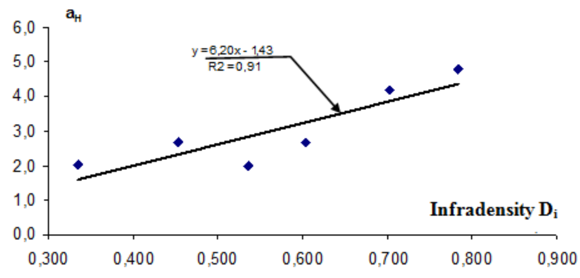
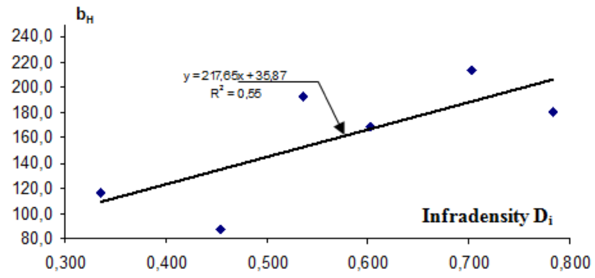


Fig. 10 : Variations of coefficient $a_H(D_i)$ with the infradensity D_i



11 : Variations of coefficient $b_H(D_i)$ with the infradensity D_i

With the linear forms $a_D(H) = \alpha_1 H + \beta_1$, $b_D(H) = \gamma_1 H + \delta_1$, $a_H(D_i) = \alpha_2 D_i + \beta_2$ and $b_H(D_i) = \gamma_2 D_i + \delta_2$, equations (10) and (11) respectively give the following ones:

$$\lambda_T(H, D_i) = \alpha_1 H D_i + \beta_1 D_i + \gamma_1 H + \delta_1 \quad (14)$$

and

$$\lambda_T(H, D_i) = \alpha_2 H D_i + \gamma_2 D_i + \beta_2 H + \delta_2 \quad (15)$$

It's then needed, by identification, the following factors be nearer in pairs: α_1 and α_2, β_1 and γ_2, γ_1 and β_2, and δ_1 and δ_2. We observe, from figures 8, 9, 10 and 11, the following values: α_1= 5.55, α_2= 6.20 , β_1= 212.54, γ_2= 217.65, γ_1= -1.15, β_2= -1.49, δ_1= 23.35 et δ_2= 35.87 ; These values lead to the following mean values for the coefficients: $\alpha=(\alpha_1+\alpha_2)/2=5.9$, $\beta=(\beta_1+\gamma_2)/2 = 215.1$, $\gamma = (\gamma_1+\beta_2)/2 = -1.29$ and $\delta=(\delta_1+\delta_2)/2 = 24.2$ with respective relative gaps of 11%, 3%, 22% and 7%. We then conclude with the following expression for the bilinear model of the conductivity of tropical woods, in the transversal direction:

$$\lambda_T (H,D_i) = 5.9\, HD_i + 215.1\, D_i - 1.29\, H + 24.2 \quad (16)$$

where λ_T is in 10^{-3} $W.m^{-1}.K^{-1}$, H in % and D_i without unit.

Figures 12, 13, 14, 15 and 16, show the comparison of the experimental measures with both bilinear model and Siau's model above.

International Journal of Engineering Research & Technology (IJERT)
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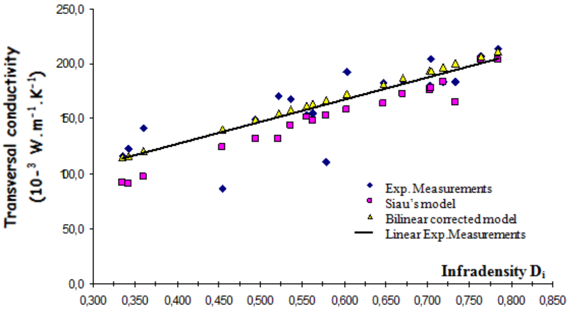

Fig.12 : Comparison graphic of experimental transversal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 0%

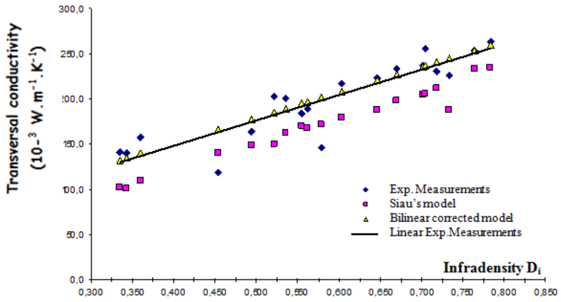

Fig. 13 : Comparison graphic of experimental transversal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 12%

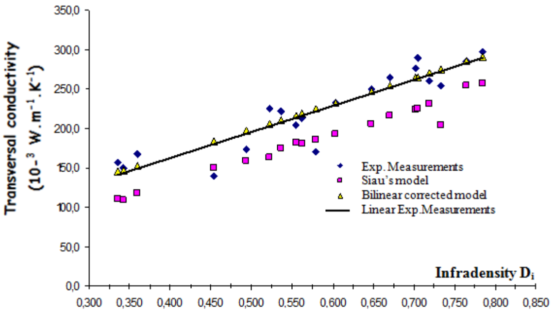

Fig. 14 : Comparison graphic of experimental transversal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 20%

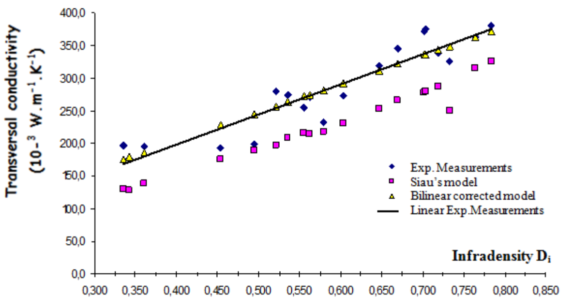

Fig. 15: Comparison graphic of experimental transversal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 40%

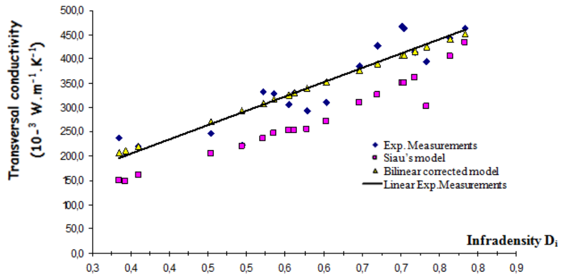

Fig. 16: Comparison graphic of experimental transversal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 60%

These five figures show an increasing difference between the bilinear model and the experimental values when moisture content increases. We then conclude that coefficients α and γ have to be rectified, so that the model given under the form (16) considerably fits the experimental measures. Even the relative gaps of 11% and 22% that we got above for these coefficients could have suggested their rectification. We then require the solver of Excel 2003, in which coefficients α and γ are variables and we choose as target, that the mean value of the sum of all relative gaps must be minimum. This gap is between the values given by the bilinear model and those got experimentally.

The solver so generates new values of these coefficients, with the mean value of relative gap of 8%, when we also allow coefficient δ to vary. The rectified bilinear model is then the following:

$$\lambda_T\,(H,D_i) = 555.0\ HD_i + 215.1\ D_i - 33.7\ H + 42.6 \qquad (17)$$

B – 2 Conduction in the longitudinal direction

The same work done for transversal conductivity is carried up for the samples cut up in the longitudinal or axial direction; table 4-a gives the values of $\lambda_{//}$ coming from Siau's model (equation 6-b) for several values of H and Di, and table 4-b those obtained experimentally.

Table 4-a : Longitudinal conductivity of tropical wood $\lambda_{//}$ ($10^{-3}W.m^{-1}K^{-1}$) as a function of basal density and moisture content (Siau's model)

Moisture content H(%)	D_i (infradensity)							
	0,330	0,359	0,710	0,568	0,610	0,511	0,517	0,782
0	231,6	254,6	474,1	377,0	441,5	367,9	345,5	530,5
12	264,3	289,7	538,7	431,8	494,0	408,2	393,6	596,5
15	272,4	298,5	554,7	445,4	506,9	418,0	405,5	612,7
20	286,0	313,0	581,2	468,0	528,1	434,2	425,3	639,5
40	340,2	370,8	685,8	557,9	611,2	497,0	504,0	744,5
60	394,2	428,3	788,9	647,1	691,9	557,6	581,9	846,8

Tableau 4-b : Longitudinal conductivity of tropical wood $\lambda_{//}$ ($10^{-3}W.m^{-1}K^{-1}$) as a function of moisture content and basal density (experimental values)

Moisture content H(%)	D_i (infradensity)							
	0,330	0,359	0,710	0,568	0,610	0,511	0,517	0,782
0	268,8	259,5	437,7	354,9	427,6	352,2	313,7	527,7
12	288,7	283,5	489,1	389,3	464,2	388,1	343,5	584,3
15	293,7	289,5	501,9	398,0	473,4	397,1	350,9	598,5
20	302,0	299,5	523,3	412,3	488,6	412,0	363,3	622,1
40	335,2	339,5	608,9	469,7	549,6	471,8	412,9	716,5
60	368,4	379,5	694,5	527,1	610,6	531,6	462,5	810,9

The bilinear model is established, this time, with the coefficients written with a superscript << ' >> ; that is for example $\lambda_{//} = \lambda_{//}(D_i) = a'_D(H)D_i + b'_D(H)$ for a fixed H. Tables 5 and 6 give the variations of these new coefficients respectively as a function of the moisture content and the infradensity.

Table 5 : Variations of coefficients $a'_D(H)$ and $b'_D(H)$

Coefficients	Moisture content H (%)					
	0	12	15	20	40	60
$a'_D(H)$	564,9	642,7	662,19	694,62	824,34	954,06
$b'_D(H)$	57,99	51,38	49,73	46,97	35,96	24,95

Table 6 : Variations of coefficients $a'_H(D_i)$ and $b'_H(D_i)$

Coefficients	D_i (infradensity)							
	0,33	0,359	0,511	0,517	0,568	0,61	0,71	0,782
$a'_H(D_i)$	1,66	2	2,99	2,48	2,87	3,05	4,28	4,72
$b'_H(D_i)$	268,8	259,5	352,2	313,7	354,9	427,6	437,7	527,7

On figures 17 to 20 we obtain the evolution laws of these coefficients.

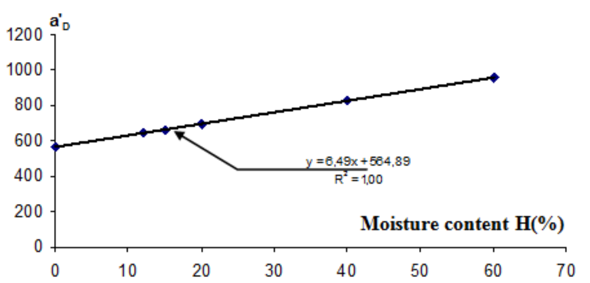

Fig. 17: Variations of coefficient $a'_D(H)$ with the moisture content

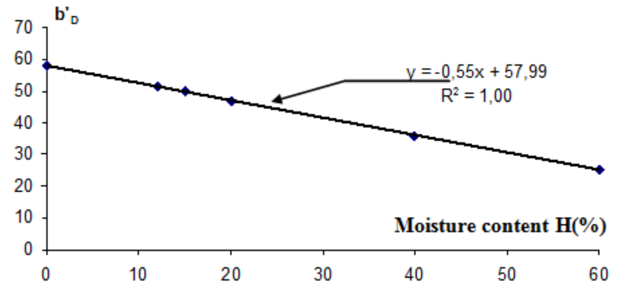

Fig. 18 : Variations of coefficient $b'_D(H)$ with the moisture content

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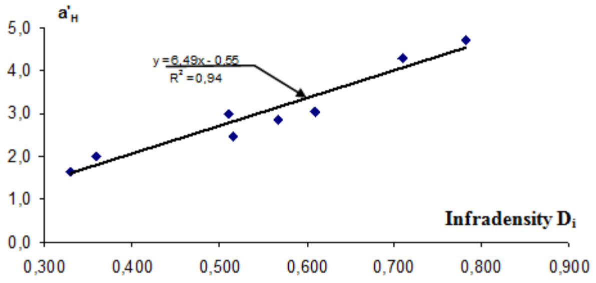

Fig. 19 : Variations of coefficient $a'_H(D_i)$ with infradensity

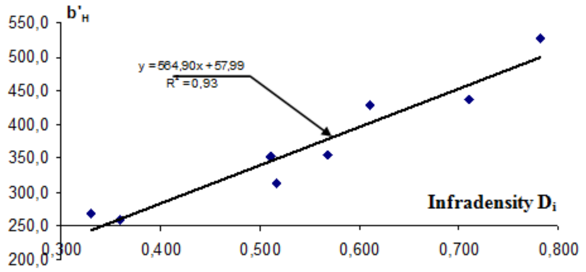

Fig. 20 : Variations of coefficient $b'_H(D_i)$ with infradensity

The cross derivatives of the longitudinal conductivity are equal almost with a relative gap of 3%, and we obtain their mean values as follows: α' = 6.5, β' = 564.9, γ' = -0.55 and δ' = 58, with relative gaps of less than 5%.

Then the variation law of the thermal conductivity in the longitudinal direction is given by:

$$\lambda_{//}(H,D_i) = 6.5\, HD_i + 564{,}9\, D_i - 0.55\, H + 58 \qquad (18)$$

with $\lambda_{//}$ in $10^{-3}\ W.m^{-1}.K^{-1}$, H in % and D_i without unit.

A rectification of this model with the Excel 2003's solver seems not necessary, since the mean relative gap obtained from the values given by the model of equation (18) with respect to the experimental ones is lower than 9%. Figures 21 to 25 below show its various comparisons with the experimental values and those coming from the Siau's model.

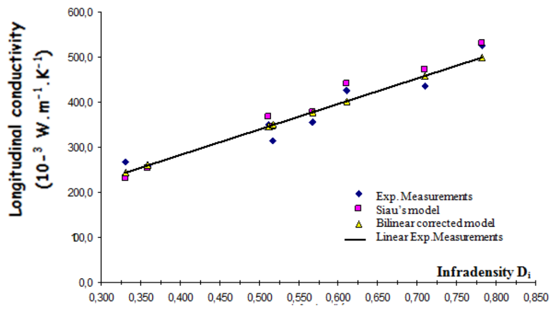

Fig. 21 : Comparison graphic of experimental longitudinal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 0%

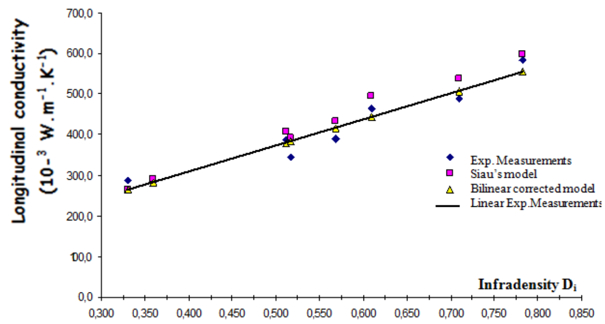

Fig. 22 : Comparison graphic of experimental longitudinal conductivities with those given by Siau's model and those of the bilinear model for a moisture content H = 12%

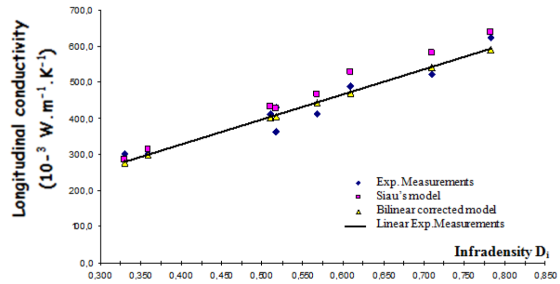

Fig. 23 : Comparison graphic of experimental longitudinal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 20%

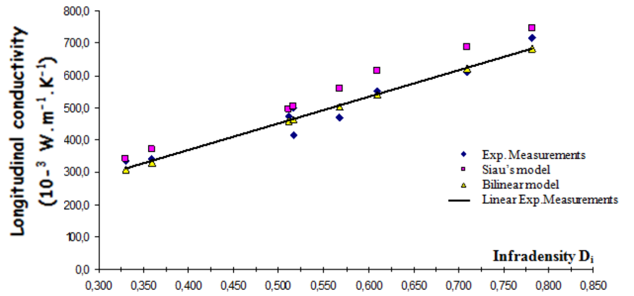

Fig. 24 : Comparison graphic of experimental longitudinal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 40%

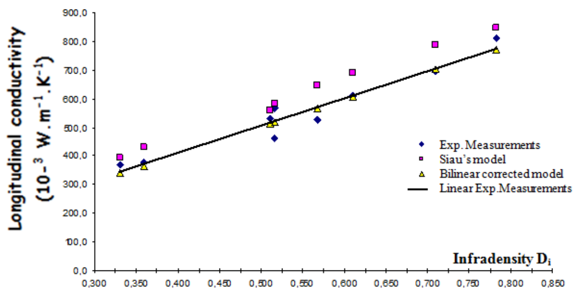

Fig. 25 : Comparison graphic of experimental longitudinal conductivities with those given by the Siau's model and those of the bilinear model for a moisture content H = 60%

III. RESULTS AND DISCUSSION

The significant set of results to which we end consists of two mathematical empirical models presented on equations (17) and (18). Their validation is made by comparison with values of tables 1 to 4. The graphics coming from this are presented in figures 12 to 16 for the transversal conductivity and figures 21 to 25 for the longitudinal one.

From these graphics the following comments can be done:

- The model of equation (18) represents the longitudinal thermal conductivity of tropical woods in a very good way, up to 60 % moisture content and with basal density covering almost all the tropical woods: we observe, in fact, a relative deviation always less than 9%.

- The model of equation (17) is enough representative of the transversal thermal conductivity of tropical woods, for moisture content going up to 60% and basal density covering almost all tropical woods: the relative deviation is less than 16%; this high deviation compared to the one obtained for the longitudinal direction, is probably coming from the fact that the radial and orthoradial directions have been combined to constitute the transversal one.

- The graphical representations of the proposed models show once more, a higher conduction (2 to 2.5 times) more in the longitudinal direction than in the transversal one, a result which fits with the observations of Mac Lean in [13] and Ngohe-Ekam in [14] and [15].

On the other hand, with comparison to the model of Siau [7] given by equation (5) and which requires, at any moisture content, to first compute the specific weight of the sample before getting it's porosity and after that its conductivity, the bilinear models presented on equations (17) and (18) have the advantage that, as soon as the basal density of the sample is known, one can directly predict its conductivity at any moisture content.

IV. CONCLUSION

The work presented below aimed to propose a mathematical model allowing an easier prediction of the thermal conductivity of tropical woods both in the axial and the transversal directions (i.e. parallel and perpendicular directions to the grain). It has led to a bilinear law whose comparison to the calculations stem from experimental values has given very low deviations, allowing us then to confirm the validity of the presented model, for tropical woods, in the ranges of all basal density and moisture contents up to 60%.

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ETHICS

This article is original and contains unpublished materials. The corresponding author confirms that all of the other authors have read and approved the manuscript and there are no ethical issues involved.

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