International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 07, July-2020

Biharmonic Solution for the Forcing Term in a Non-Homogeous Equation of Statics in the Theory of Elastic Mixture

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Abstract:- **In this paper, we analysed the problem of plane elasticity for a doubly connected body with outer and inner boundaries in the form of a regular polygon with a common centre and parallel sides. The sides of the polygon are exposed to external gravitational force and the biharmonic solution of the forcing term is determined. This is achieved by defining the forcing term in a non-homogeneous equation of statics, using complex variable theory. The forces are analysed under two-dimensional stress function, from which the equilibrium and compatibility equations were derived. Using the compatibility equation and the stress-strain relations, we derived our biharmonic equation. Our results show that the theoretical frame work of the forcing term is consistent with the previously existing results.**

Keywords: Elasticity, biharmonic, forcing term, compactibility

1 INTRODUCTION

The application of the methods of conformal mappings and boundary value problems of analytic functions has proved to be the most effective way of solving boundary value problems of elasticity and plate bending. However, for a simply-connected domain, these methods yield effective results (especially for domains mapped onto the circle by rational functions). However, these methods still remain poorly [1]. Nevertheless, for some practically important classes of doubly connected domains bounded by polygons including the polygonal domain with a curvilinear 2-gonal holes, we may succeed in constructing effectively (in the analytical form) functions conformally mapping this domain onto the circular ring [2]. In addition to this, the Kolosov-Muskhelishvili methods make it possible to decompose these problems, into two Riemann-Hilbert problems, for the circular ring and by solving the latter problem, we can construct the sought complex potentials in analytic form.

Theories of mixtures in the frame-work of rational continuum thermodynamics have been developed throughout the sixties and seventies, and subsequent development in various constitutive theories and thermodynamic analysis are too numerous to document [3]. Boundary-value problems for a finite domain with part of its boundary being unknown and the other a polygonal line were solved in [4]. A similar boundary-value problems of plane elasticity for infinite plates weakened by unknown full-strength holes with normal stresses on their boundaries and forces applied at infinity were analysed in [5, 6, 7].

A mixed problem of elasticity was solved in [8, 9] for a convex polygon and for a doubly connected domain with a polygonal boundary. Also, linear and non-linear static boundary-value problems for doubly or multiply connected isotropic and anisotropic elastic bodies (plates and shell) were solved by Maksimyuk and Chernyshenko [10], Liu,I-shih [7] discussed the entropy flux of transversely isotropic elastic bodies of homogeneous type [11], while [12, 13, 14] gave a solution of a non-classical problem of oscillation of two component mixtures. A fundamental solution of the system of differential equations of stationary oscillations of two-temperature elastic mixtures theory was provided by [15].

The problem of plane elasticity for a doubly connected body with outer and inner boundaries in the form of a regular polygon with a common centre and parallel sides had been reported in literature; thereby motivating this research.

2 MATHEMATICAL FORMULATION

We consider a homogenous isotropic elastic body occupying a doubly connected domain on the complex plane $z = x + iy$. It's outer and inner boundaries are L_0 and L_1 respectively. The body is a rectangle whose centre is at

z = 0. The neighborhood of the vertices of the inner rectangle symmetric angles of equidistance from the centre is shown in Fig 1.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 07, July-2020

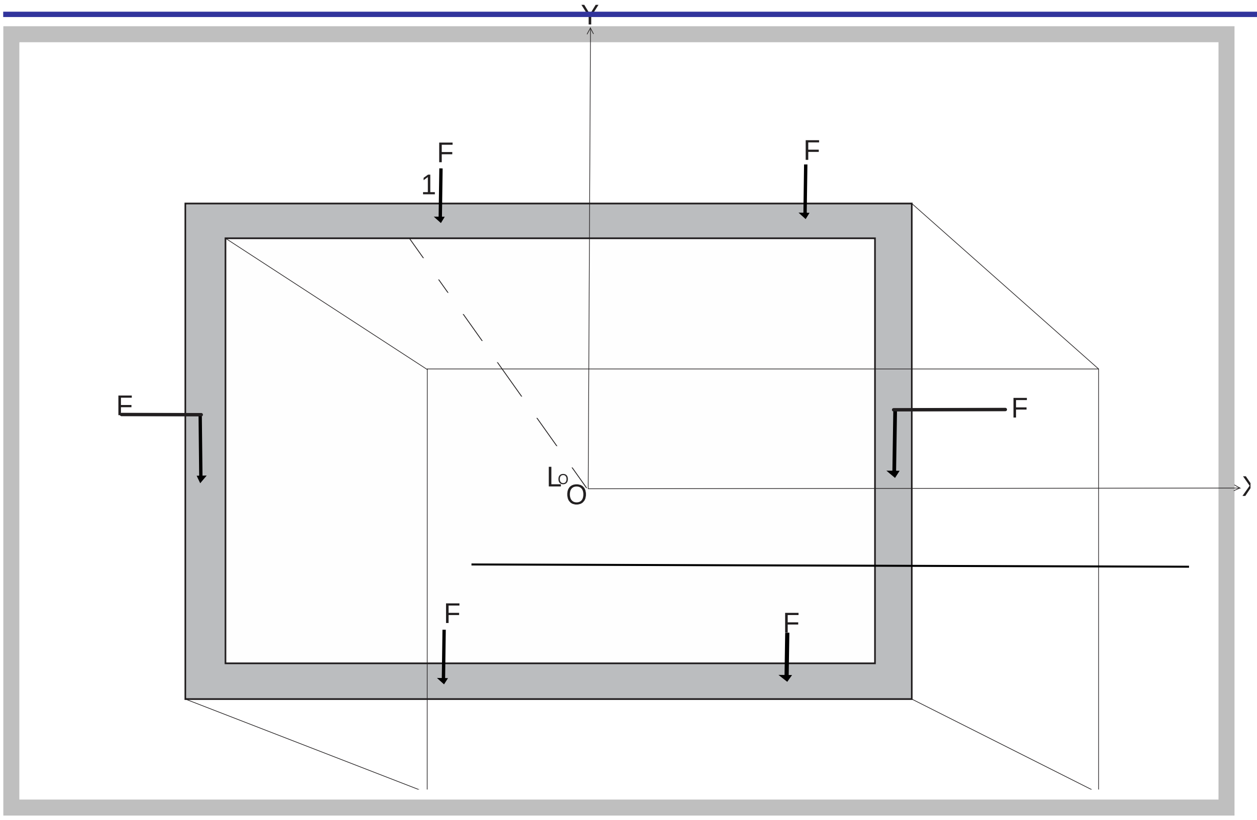

Figure 1: isotropic Elastic Body

We assumed that the edges of the isotropic elastic body is exposed to an external force in the form of load. Our aim is to determine the biharmonic solution for the force (F), and the stress state.

3. METHOD OF SOLUTION

We use non-homogeneous equation in the theory of elastic mixtures as our governing equation to define the forcing term (F) [16-18]. It is shown that the displacement vector components are represented in this theory by means of four arbitrary analytic functions. In the two-dimensional case, the basic non-homogenous governing equation [16] of the theory of elastic mixture has the form:

$$a\Delta u' + b\,\mathrm{grad}\,\mathrm{div}\,u' + c\Delta u'' + d\,\mathrm{grad}\,\mathrm{div}\,u'' = \ell F \quad (1)$$

Where Δ is the two-dimensional laplacian, grad and div are the principal operators of the field theory, ℓ is the partial density (positive constant of the mixture), F is the mass force, u' = w' and u'' = w'' are the displacement vectors, a, b, c, and d are combination of constitutive constants, characterizing the physical properties of the mixture[19].

3.1 **Theory of Complex Variables**

We solve equation (1), using complex variable as follows:

$$z = x + iy \text{ with its conjugate given as } \bar{z} = x - iy \quad (2)$$

which in addition results to

$$2x = z + \bar{z} \quad (3)$$

Expressing equation (3) in partial differential equation

$$2\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} \quad (4)$$

and by subtraction, equation (2) becomes,

$$2iy = z - \bar{z} \quad (5)$$

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Expressing equation (5) in partial differential equation

$$2i\frac{\partial}{\partial y} = \frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}} \qquad (6)$$

Adding equation (4) and (6); we have,

$$2\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) = 2\frac{\partial}{\partial z} \qquad (7)$$

Subtracting equation (6) from (4); we have,

$$2\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) = 2\frac{\partial}{\partial \bar{z}} \qquad (8)$$

Multiplying equation (7) and (8); we obtain,

$$4\frac{\partial^2}{\partial z\partial \bar{z}} = 4\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + 4\mathrm{i}\left(\frac{\partial^2}{\partial x\partial y} - \frac{\partial^2}{\partial x\partial y}\right) \qquad (9)$$

Equating the two right terms of equation (9) to the real and imaginary parts of equation (4) and (6) respectively; to have,

$$4\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = 4\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}\right) \qquad (10i)$$

$$4\mathrm{i}\left(\frac{\partial^2}{\partial x\partial y} - \frac{\partial^2}{\partial x\partial y}\right) = -4i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right) \qquad (10ii)$$

Replacing the two right hand terms of equation (9) by the two right hand terms of equation (10i) and (10ii); to have,

$$4\frac{\partial^2}{\partial z\partial \bar{z}} = 4\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}\right) - 4i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right) \qquad (11)$$

Let the displacement vectors u' and u'' in their complex form be represented by

$$\omega' = u_1' + iu_2' \text{ and } \bar{\omega}' = u_1' + iu_2' \qquad (12)$$

Introducing the displacement vectors into equation (11); we have,

$$4\frac{\partial^2\omega\prime}{\partial z\partial \bar{z}} = 4\left(\frac{\partial\omega\prime}{\partial z} + \frac{\partial\overline{\omega\prime}}{\partial \bar{z}}\right) - 4i\left(\frac{\partial\omega\prime}{\partial z} - \frac{\partial\overline{\omega\prime}}{\partial \bar{z}}\right) \qquad (13)$$

Note: The displacement vector (ω) depends on the elastic (z) and plastic ($\bar{z}$) regions respectively.

In order make equation (1) solvable, we adopt the method used in [1] and let

$$\Delta u' = 4\frac{\partial^2\omega\prime}{\partial z\partial \bar{z}} \quad and \quad \Delta u'' = 4\frac{\partial^2\omega\prime\prime}{\partial z\partial \bar{z}} \qquad (14)$$

and

$$\frac{\partial\omega\prime}{\partial z} + \frac{\partial\bar{\omega}}{\partial \bar{z}} = 2\left(\frac{\partial u\prime}{\partial x} + \frac{\partial u\prime}{\partial y}\right) = 2divu' = 2\theta' \qquad (15)$$

Substituting equation (14) and (16) for $\Delta u' and\ divu'$ in equation (1) ; we have,

$$4a\frac{\partial^2\omega'}{\partial z\partial \bar{z}} + 4c\frac{\partial^2\omega\prime\prime}{\partial z\partial \bar{z}} + 2bgrad\theta' + 2dgrad\theta'' = \psi' \qquad (16)$$

Note: Our laplacian here is define as

$$\Delta = \nabla.\nabla = \frac{\partial}{\partial z}.\frac{\partial}{\partial \bar{z}}$$

where

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$$\nabla = \quad grad \;=\; \frac{\partial}{\partial \bar{z}}$$

Thus, substituting for grad with $\frac{\partial}{\partial \bar{z}}$ in equation (16); we have,

$$4a\frac{\partial^2 \omega\prime}{\partial z\partial \bar{z}} + 4c\frac{\partial^2 \omega\prime\prime}{\partial z\partial \bar{z}} + 2b\frac{\partial \theta\prime}{\partial \bar{z}} + 2d\frac{\partial \theta\prime\prime}{\partial \bar{z}} = \psi' \qquad (17)$$

$$\frac{\partial}{\partial \bar{z}}\left(4a\frac{\partial \omega\prime}{\partial z} + 4c\frac{\partial \omega\prime\prime}{\partial z} + 2b\theta' + 2d\theta''\right) = \psi' \qquad (18)$$

$$\int d\left(4a\frac{\partial \omega\prime}{\partial z} + 4c\frac{\partial \omega\prime\prime}{\partial z} + 2b\theta' + 2\theta''\right) = \int \psi' d\overline{z} \qquad (19)$$

$$4a\frac{\partial \omega\prime}{\partial z} + 4c\frac{\partial \omega\prime\prime}{\partial z} + 2b\theta' + 2d\theta'' = \int \psi' d\overline{z} \qquad (20)$$

From Pompeiu formula, the integral [11] of (19); gives,

$$4a\frac{\partial \omega\prime}{\partial z} + 4c\frac{\partial \omega\prime\prime}{\partial z} + 2b\theta' + 2d\theta'' = \frac{\partial \dot{\psi}\prime}{\partial \bar{z}} \qquad (21)$$

where $\frac{\partial \dot{\psi}\prime}{\partial \bar{z}}$ is the analytic non-homogeneous terms, define as,

Ψ' = u + iv is the displacement vector component at the transformed state, as a result of the contact with external force.

3.2 **Non-homogeneous Part**

Equating the non-homogeneous parts of equations (21) and (1)

That is,

$$\frac{\partial \dot{\psi}\prime}{\partial \bar{z}} = \ell F = \ell(F_1 + iF_2) \qquad (21^*)$$

So that

$$\frac{\partial \dot{\psi}\prime}{\partial \bar{z}} = \frac{\partial(u+iv)}{\partial(x+iy)} = \ell F_1 + i\ell F_2$$

which gives

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} + i\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial x} = \ell F_1 + i\ell F_2 \qquad (22)$$

Equating coefficient in equation (22); to have,

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = \ell F_1 \qquad (23)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \ell F_2 \qquad (24)$$

Introducing new variables φ *and* ŋ [15]; such that,

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial ŋ}{\partial y}$$

$$v = -\frac{\partial \varphi}{\partial y} + \frac{\partial ŋ}{\partial x}$$

Substituting for u and v in equation (22) and (23) gives

$$\frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial x} + \frac{\partial ŋ}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \varphi}{\partial y} + \frac{\partial ŋ}{\partial x}\right) = \ell F_1$$

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which implies

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \ell F_1$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi = \ell F_1$$

$$\nabla^2\varphi = \ell F_1 \qquad (25)$$

a similar procedure gives:

$$\nabla^2 \eta = \ell F_2 \qquad (26)$$

hence, from equation (25) and (26); we have,

$$\nabla^2\varphi + i\nabla^2\eta = \ell F_1 + i\ell F_2 \qquad (27)$$

$$\nabla^2(\varphi + i\eta) = \ell(F_1 + iF_2)$$

$$\nabla^2\varphi = \ell F \qquad (28)$$

Note: The classical field theory describing gravity is the Newtonian gravitation, which describes the gravitational force F, as a mutual interaction between two masses M, and m. That is,

$$F = -\frac{GMm}{r^2} \qquad (29)$$

where G is the earth gravitational constant, and r is the radius of the earth.

The massive body M has a gravitational field g. Since the gravitational force is conservative, the field g, can be written as a gradient of gravitational scalar potential φ; that is,

$$G = -\nabla\varphi \qquad (30)$$

Also, in the case of gravitational field due to an attracting massive object of density ℓ, Gauss' law for gravity in differential form can be used to obtain the corresponding Poisson equation for gravity[14] as,

$$\nabla . g = -4\pi G\ell \qquad (31)$$

Substituting for g in equation (31) using equation ((30)

$$\nabla .(-\nabla\varphi) = -4\pi G\ell \qquad (32)$$

$$\nabla^2\varphi = 4\pi G\ell \qquad (33)$$

Equation (33) is called Poisson equation for gravity[2]. Hence, equation (28) is equivalent to equation (33); because, they both involve the mutual interaction between two masses (M and m). That is

$$\nabla^2\varphi = \ell F = \nabla^2\varphi = 4\pi G\ell$$

$$\ell F = 4\pi G\ell$$

$$F = 4\pi G \qquad (34)$$

Thus, our forcing term (F) is the gravitational force, which is Poisson in nature; as such, it is restricted to a plane.

*3.3 **Biharmonic Solution for the Forcing Term (F)***

We determine the stress state of the forcing term, by considering Euler Cauchy Stress Principle [20], which states that "upon any surface (real or imaginary) that divides a body, the action of one part of the body on another is equivalent to the system of distributed forces, and is represented by a field T_n, called stress vector, defined on the surface A, assumed to depend continuously on the unit vector n [21].

This is mathematically as:

$$F = \oint_A T_n dA \quad (35)$$

Thus,

$$dF = T_n dA$$

$$\frac{dF}{dA} = T_n \quad (36)$$

where T_n is the resultant stress vector, components σ_n and τ_n are the normal and shear stresses respectively. Hence, the stress distribution diagram becomes

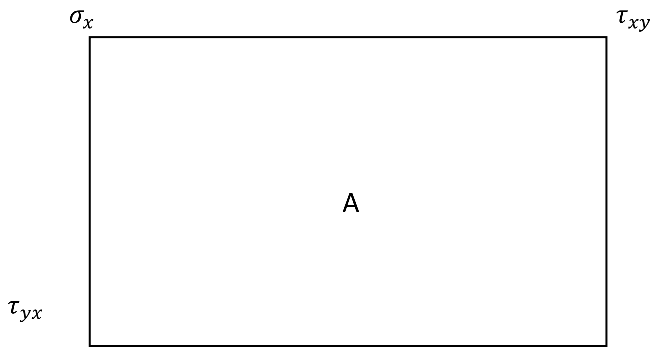

Figure 2: Stress distribution on a plane

where, σ_x and σ_y are the normal stresses in the x and y directions.

$\tau_{xy} = \tau_{yx}$, is the symmetric shear stress in the x and y directions

Hence, we deduce the equilibrium equation from figure 2, as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad (37i)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \quad (37ii)$$

equations (37) is called the equilibrium equation.

Equation (37), involves two equations with three unknown,($\sigma_{x,}\ \sigma_y$ *and* τ_{xy}). For compatibility, we introduce an extra equation from the strain-displacement relation of the deformation process [22].

$$\epsilon_x = \frac{\partial u}{\partial x} \quad (38i)$$

$$\epsilon_y = \frac{\partial v}{\partial y} \quad (38ii)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (38iii)$$

where u = u(x, y) and v = v(x, y) are the displacement vectors in the transformed state.

Differentiating equation (38) twice with respect to $y, x,$ *and* xy respectively, result to

$$\frac{\partial^2}{\partial y^2} \cdot \frac{\partial u}{\partial x} = \frac{\partial^2 \epsilon_x}{\partial y^2} \quad (39i)$$

$$\frac{\partial^2}{\partial x^2} \cdot \frac{\partial v}{\partial y} = \frac{\partial^2 \epsilon_y}{\partial x^2} \quad (39ii)$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2}{\partial y^2} \cdot \frac{\partial u}{\partial x} + \frac{\partial^2 \epsilon_x}{\partial x^2} \cdot \frac{\partial v}{\partial y} = \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} \quad (39iii)$$

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Hence, adding equations (39i), (39ii) and (39iii); we have,

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \quad (40)$$

equation (40) is called compatibility equation.

To solve equation (40), we use stress-strain relationship [22] for stress:

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - v\sigma_y\right) \quad (41i)$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) \quad (41ii)$$

$$\gamma_{xy} = \frac{2}{E}(1+v)\tau_{xy} = -\frac{\tau_{xy}}{G} \quad (41iii)$$

where:

v = Poisson ratio

E = Young modulus

G = Modulus of rigidity

Substituting for ϵ_x, ϵ_y *and* γ_{xy} into equation (40); we have,

$$\frac{1}{E}\left[\frac{\partial^2}{\partial y^2}\left(\sigma_x - v\sigma_y\right) + \frac{\partial^2}{\partial x^2}(\sigma_y - v\sigma_x)\right] = \frac{2}{E}(1+v)\frac{\partial^2 \tau_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2}{\partial x^2}\left(\sigma_x - v\sigma_y\right) + \frac{\partial^2}{\partial x^2}\left(\sigma_y - v\sigma_x\right) = 2(1+v)\frac{\partial^2 \tau_{xy}}{\partial x \partial y} \quad (42)$$

To eliminate the shearing term (τ_{xy}) in equation (42), we differentiate the equilibrium equation (37i) and (37ii) w.r.t. x *and* y respectively. That is,

$$\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \tau_{xy}}{\partial x \partial y} = 0 \quad (43i)$$

$$\frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \tau_{xy}}{\partial x \partial y} = 0 \quad (43ii)$$

adding equation (43i) and (43ii); we have,

$$\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2} + 2\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = 0 \quad (44)$$

$$\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = -\frac{1}{2}\left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}\right) \quad (45)$$

Substituting for $\frac{\partial^2 \tau_{xy}}{\partial x \partial y}$ in equation (42); give,

$$\frac{\partial^2}{\partial y^2}\left(\sigma_x - v\sigma_y\right) + \frac{\partial^2}{\partial x^2}\left(\sigma_y - v\sigma_x\right) = 2(1+v)\frac{1}{2}\left(-\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2}\right)$$

$$\frac{\partial^2}{\partial y^2}\left(\sigma_x - v\sigma_y\right) + \frac{\partial^2}{\partial x^2}\left(\sigma_y - v\sigma_x\right) = (1+v)\left(-\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2}\right)$$

Expanding and simplifying the equation above; we have,

$$\frac{\partial^2 \sigma_x}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} = -\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2}$$

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$$\frac{\partial^2\sigma_x}{\partial y^2} + \frac{\partial^2\sigma_y}{\partial x^2} + \frac{\partial^2\sigma_x}{\partial x^2} + \frac{\partial^2\sigma_y}{\partial y^2} = 0 \qquad (46)$$

The solution to equation (46), can be obtained by introducing a new function Φ, called Airy's stress function [22].

For the case under consideration, we can define Φ; so that,

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\Phi}{\partial x^2}\ , and \ \ \tau_{xy} = \frac{\partial^2\Phi}{\partial x\partial y}$$

Substituting for $\sigma_x, and\ \sigma_y$ in equation (46); that is,

$$\frac{\partial^2}{\partial y^2}\frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2}{\partial x^2}\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2}{\partial y^2}\frac{\partial^2\Phi}{\partial x^2} = 0$$

$$\frac{\partial^4\Phi}{\partial y^4} + \frac{\partial^4\Phi}{\partial x^4} + 2\frac{\partial^4\Phi}{\partial x^2\partial y^2} = 0$$

$$\frac{\partial^4\Phi}{\partial x^4} + 2\frac{\partial^4\Phi}{\partial x^2\partial y^2} + \frac{\partial^4\Phi}{\partial y^4} = 0$$

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right]\Phi = 0$$

$$\nabla^2.\nabla^2\Phi = 0$$

$$\nabla^4\Phi = 0 \qquad (47)$$

equation (47) is called biharmonic equation. Hence, our forcing term in equation (34) is,

$$F = 4\pi G = \nabla^4\Phi = 0.$$

3.4 *Stress State of the Forcing Term on the Plane*

Stress is force per unit area; that is,

$Stress = \frac{Force}{Area}$ Hence,

Force = Area × Stress

With the above definition, figures 3-6 illustrate the stress distribution on the plane.

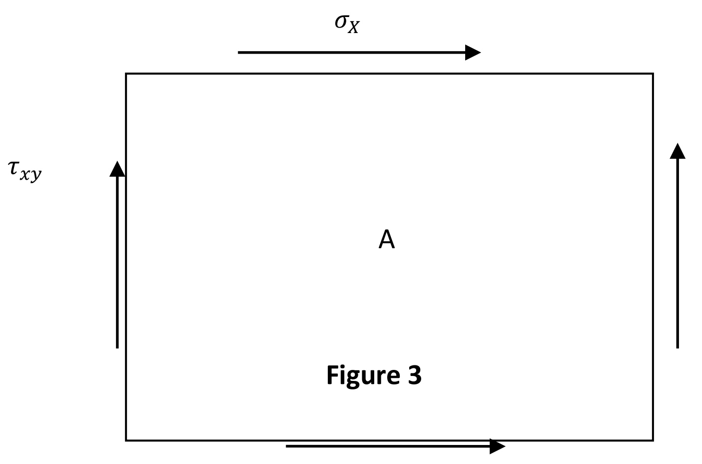

Figure 3

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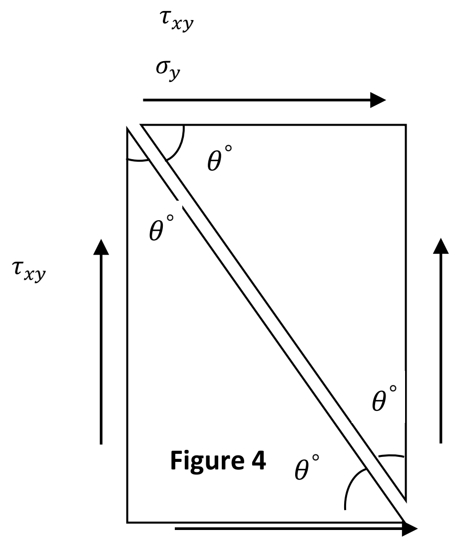

Deducing from figure 4, the force impact on the plane in x-direction is,

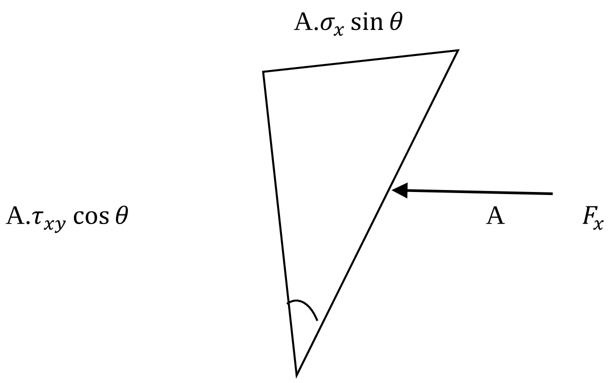

Figure 5: Stress distribution in the x-direction.

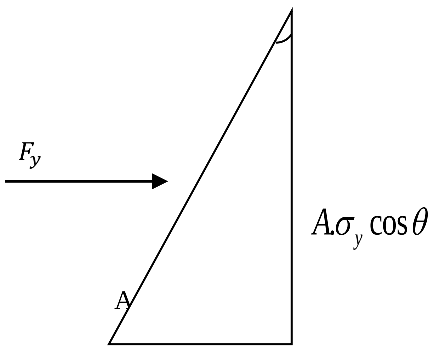

Figure 6: Stress distribution in y-direction.

$$F_x = A(\tau_{xy} \cos\theta + {}_{\sigma_x} \sin\theta) \quad (48)$$

$$\frac{F_x}{A} = \tau_{xy} \cos\theta + \sigma_x \sin\theta \quad (49)$$

Also, deducing from figure 6, the force impact on the plane in the y-direction is,

$$F_y = A\tau_{xy} \sin\theta + \sigma_y \cos\theta \quad (50)$$

$$\frac{F_y}{A} = (\tau_{xy} \sin\theta + \sigma_y \cos\theta) \quad (51)$$

Thus, total stress impact on the plane is,

$$\frac{F_x}{A} = \frac{F_y}{A} = \frac{F}{A} = \begin{pmatrix} \tau_{xy} \cos\theta & \sigma_x \sin\theta \\ \tau_{xy \sin\theta} & \sigma_{y \cos\theta} \end{pmatrix}$$

Recall from our main result,

$$F = 4\pi G = \nabla^4 \varphi = 0$$

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$$0 \quad = det\begin{pmatrix} \tau_{xy}\cos\theta & \sigma_x\sin\theta \\ \tau_{xy\,\sin\theta} & \sigma_{y\,\cos\theta} \end{pmatrix}$$

$$0 \;=\; \sigma\tau_{xy}\,cos^2(\theta) - \;\sigma\tau_{xy}sin^2(\theta)$$

$$\sigma\tau_{xy}\,cos^2(\theta) \;=\; \sigma\tau_{xy}sin^2(\theta)$$

$$\frac{sin^2\theta}{cos^2\theta} = \frac{\sigma\tau_{xy}}{\sigma\tau_{yx}} = T_n^2(\theta) \qquad (52)$$

Recall that T_n is our resultant stress vector earlier mentioned in equation (36)

$$Tan^2\theta \;=\; 1 \;=\; T_n^2(\theta) \qquad (53)$$

So that for

$$Tan^2\theta \;=\; 1$$

$$\theta \;=\; tan^{-1}1 \;=\; 45^{\circ}$$

$$2\theta \;=\; 90^{\circ}$$

For

$$Tan^2\theta \;=\; T_n^2(\theta)$$

$$T_{n\theta} = tan\theta \qquad (54)$$

With the result of equation (54), we can generate the following table to ascertain the stress state of the following angles θ

Table 1: Relationship between θ and $T_{n(\theta)}$

θ°	0	30	60	90	120	150
$T_{n(\theta)}$	0.00	0.58	1.73	0.00	-0.58	-1.73

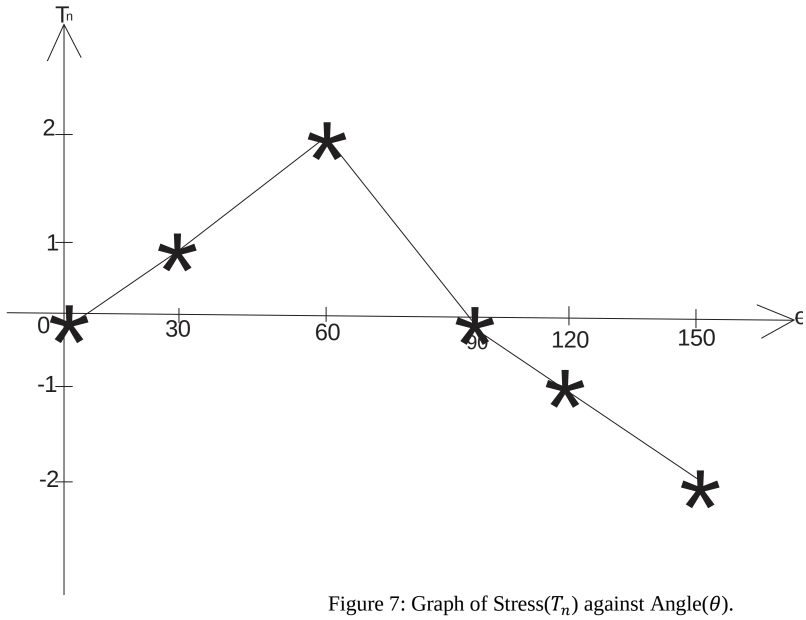

Figure 7: Graph of Stress(T_n) against Angle(θ).

3.5 Discussion of Result

The graph in Fig.7, show the relationship between the angle (θ) and the stress state of the isotropic elastic body. It is shown that the body attain stability at angle 90°; where, the resultant stresses resolved to zero. At angle 30° $t0$ $60,^{\circ}$the body undergoes an elastic motion; while, at angle 120° to 150°, the body suffers a plastic deformation.

3.6 Conclusion

In this paper, we considered the biharmonic solution for the forcing term (F) in a non-homogeneous equation of statics in the theory of elastic mixture. It was found that our theoretical solution for the forcing term of the isotropic elastic body (Fig.1) is consistent with the existing result of [23]. In this paper, we analysed the problem of plane elasticity for a doubly connected body with outer and inner boundaries in the form of a regular polygon with a common centre and parallel sides. The sides of the polygon are exposed to external gravitational force and the biharmonic solution of the forcing term is determined. This is achieved by defining the forcing term in a non-homogeneous equation of statics, using complex variable theory. The forces are analysed under two-dimensional stress function, from which the equilibrium and compatibility equations were derived. Using the compatibility equation and the stress-strain relations, we derived our biharmonic equation. Our results show that the theoretical frame work of the forcing term is consistent with the previously existing results.

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