Bifurcation Study in Discretized Sliding Mode Controlled DC-DC Buck Converter

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Abstract

This paper presents the study of bifurcation and chaos in a discretized sliding mode controlled for buck converters operating in continuous conduction mode. The discretization behaviors of equivalent SMC of dc-dc buck converter are investigated. In particular, one of the most frequently used discretization schemes for digital controller implementation, the zero-order-holder discretization, is studied. The different parameters such as input voltage, load resistance, and inductor are used as bifurcation parameter to observe such unusual dynamic behaviors. Some inherent dynamical properties of the discretized SMC systems are studied. All the simulation work is done in the MATLAB environment.

"1. Introduction"

Nonlinear dynamics can be described in terms of chaos, bifurcation, fractal, Lyapunov exponent, period doubling, Poincare map, strange attractors. The chaos theory has been applied to all the branches of science. This nonlinear dynamics is applied to the power electronics to explore it completely. Since the power electronics systems are mostly nonlinear systems, it can exhibit four basic states such as fixed point, limit cycle, quasi-periodic state and chaotic state. Chaotic state is the last state appearing after all other three states. The nonlinear system can undergo to chaos state in two ways which are local bifurcation to chaos and global bifurcation to chaos. In local bifurcation, one limit cycle loses its stability whereas in global bifurcation, more than one limit cycles lose its stability. Local bifurcation has three subclasses such as period doubling, quasi-periodicity and intermittency. Global bifurcation has two subclasses such as chaotic transient and the crisis. In our study, we are considering the local bifurcations and the subclass is period doubling. The nonlinear systems can be described by nonlinear differential equations and represents in state space.

This chaos and bifurcation has been studied in different converters by applying different control methods. There are mainly six parameters in a power electronics dc-dc converter system. Those are load resistance, input voltage, inductance, load capacitance, frequency and amplitude of the triangular wave. The behavior of a power electronic converter is sensitively dependent on the parameter. So detail knowledge about the chaos and bifurcation in the parameter space is required while choosing the parameter values depending upon the desired output behavior.

"2. Literature Review"

The bifurcation behavior of the buck converter has been studied by applying current feedback control [2]. It has taken load resistance, inductance and frequency amplitude of triangular wave as bifurcation parameters. It has been observed that the system directly undergoes from period-1 behavior to chaos. Chaos has been observed in the buck converter while applying constant-frequency Pulsewidth modulation in continuous conduction mode [3]. In chaotic region, a strange attractor was found. The goal was to design a converter with reliability and predictability in their performance, when operating under instability or even chaotically. Sub-harmonics, bifurcations and chaos in a sliding-mode controlled boost switching regulator have been studied [4]. Hysteresis band theory has been applied to observe the unusual behavior. A discrete time mapping for the analysis of nonlinear phenomena in current and voltage controlled pulse width modulation dc/dc buck converters has been studied [5]. Period-doubling bifurcations have been explained with detailed mathematical analysis. This work is extended by systematic classification of discrete-time models for different dc-dc converter [6]. A Jacobian matrix of the map is used for the analysis of several bifurcations and chaotic behaviors. Complex discretization behaviors have been observed in a simple sliding-mode control system [7]. The system is analyzed with the Euler's discretization. This work is extended into two-dimensional sliding mode control systems to study the dynamic behavior with the help of zero-order discretization [8]. A chattering phenomenon is observed and for certain parameter values arbitrarily, long periodic orbits exist for small discretization steps. Chattering phenomenon is studied in a buck-boost converter by applying fuzzy sliding mode control [9]. Fixed frequency PWM based fuzzy sliding mode control is compared with a classical sliding mode control in terms of start-up behavior and robustness to disturbances. The complex behavior of a voltage feedback buck converter is experimentally is observed with wide variation of input voltage and different resistances [10].

"3. Basic Principle of Buck Converter"

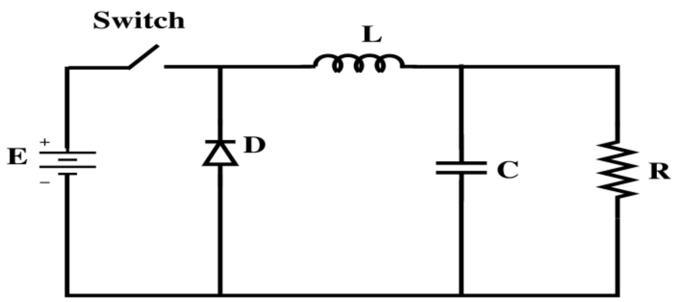

Fig. 1 Buck converter

A buck converter is a basic converter for steeping down the input voltage to the desired output voltage. The components used in this converter are MOSFET, diode, inductor, capacitor and load resistance. Depending upon this switching state of the MOSFET, the converter changes its mode of operation. There are two mode of operation: Switch ON where input voltage is transferred to the output voltage, and Switch OFF where input voltage is disconnected and inductor is discharged to the load.

During switch ON: u=1   During switch OFF: u=0

$$E - L\frac{di}{dt} = v,$$

$$\frac{di}{dt} = \frac{-v}{L} + \frac{E}{L}, \qquad \frac{di}{dt} = \frac{-v}{L} + 0,$$

$$i = C\frac{dv}{dt} + \frac{v}{R}, \qquad \frac{dv}{dt} = \frac{i}{C} - \frac{v}{CR}$$

$$\frac{dv}{dt} = \frac{i}{C} - \frac{v}{CR}$$

In compact form, it can be written as,

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{-1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix} u$$

Where u∈(0, 1). E is input voltage, *v* is output voltage, L is inductor, C is capacitor, *i* is inductor current, R is resistance.

"4. Discretized Sliding Mode Controller"

The basic principle of sliding mode control is to design a certain sliding surface in its control law that will direct the trajectory of the state variables toward a desired origin when coincided. SMC systems are usually discretized using the Euler method, while a zero-order holder (ZOH) is used in practical implementation of the SMC strategy. Discretization of SMC may cause irregular behaviors such as periodic trajectories and strange attractors. In Euler method, the sampled SMC suffers more from the sampling process, as it would lose the high gain property nearby the vicinity of the switching surface. To compensate for this, disturbance prediction is indispensable, which is feasible under the hypothesis that the disturbance is slow time-varying.

"5. DSMC Applied to Buck Converter"

We are applying DSMC to the DC-DC buck converter. First the state equations of buck converter have to be formed.
For ON state dynamic,

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v_{ref} - E}{LC} \end{bmatrix} = f^+$$

For OFF state dynamic,

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v_{ref}}{LC} \end{bmatrix} = f^-$$

The system is defined as $\dot{x} = Ax + Bu + D$

Where $A = \begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-1}{RC} \end{bmatrix}, D = \begin{bmatrix} 0 \\ \frac{v_{ref}}{LC} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{-E}{LC} \end{bmatrix}$

Sliding surface S(x) is defined as

$$S(x) = c_1 x_1 + c_2 x_2 = Cx$$

$$= c_1(v_{ref} - v) + c_2\left(\frac{-i}{C} + \frac{v}{CR}\right).$$

For existence condition, $\dot{S}(x) = C^T \dot{x} = 0$

$$\dot{S}(x) = C^T Ax + C^T Bu + C^T D$$

After a detailed mathematical derivation,

The resulting dynamics can be written as

$$\dot{x} = Ax + B[u_{eq} + u_s] + D.$$

The system is simulated with the help of MATLAB. The combination of SIMULINK model and program runs the system to study its dynamic behaviors.

"6. Simulation Results"

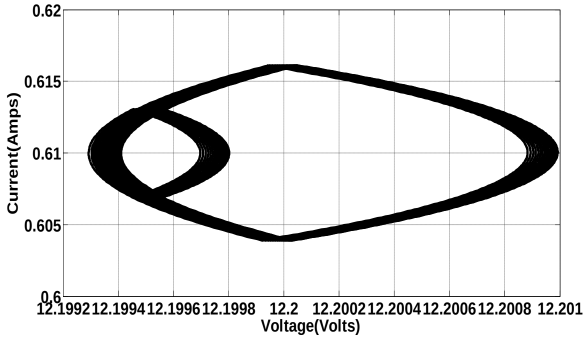

Fig. 2 Limit cycle with R=50Ω

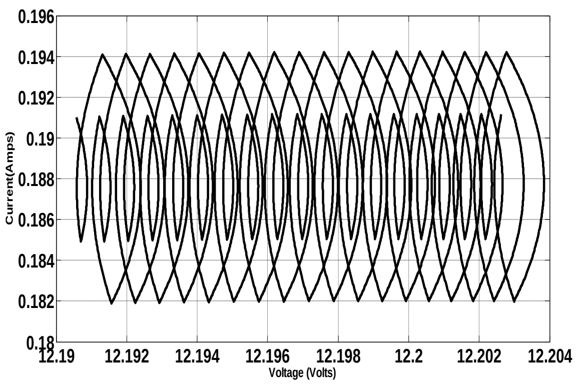

Fig. 3 Limit cycle with R=65Ω

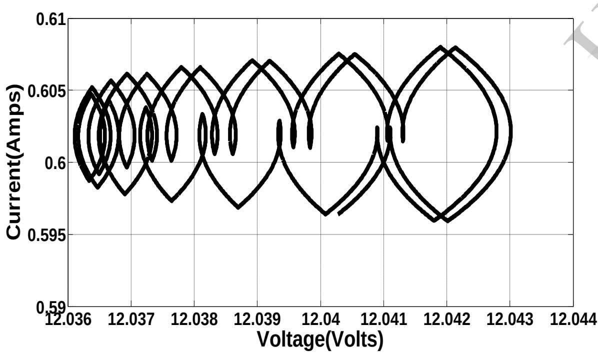

Fig. 4 Limit cycle with V_{in}=25V

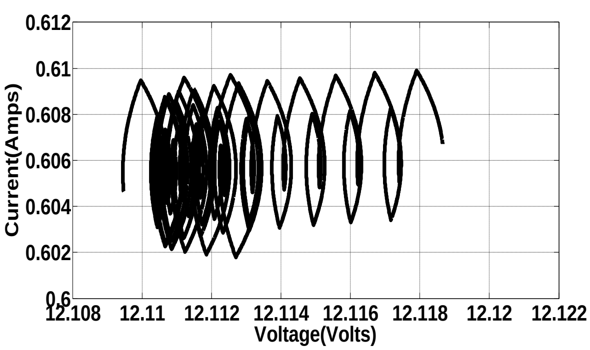

Fig. 5 Limit cycle with L=38mH

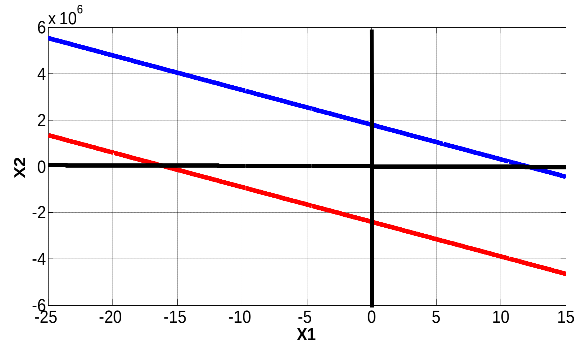

Fig. 6 Phase Plane with the two boundary line

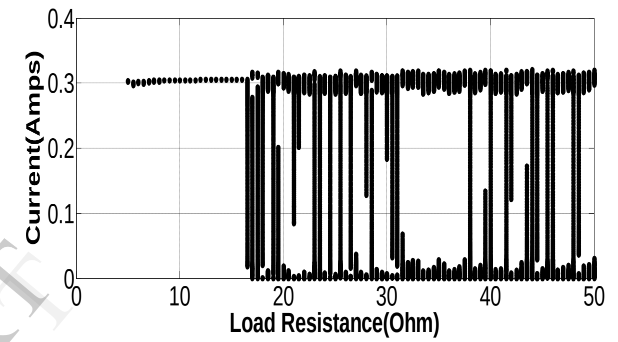

Fig. 7 Bifurcation diagram with load resistance as the bifurcation parameter

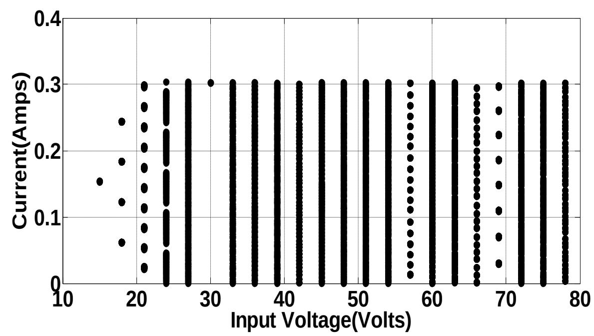

Fig. 8 Bifurcation diagram with input voltage as the bifurcation parameter

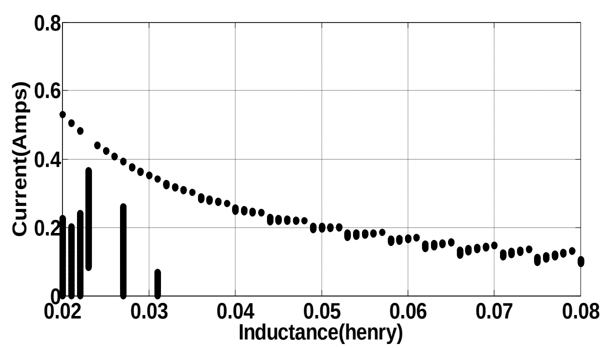

Fig. 9 Bifurcation diagram with inductor as the bifurcation parameter

The converter is modeled in the discrete domain. The combination of both SIMULINK and program helps to simulate this converter. The sample and hold circuit and zero-order hold block are used for discretization. There are three parameters such as input voltage, load resistance and inductor which are using for the bifurcation parameters. The limit cycles for different parameters are shown in fig. 2, fig. 3, fig. 4, and fig. 5. The boundaries lines for phase plane are shown in fig. 6. The bifurcation diagrams are shown in fig. 7, fig. 8 and fig. 9. The current is compared with the bifurcation parameters. The converter goes under instability which is show by the bifurcation diagram. It has been observed that the system does not go through the period doubling bifurcations, but it is leading from period one to chaos. There are wide regions where the converter behaves chaotically in the parameter space. The results have been studied in the discrete domain with sliding mode control.

"7. Conclusion"

We have discussed the use of DSMC in this paper. Discretization of SMC using zero-order-hold is studied in details. The simulation results are mentioned in discrete domain by varying parameters to study the behavioral changes in discrete domain. We can observe that by varying parameter, system goes from period one to chaos. The simulations results for different bifurcations parameters are shown. The previous work has been extended and unusual behavior has been discussed. This work is done under continuous conduction mode and this can be extended to discontinuous mode and for different converters.

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