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Bending Stress & Contact Stress Analysis of Spur Gear Tooth using 'MATLAB'

(Role of Spur gear in Rural Development)

Ashish Kumar Singh
P.G. Scholar [Machine Design]
Shri Shankaracharya Technical Campus
Bhilai, India

Mahesh Dewangan
Associate Professor, Deptt. of Mechanical Engineering
Shri Shankaracharya Technical Campus
Bhilai, India

***Abstract*— Gearing is one of the most critical components in a mechanical power transmission system. In rural areas spur gear are playing very important role for producing energy and other rural activities. Due to their high degree of reliability and compactness of gears it will predominate as the most effective means of transmitting power in machines. In addition the rapid shift in the heavy industries such as shipbuilding to industries such as automobile manufacture and office automation tools will necessitate a refined application of spur gear system. In the design process of gear tooth the induced stresses are of paramount importance and needs experimental evaluation and validation. Lewis equation is used for finding out the bending stress in a spur gear tooth. This equation is based on certain assumptions. Bending strength of gear tooth can also be found out by modified Lewis equation, AGMA standards etc. In this paper a comparison has been made between the results obtained by Lewis equations, hertz contact stress and ANSYS. A computational tool MATLAB is used to evaluate the bending stress and contact stress.**

Keywords- Bending Stress, MATLAB, Contact stress,

I. INTRODUCTION

Gear transmission systems play an important role in many industries. The knowledge and understanding of gear behavior in mesh such as stress distribution, work condition and distortion is critical to monitoring and controlling the gear transmission system. Gears are often required to operate at high torque and speed, while remaining competitively priced and highly reliable. In general, gears are used to transmit rotary motion between two shafts, normally with a constant ratio.

There have been vast researches on gear design analysis, and a large body of literature on gear modelling has been published. The gear stress analysis, transmission errors, and the prediction of gear dynamic loads, vibration, gear noise, and the optimal design for gear sets are always major concerns in gear design.

Until the mid 20th century all gear design was based upon Lewis original bending equation. Lewis based his analysis on a cantilever beam and assumed that failure will occur at the weakest point of this beam. Lewis considered the weakest point as the cross-section at the base of the spur gear.

Hertz calculated the contact pressure between two deformable cylinders. The contact pressure is mainly a function of the type of material in contact and the radius of curvature.

With this continuing trend of experimental bending stress analysis the American Gear Manufacturers Association (AGMA) published their own standard based on Lewis' original equation. Established in 1982 this equation is still widely used in gear design today. The bending stress is dependent on the geometry and shape of the gear tooth [1].

Buckingham showed that two contacting parallel cylinders can be used to study contact stresses of spur gears with fair accuracy [2].

Ramamurti and Rao 1988 use FEM and cyclic symmetry approach for the stress analysis of spur gear teeth. The contact line load at one such substructure leads to an asymmetric loading of the wheel as a whole. This force system is resolved into a finite Fourier series to calculate the static stresses. [3]

Vijayarangan and Ganesan 1993 uses 3 D FEA approach to obtain static stress analysis of composite gears and compared with mild steel gear and conclude that composite material is better for power transmission gears. [4]

Lu and Litvin [5] analyze the tooth surface contact and stresses for double circular-arc helical gear drives and FE method is use to investigate load share and contact ratio for aligned and misaligned gear.

Daniewicz and Moore 1998 increases fatigue life of gear by introducing compressive residual stresses is prestressing or presetting nad applied to AISI 1040 steel spur gear teeth were individually preset using a single tooth bending fatigue fixture. [6]

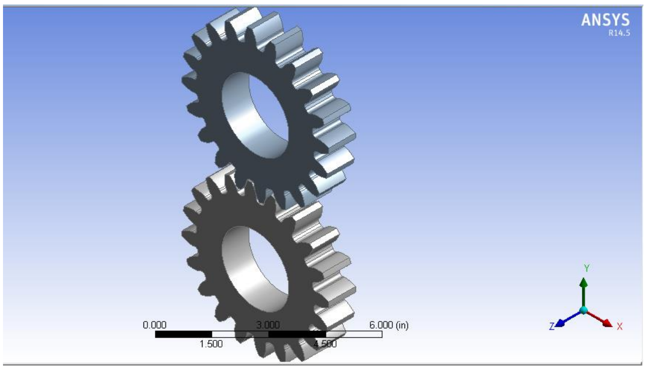
Woods and Daniewicz 1999 increases bending fatigue strength of carburized spur gear teeth using presetting and develop a model FEM in order to evaluate presetting on a gear tooth; his model is namely elastic–perfectly plastic.[7] The model is been verified experimentally and analytically. The place where Fatigue cracks originate this model helps to determine stress

Chien et. al 2002 Similar use roller test machine to study the spalling mechanism of spur gears for testing helical gearand explain explains how a subsurface crack is initiated and the influence of material properties on gear spalling life.[8]

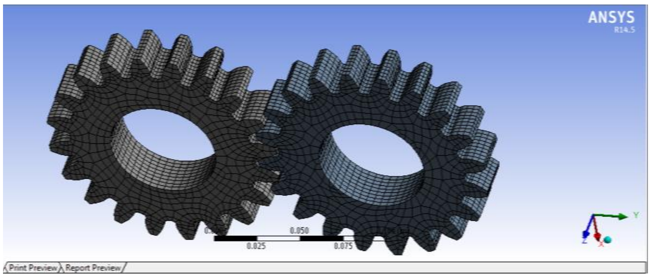
Faydor et. al 2005 presents new computerized developments in design, generation, simulation of meshing, and stress analysis of gear drives and give numerical example for a developed theory.[9]

Yahya et.al [10] designed S and C shaped transition curve and applied to design spur gear tooth. The design will be analyzed by using Finite Element Analysis (FEA). This analysis is used to find out the applicability of the tooth design and the gear material that chosen.

II. MATHEMATICAL MODEL



Spur gear



Mesh Model

Figure 1 Model Configuration

The FEM Formulation

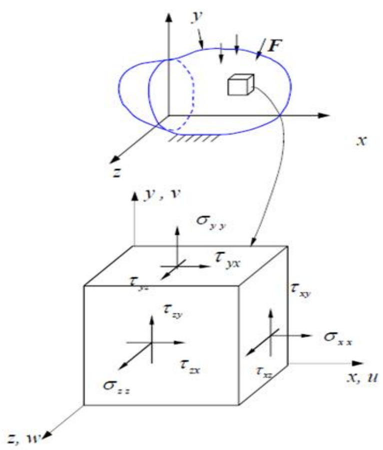


Figure 2 Infinitesimal element showing stress state [12].

Displacement

$$U=\{u(x,y,z),v(x,y,z),w(x,y,z)\}$$

Cauchy's Stress tensor =

$$\sigma=\begin{bmatrix}\sigma_{xx}\tau_{xy}\tau_{xz}\\ \tau_{yx}\sigma_{yy}\tau_{yz}\\ \tau_{zx}\tau_{zy}\sigma_{zz}\end{bmatrix}$$

The strain-stress relations (Hooke's law)for isotropic materials are given by:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)\end{bmatrix}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix}$$

Strain-Displacement relations are:

$$\gamma_{yz}=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z},\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}$$

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}+X=0$$

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}+Y=0$$

$$\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+Z=0$$

$$(\lambda+G)\frac{\partial e}{\partial x}+G\nabla^2u+X=0$$

$$e=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

$$b=\sqrt{\frac{4F\left\{\frac{\left[1-\mu_1^2\right]}{E_1}+\frac{\left[1-\mu_2^2\right]}{E_2}\right\}}{\pi l\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

Stresses internal to the cylinder are given by

$$\sigma_x=-P_{\max}\left\{\left[1+\left(\frac{y}{b}\right)^2\right]^{\frac{1}{2}}\left[2-\left(1+\left(\frac{y}{b}\right)^2\right)^{-1}\right]-2.\frac{y}{b}\right\}$$

$$\sigma_y=-P_{\max}\left[1+\left(\frac{y}{b}\right)^2\right]^{\frac{-1}{2}}$$

Von-mises stress is given by

$$\sigma_{von}=\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

The equation below is the AGMA bending stress equation for S.I specification of gears.

$$\sigma_b = \frac{F_t k_v k_m k_o}{bmj}$$

Lewis,

$$\sigma = \frac{F_t}{bpy}$$

According to Shigley [11], the fundamental equation for pitting resistance (contact stress) is

$$S_c = C_p \sqrt{\frac{W_t k_a k_s k_m k_f}{k_v dFI}}$$

$$CR = \frac{\sqrt{(r_p + \phi)^2 - r_p^2 \cos^2\phi}}{\pi m \cos\phi} + \frac{\sqrt{(r_g + \phi)^2 - r_g^2 \cos^2\phi} - (r_p + r_g)\sin\phi}{\pi m \cos\phi}$$

$$C_p = \sqrt{\frac{1}{\pi\left(\frac{[1-\mu_1^2]}{E_1} + \frac{[1-\mu_2^2]}{E_2}\right)}}$$

In figure 1 the spur gear model configuration has been shown along with the mesh model. The gear model is discretisized in 39127 nodes and 7208 elements and the boundary conditions are applied and the von-mises (Bending) stress is evaluated.

III. Results and Discussions

This chapter gives results on Stress analysis of Spur gear with using ANSYS and Analytical MATLAB calculation. The parametric study of effect of face width, Pressure Angle, varying load, no. of teeth on Spur gear is carried out.

The MATLAB results are validated with literature and by Analytical calculation for a few cases are also illustrated.

Table 1 Validation of Von-Mises Stresses for Spur gear Models

Load (MN)	Reference [10] (MPa)	Present MATLAB (MPa)
5	0.69	0.689
70	9.7	9.689
800	112	112.1
900	125	122.76
1000	139	137.89

Table 2: Validation of Von-Mises (Bending) Stresses for Spur gear Models

No of teeth(N)	MATLAB Stresses(MPA)	3DStresses (ANSYS)(MPA)
22	130.1847	131.53
23	126.8841	127.04
25	122.2941	123.89
28	120.4364	122.94
30	119.0751	120.13
34	117.4243	118.57

For the number of teeth (Z) = 22

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43 \times 4 \times 0.36174} \times 1.2 \times 1.25 \times 1.8 = 130.1847 MPa$$

For number of teeth (Z) = 23

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43 \times 4 \times 0.37115} \times 1.2 \times 1.25 \times 1.8 = 126.8841 MPa$$

For number of teeth (Z) = 28

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43 \times 4 \times .39102} \times 1.22 \times 1.25 \times 1.8 = 120.436 MPa$$

For number of teeth (Z) = 30

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43 \times 4 \times 0.39549} \times 1.22 \times 1.25 \times 1.8 = 119.075 MPa$$

For number of teeth (Z) =34

$$\sigma_b = \frac{F_t}{bmJ} k_v k_o k_m = \frac{3000}{43 \times 4 \times 0.401056} \times 1.22 \times 1.25 \times 1.8 = 117.4242 MPa$$

From Table 1, 2 and Figure 3 shows the stress distribution in spur gear and Shows the comparison of results for different 3-D models and the corresponding MATLAB stress values and Present FEM values. From this it can be revealed that on comparing Analytical result with computational result shows good agreement. And it can also be concluded that on increasing number of teeth of spur gear Von-Mises (Bending) Stresses decreases.

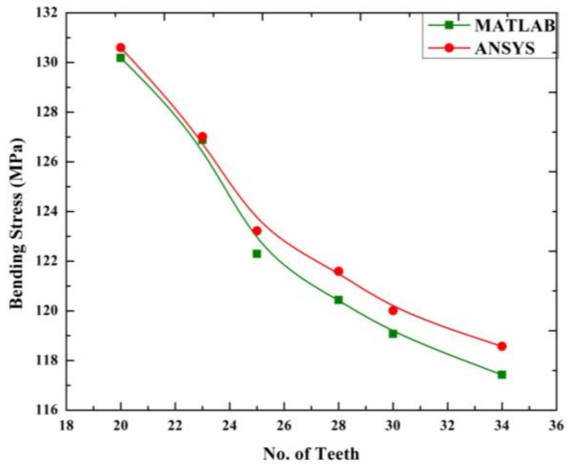


Figure 3-Variation of Stress with Number of Teeth of Spur Gear.

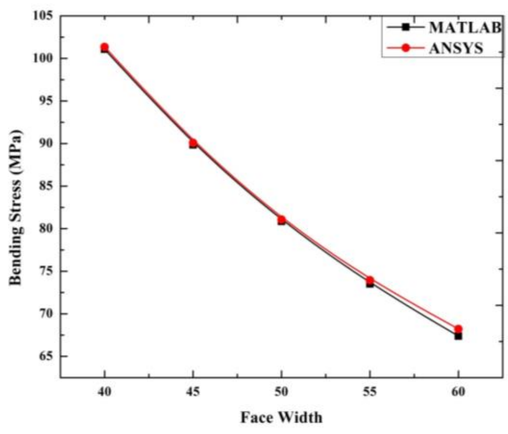


Figure 4 Variation of Bending Stress with respect to face width

Figure 4 shows the Variation of Bending Stress with respect to face width. It can see the bending stress significantly decreases as the gear face width increase. The FEA Bending stress results shows good agreement with the MATLAB result the variation of ±0.00263% is there.

It can also be conclude that on increasing face width, the bending stress spreads in more area and the load bearing capacity of the gear increases but on other hand the gear system become heavier.

IV. Conclusion

It was observed that the stresses generated on spur gear teeth changes with the number of teeth.

A comparison of the results obtained from the FEM with those using the MATLAB (maximum bending stresses) and Hertz theory (maximum contact stresses) reveals that the maximum stresses predicted by the FEM are slightly higher than those predicted by the AGMA (MATLAB) and Hertz theory.

The variation between the MATLAB and ANSYS result is in the range of ±.0122 to ±.02014.

It is also conclude that the variation of bending stress with respect to face width. The bending stress significantly decreases as the gear face width increase. The FEA Bending stress results shows good agreement with the MATLAB result the variation of ±0.00263% is there.

V. APPLICATION

Application of spur gear in rural area such as one example is Bull as Prime Mover of Eco-Generation Power Plant, by using some mechanical drives (such as spur gear, belt drive) which convert High Torque & Low speed bull input into high speed & Low torque output to Synchronous generators. This is the advantage of mechanical drives to achieve synchronous speed from at generator shaft, due to this arrangement three phase ac power output produced from plant and other application is animal driven water pump with gear box and bullocks. Spur gear application in chaff cutting machine. Spur gear is also use in channel gate of biogas plant for drain out the wastage of plant.

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