

# Beamforming with Per-Antenna Power Constraint and Transmit Antenna Selection using Convex Optimization Technique

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**Abstract** - In this paper, transmit beamforming and antenna selection techniques are presented for the Cooperative Distributed Antenna System. The beamforming technique with minimum total weighted transmit power satisfying threshold SINR and Per-Antenna Power constraints using convex optimization is presented for the efficient performance of Distributed Antenna System (DAS). Antenna Selection technique is used to select the optimum antennas from all the available ones. This achieves the best compromise between capacity and system complexity. Simulation results prove that integrating Beamforming with DAS enhances the performance of DAS. Also antenna selection reduces the cost of RF front end and its complexity.

**Keywords**— Distributed Antenna System, Beamforming, Convex Optimization, frequency flat quasi-static channel.

## I. INTRODUCTION

Wireless communication systems have been developing and evolving in a furious pace in these recent years. The number of mobile subscribers is growing tremendously in the past decades. The early wireless systems consisted of a base station with a high-power transmitter which served a large geographic area. Each base station could serve only a small number of users and was costly as well. Today, due to the advancement in technology, the cellular system consists of a cluster of base stations with low-power radio transmitters. Now the total number of users served is increased because of channel reuse and larger frequency bandwidth.

Next generation broadband wireless access systems are evolving towards distributed architectures as a promising solution to meet the ever increasing demand for the wireless connectivity. With limited transmit power and bandwidth; one way to achieve the high data rate is to reduce the radio transmission distance between the transmitter and the receiver which is made possible by using the distributed architectures [1]. It is well established that multiple-input multiple-output (MIMO) technology provides high data rates and link reliability without additional bandwidth or power [2]. Efficient allocation of the transmit power over the coverage area is possible by introducing the MIMO technologies into new radio architectures especially for future mobile communication systems [3]. Distributed Antenna System (DAS) is an evolving architecture which serves the need of the future wireless communication systems.

Beamforming or spatial filtering is a powerful signal processing technique used in antenna arrays or sensor arrays

for directional signal transmission or reception. Beamforming helps in main lobe enhancement, side lobe reduction and removal of interference caused by unwanted transmitters.

There are also a set of limitations that result from practical system design such as a Per-Antenna power constraint that keeps the amplifier at each transmit antenna in its linear range [4]. In designing the communication system, these types of constraints should all be taken into account. Instead of Sum power constraint, in this paper we consider the more realistic Per-Antenna power constraint on each of the transmitters at the Remote Antenna Units (RAUs), since in practice each antenna is limited individually by its equipped power amplifier.

The use of multiple antennas in DAS can increase the capacity linearly with the number of antennas. Also it promises to increase the diversity gain. However, the radio-frequency (RF) chain (composed of amplifier, mixer and analog-to-digital converter) associated with each RAU increases the system hardware cost and complexity. Hence an optimum number of Remote Antenna Units from the available set of antennas has to be selected. Antenna Selection is a powerful signal processing technique that is used to select the best antennas out of all the available ones. Selection criteria can be maximization of channel capacity, Signal to Interference-plus-Noise ratio (SINR) or minimization of Bit Error Rate.

In paper [5], the transmitter optimization problem for multiuser downlink channels with multiple transmit antennas at the base-station is considered. Assuming perfect channel knowledge at the transmitter, this paper investigates two different transmission schemes under the per-antenna power constraint: a minimum-power beamforming design for downlink channels with a single antenna at each remote user and a capacity-achieving transmitter design for downlink channels with multiple antennas at each remote user. It is shown that in both cases, the Per-Antenna downlink transmitter optimization problem may be transformed into a dual uplink problem with an uncertain noise. This new interpretation of duality gives rise to efficient numerical optimization techniques for solving the downlink Per-Antenna transmitter optimization problem.

The benefit of coordinating base-stations across multiple cells in a multi-antenna beamforming system, where multiple base-stations may jointly optimize their respective beamformers to improve the overall system performance is considered in [6]. This paper focuses on the design criteria of

minimizing either the total weighted transmitted power or the maximum per-antenna power across the base-stations subject to Signal-to-Interference-and-Noise-ratio (SINR) constraints at the remote users. The main contribution of this paper is an efficient algorithm, which is capable of finding the globally optimal downlink beamforming vector across all base-stations.

## II. COOPERATIVE DISTRIBUTED ANTENNA SYSTEM

Cooperative Distributed Antenna Systems have drawn considerable attention in recent years due to their potential to enhance both the coverage and the spectral efficiency of mobile communication systems. In a traditional cellular model, the multiple antennas are centrally located at the Base Station, while in the cooperative DAS cellular model shown in Fig.1, the multiple antennas of a Base Station, called Remote Antenna Units (RAUs), are distributed in the cell. All RAUs within a cell are connected to the Base Station by using a high-quality bidirectional wired (e.g., Radio over Fiber) or wireless (e.g., microwave repeater) links. The RAUs are simple transceiver of the Base Station, whereas the Base Station has all the baseband processing capability.

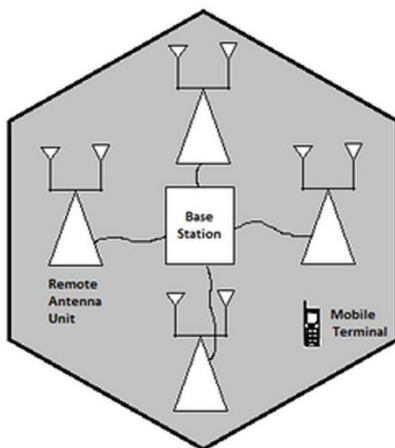


Fig.1. DAS cellular model

DAS was originally proposed to improve the indoor performance of wireless communication systems. In order to exploit the advantages of both the collocated MIMO and the DAS, a cooperative DAS based on a radio over fiber (RoF) technique was introduced in China's beyond 3G FuTURE project [8, 9] and tested via field experiments. Research on distributed MIMO has been carried out in [10–12] by exploiting both microscopic and macroscopic spatial diversity. In [3] an analytical capacity study was presented for DASs. In a Conventional Antenna System, the power of a radio signal exponentially decreases as the transmission distance between the Base Station and the Mobile Terminal increases. This leads to a serious edge problem i.e. the Mobile Terminals at the edges of the cellular sector will receive a lower data rate while the Mobile Terminals closer to the Base Station gets a higher data rate. But in a Cooperative DAS as the RAUs are distributed in the cell, the exhaustion of the radio signal power is reduced and hence the coverage is uniform throughout the cell.

DAS is often used in scenarios where alternate technologies are infeasible due to terrain or zoning challenges. The idea works because less power is wasted in overcoming penetration and shadowing losses, and because a line-of-sight channel is present more frequently, leading to reduced fade depths and reduced delay spread. In the cooperative DAS the transmitted signals of a few RAUs, instantaneously linked to a reference MT, are considered useful signals rather than interference. A set of RAUs within reachable distances of a reference MT can cooperatively communicate with this MT to form a MIMO system. Note that in cooperative DAS the distributed MIMO system differs from the traditional centralized MIMO since each of the RAUs experiences independent macroscopic fading from each other [11]. Therefore, the distributed MIMO formed by several RAUs at different geometric locations can make use of the statistically independent properties of the channels more efficiently, and larger channel capacity would be expected than in the traditional collocated MIMO. The theoretic and Monte Carlo simulation results presented in [3] shows the benefits of the distributed MIMO.

## III. BEAMFORMING AND PER-ANTENNA POWER CONSTRAINT

Beamforming is a signal processing technique used in antenna arrays for directional transmission and reception. It steers all the energy in one direction by directing a transmitting element in an antenna array to reach a desired receiver in a given direction. Using Beamforming technique, weight vectors are assigned for each channel. The following assumptions are made 1) Perfect Channel State Information at the transmitter (CSIT) 2) The channel is assumed to be a frequency flat quasi static channel 3) The number of users are greater than the number of transmit antennas at the RAU.

Depending upon the CSIT, Beamforming technique allocates weight vectors for each channel. Now, the transmit antenna element in the RAU whose channel is assigned a high value of weight vector is chosen from among the available set of antenna elements and transmission takes place between the selected antenna terminal in the RAU and the mobile terminal through this channel.

Also in practical multi-antenna implementations, each transmit antenna is usually equipped with its own power amplifier. Thus, an individual power constraint on each antenna individually is more realistic than a sum power constraint across all the antennas. There are certain benefits in considering the Per-Antenna power constraint. They are as follows

- It helps in joint transmission and reception.
- Since the power is equally divided among the transmit antenna elements, all the antennas will be active at a particular time.

Hence in this paper we consider the Per-Antenna power constraint.

## IV. SYSTEM MODEL

Consider a DAS scenario where a Base Station (BS) is located at the center of the macro cell and the multiple antennas of the BS, the Remote Antenna Units (RAUs) are distributed throughout the cell. The RAUs are connected to the BS by an optical fiber or cable. Each RAU consists of one

or more antenna terminals. Mobile Terminals (MTs) can be located anywhere in the cell. The system model is shown in Fig.2.

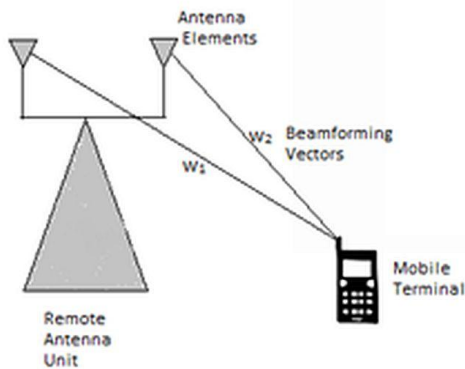


Fig.2. System Model (showing a RAU with 2 antenna elements and a MT with 1 antenna)

The general system model for a RAU with N antenna elements and K receivers, each with one antenna can be given as,

$$y_k = \mathbf{w}_k^H \mathbf{h}_k + n_k, \quad k=1,2,\dots,K$$

where,  $y_k$  represents the received signal (1x1),  $\mathbf{w}_k^H$  is a (1xN) beamforming vector,  $\mathbf{h}_k$  is a (Nx1) channel response vector and  $n_k$  represents the noise (1x1). The channels are allocated with weight vectors using the Beamforming technique. The capacity of the distributed antenna system is represented as

$$C(G)=\log_2 \det(I_M + c_k M D)$$

Where G is the downlink propagation channel,  $c_k$  is the SINR, D is the Large Scale Fading matrix, M is the number of RAUs.

### V. BEAMFORMING IN DAS AS A CONVEX OPTIMIZATION PROBLEM

Let  $\mathbf{w}_k$  denote the weight vectors ( $k=1,2,\dots,K$ ). The total weighted power at the transmitter is given as  $\sum_{k=1}^K \|\mathbf{w}_k\|^2$ . Increasing the transmit power affects the linearity of devices (e.g. amplifier) present in the RAU. This is an undesirable effect. Hence the aim is to reduce the total weighted transmit power but at the same time ensure that the Signal to Interference-Noise ratio (SINR) target is met at the receiver. We formulate the Beamforming in cooperative DAS as a convex optimization problem to get the solution. Convex optimization technique finds the weight vector that minimizes the total transmit power and also meets the SINR target. The two constraints to be satisfied are the Per-Antenna power constraint and multi-user transmit beamforming under individual SINR constraint. The problem can be expressed as follows

$$\min_{\{\mathbf{w}_k \in C^N\}_{k=1}^K} \sum_{k=1}^K \|\mathbf{w}_k\|^2 \tag{1}$$

$$\text{subject to } \frac{|\mathbf{w}_k^H \mathbf{h}_k|^2}{\sum_{l \neq k} |\mathbf{w}_l^H \mathbf{h}_k|^2 + \sigma_k^2} \geq c_k, \quad k=1,2,\dots,K \tag{2}$$

$$\mathbf{w}_k = 1/N \tag{3}$$

Eqn (1) defines the objective function (minimize transmit power), (2) defines the SINR constraint and (3) defines the Per-Antenna power constraint. N denotes the number of transmit antennas,  $c_k$  denotes the target SINR,  $\sigma_k^2$  denotes the noise power. The problem becomes infeasible when 1) the channel vectors of two or more channels are co-linear or highly correlated 2) SINR target is too high 3) the number of users, K is much greater than the number of antenna terminals, N in the RAU.

### VI. TRANSMIT ANTENNA SELECTION IN DAS AS A CONVEX OPTIMIZATION PROBLEM

The transmit antenna selection problem in DAS can be formulated as a linear programming problem and can be solved using the Interior Point algorithm. Linear programming is a mathematical optimization technique to solve a linear objective function, subjected to the linear equality and linear inequality constraints. Define  $\Delta_i$  ( $i=1,2,\dots,M$ ) as the antenna selection variable for each RAU such that,

$$\Delta_i = \begin{cases} 1, & i^{\text{th}} \text{ RAU is selected} \\ 0, & \text{otherwise} \end{cases}$$

Define a M X M diagonal matrix  $\Delta$  for RAU selection at the Base Station which has  $\Delta_i$  as its diagonal entries. The diagonal matrix  $\Delta$  is represented as follows,

$$\Delta = \begin{bmatrix} \Delta_1 & & & \\ & \Delta_2 & & \\ & & \ddots & \\ & & & \Delta_M \end{bmatrix}_{M \times M}$$

The modified channel capacity equation as a function of antenna selection variable can be written as

$$C(\Delta)=\log_2 \det(I_M + c_k M \Delta D)$$

The set of RAUs that maximizes the capacity of the system must be selected out of all the available RAUs. The antenna selection problem is formulated as a convex optimization problem. The objective function is to maximize the channel capacity. The two constraints defined are: 1)  $0 \leq \Delta \leq 1$ , the antenna selection variable is relaxed to a weaker constraint so that it can be solved in polynomial time. 2) the sum of the diagonal elements of  $\Delta$  must be equal to N, the total number of RAUs selected. The problem can be expressed as follows,

$$\text{maximize } C(\Delta) = \log_2 \det ( I + c_k M \Delta D)$$

$$\text{subject to } 0 \leq \Delta \leq 1$$

$$\text{trace}(\Delta) = \sum_{i=1}^M \Delta_i = N$$

The solution to this problem would be that N optimal RAUs will be selected from the available set to serve a particular user.

VII. RESULTS AND DISCUSSION

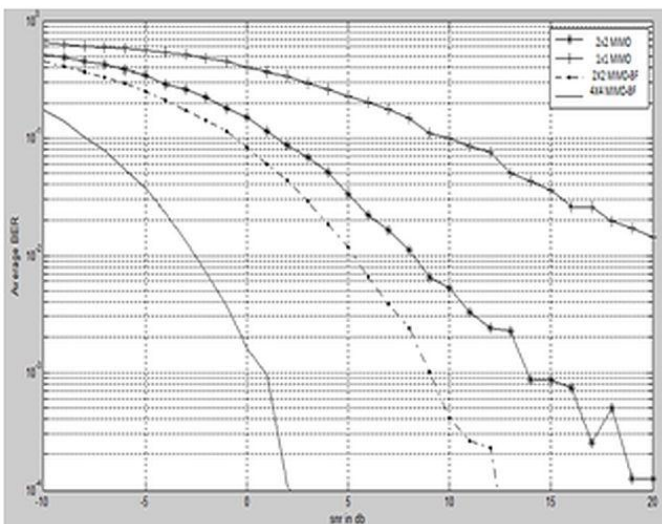
For the performance evaluation, the simulation parameters are set as follows: Number of users K=2, Number of transmit antennas in the RAU M<sub>t</sub>=2, Number of receive antennas M<sub>r</sub>=1, SINR target c<sub>k</sub>=2 dB. Two cases are taken to evaluate the formulated problem. Case 1: Two orthogonal channels are taken. Case 2: Two highly correlated channels are taken. In Case 1, weight vectors are found for link 1 and link 2. By using convex optimization technique, the optimized value of the weight vector is found to be 0.7068. Optimal value of transmit power is found to be 2.8280 dB. Table 1 tabulates the optimized values obtained for Case 1.

TABLE I  
OPTIMAL VALUES OBTAINED FOR CASE 1

S.NO	PARAMETERS	VALUES
1	Optimal Value (cvx_optval)	+2.82805
2	SINR	Link 1 1.9985      Link 2 1.9985
3	Weight Vectors	Link 1 0.7068      Link 2 0.7068 -0.7068
4	Total transmit power	2.8280

In Case 2, two channels which are highly correlated are taken. As discussed before, this case becomes infeasible. Hence no weight vectors are assigned. Simulation results for SNR in dB vs. average Bit Error Rate (BER) is shown in Fig.3. Graphs are obtained for four different cases i) 1x1 MIMO ii) 2x2 MIMO iii) 2x2 MIMO-BF (Beamforming) iv) 4x4 MIMO-BF.

Fig.3. Plot of SNR in dB vs. Average BER



It is evident from the graph that to attain a minimum average BER of 10<sup>-1</sup>, SNR needed by the 1x1 MIMO and 2x2 MIMO are 10 dB and 2dB respectively. Thus by increasing the number of transmit and receive antennas, a minimized average BER can be obtained at a reduced SNR. Also including Beamforming technique with MIMO further reduces the SNR (and hence the transmit signal power) needed to achieve a minimized average BER. From Fig.3 we can see that to get a minimum average BER of 10<sup>-1</sup>, SNR needed by 2x2 MIMO-BF and 4x4 MIMO-BF are -1 dB and -7.5 dB. Table 2 shows the SNR values obtained for different system configurations.

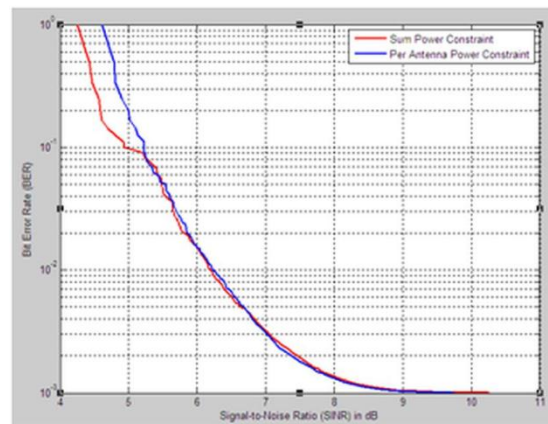
TABLE 2  
COMPARISON OF SNR VALUES FOR DIFFERENT SYSTEM CONFIGURATIONS

S.NO	SYSTEM	MIMO (Without Beamforming) (SNR dB)	MIMO-BF (With Beamforming) (SNR dB)
1	1X1	10	-
2	2X2	1	-1
3	4X4	-	-7.5

(BER=10<sup>-1</sup>)

Simulation result for SINR in dB vs. BER for two methods- sum-power constraint and per-antenna power constraint is shown in Fig.4. From the plot it is seen that the performance of per-antenna power constraint is better than the sum-power constraint. Therefore by implementing the more realistic Per-Antenna power constraint, the amplifiers at the RAU are kept in the linear range. Thus it is clear that by integrating Beamforming techniques with DAS, a reduced total transmit power can be achieved with target SINR constraint. Also, it is clear that Beamforming technique in DAS with per-antenna power constraint achieves a better performance

Fig.4. Plot of SINR in dB vs. BER



VIII .CONCLUSION

The problem to find the optimal beamforming weights to minimize the total weighted transmit power, satisfying the SINR constraint and Per-Antenna power constraint, is formulated as a convex optimization problem and is solved using convex optimization tools (e.g. CVX). Transmit beamforming is achieved with minimum transmit power. The



target SINR constraint and the Per-Antenna power constraint are met. Two different cases- orthogonal channels and highly correlated channels are discussed. Comparison between Sum power and Per-Antenna power constraint is done. The results and graphs obtained prove that the performance of the system is improved by integrating beamforming technique with Cooperative Distributed Antenna System (DAS).

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