Batch Arrival Retrial Queueing Model with Starting Failures, Customer Impatience, Multi Optional Second Phase and Orbital Search

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Abstract — **In this paper, we consider a single server batch arrival retrial queueing model with starting failures, customer impatience and multi optional second phase. Batch arrival of customers is in accordance with Poisson process. The server is subject to starting failure. The customers balk and renege at particular times. The server provides two phases of service, first essential and second multi optional. On each service completion, the server searches for customers in the orbit if any or remains idle. We derive the system performance measures for the prescribed model.**

Keywords— Retrial queue, Starting failure, Customer impatience, Multi optional second phase, Orbital search

I. INTRODUCTION

Retrial queueing system is characterized by the feature that the customers who find the server busy on arrival, join the retrial queue (orbit) and try again after some random time. It has been widely used in switching systems, telecommunication networks, local area networks and daily life situations. There are several contributions considering queueing system with two phases of service, customer impatience and starting failures. Ke and Chang (2009) studied the batch arrival retrial queue with starting failures, two phases of heterogeneous service and Bernoulli vacation. Sumitha and Udaya Chandrika (2012) discussed retrial queueing system with starting failure, single vacation and orbital search. Suganthi and Madheswari (2015) considered the retrial queueing systems with customer impatience. Madheswari et al. (2016) discussed the retrial queueing system with retention of reneging customers. Krishna kumar and Suganthi (2019) dealt with M/G/1 retrial queues with second optional service and customer balking under two types of Bernoulli vacation schedule. Varalakshmi and Chandrasekaran (2017) analyzed an unreliable two phase retrial queue with immediate Bernoulli feedbacks. Madhan (2018) considered optional deterministic server vacations in a single server queueing system providing two types of additional optional service. Vanitha (2018) studied the M/G/1 feedback queue with two stage heterogeneous service and deterministic server vacations. Rajadurai et al. (2015) have examined the $M^X/G/1$ retrial queue with two phases of service with Bernoulli vacation schedule and random breakdown.

In this paper we discuss a single server retrial queue with starting failures, Customer impatience, Multi optional second phase and orbital search. In section II, we introduce the mathematical model of the system. In section III, we derive the main results of the paper.

II. DESCRIPTION OF THE MODEL

Consider a single server retrial queueing model with starting failures. Customers arrive in batches according to Poisson process with parameter λ. The batch size Y is a random variable with P{Y = k} = C_k; k ≥ 1, $\sum_{k=1}^{\infty} C_K = 1$, probability generating function C(z) and first two moments m_1 and m_2.

If the arriving batch finds the server free, one of the customers start the server which takes negligible time. If the server is started successfully, the customer gets the service and all the other customers join the orbit. The probability of successful commencement of service is α. Otherwise the repair of the server commences and the arriving batch joins the orbit and makes their retrials later. The retrial time follows general distribution with distribution function A(x). Let a(x), A*(θ) denote the respective probability density function (pdf) and Laplace stieltjes transform (LST). Repair times follow general distribution with distribution function R(x). Let γ(x), R*(θ) be the respective pdf and LST with first two moments γ_1, γ_2.

If the server is busy or unavailable, the new arriving batch may join the orbit with probability p or leave the system with complementary probability. On account of arrival of primary customer, the retrial customer cancels his attempt for service and returns to the retrial queue with probability q or leaves the system with complementary probability. The server provides two phases of service .The first phase of service is essential to all customers whereas the second phase is multi optional. After completion of essential service, the customer may leave the system with probability r_0 or choose any one of the multi optional service with probability r_i where 1 ≤ i ≤ M.

The essential service time follows general distribution with distribution function $B_0(x)$. Let $b_0(x), B_0^*(\theta)$ be the respective pdf and LST with first two moments μ_{01} *and* μ_{02}. The multi optional service time follows general distribution with distribution function $B_i(x)$. Let $b_i(x), B_i^*(\theta)$ be the respective pdf and LST with first two moments μ_{i1} and μ_{i2} where $1 \le i \le M$. On each service completion, the server searches for the customers in the orbit with probability θ or remains idle with probability $\bar{\theta}$.

Define the Markov process

$\{N(t); t \ge 0\} = \{X(t), C(t); \varepsilon_0(t), \varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t); t \ge 0\}$

X(t) - Orbit size at time t , C(t) - Server state.

C(t)---0 if the server is idle at time t.

---1 if the server is busy in first essential service at time t.

---2 if the server is busy in second multi optional service at time t.

---3 if the server is on repair at time t.

We define the respective hazard rate function for repeated attempts, first essential service, second multi optional service, and repair as

$$\eta(x) = \frac{a(x)}{1-A(x)}; \quad \mu_0(x) = \frac{b_0(x)}{1-B_0(x)};$$

$$\mu_i(x) = \frac{b_i(x)}{1-B_i(x)}, \quad 1 \le i \le M; \quad \gamma(x) = \frac{r(x)}{1-R(x)}$$

III. STEADY STATE DISTRIBUTION

Define the following probability densities

$$I_n(x, t)dx = P\{C(t)=0, X(t)=n, x \le \varepsilon_0(t) < x+dx\}; \quad t \ge 0, x \ge 0, n \ge 1$$

$$P_n(x, t)dx = P\{C(t)=1, X(t)=n, x \le \varepsilon_1(t) < x+dx\}; \quad t \ge 0, x \ge 0, n \ge 0$$

$$Q^i{}_n(x, t)dx = P\{C(t)=2, X(t)=n, x \le \varepsilon_2(t) < x+dx\}; \quad t \ge 0, x \ge 0, n \ge 0, 1 \le i \le M$$

$$R_n(x, t)dx = P\{C(t)=3, X(t)=n, x \le \varepsilon_3(t) < x+dx\}; \quad t \ge 0, x \ge 0, n \ge 1$$

The system of steady state equations that governs the model is given below

$$\lambda I_0 = r_0 \int_0^\infty P_0(x)\mu_0(x)\,dx + \int_0^\infty \sum_{i=1}^{M} Q_0^i(x)\mu_i(x)\,dx \quad (1)$$

$$\frac{d}{dx} I_n(x) = -[\lambda + \eta(x)]I_n(x), \; n \ge 1 \quad (2)$$

$$\frac{d}{dx} P_0(x) = -[p\lambda + \mu_0(x)]P_0(x) \quad (3)$$

$$\frac{d}{dx} P_n(x) = -[p\lambda + \mu_0(x)]P_n(x) + p\lambda \sum_{k=1}^{n} C_K P_{n-k}(x), \; n \ge 1 \quad (4)$$

$$\frac{d}{dx} Q_0^i(x) = -[p\lambda + \mu_i(x)]Q_0^i(x), \quad \text{for } 1 \le i \le M. \quad (5)$$

$$\frac{d}{dx} Q_n^i(x) = -[p\lambda + \mu_i(x)]Q_n^i(x) + p\lambda \sum_{k=1}^{n} C_K Q_{n-k}^i(x), \quad n \ge 1, \text{ for } 1 \le i \le M. \quad (6)$$

$$\frac{d}{dx} R_1(x) = -[p\lambda + \gamma(x)]R_1(x) \quad (7)$$

$$\frac{d}{dx} R_n(x) = -[p\lambda + \gamma(x)]R_n(x) + p\lambda \sum_{k=1}^{n} C_K R_{n-k}(x), \; n \ge 2 \quad (8)$$

with boundary conditions

$$I_n(0) = r_0(1-\theta)\int_0^\infty P_n(x)\mu_0(x)\,dx + (1-\theta)\int_0^\infty \sum_{i=1}^{M} Q_n^i(x)\mu_i(x)\,dx + \int_0^\infty R_n(x)\gamma(x)\,dx; \; n \ge 1 \quad (9)$$

$$P_0(0) = \alpha\lambda c_1 I_0 + \alpha\int_0^\infty I_1(x)\eta(x)dx + \alpha\lambda\bar{q}\int_0^\infty I_1(x)dx + r_0\theta\int_0^\infty P_1(x)\mu_0(x)\,dx + \theta\int_0^\infty \sum_{i=1}^{M} Q_1^i(x)\mu_i(x)\,dx \quad (10)$$

$$P_n(0) = \alpha\lambda c_{n+1} I_0 + \alpha\int_0^\infty I_{n+1}(x)\eta(x)dx + \alpha\lambda\bar{q}\sum_{k=1}^{n+1} C_K \int_0^\infty I_{n-k+2}(x)dx + \alpha\lambda q\sum_{k=1}^{n} C_K\int_0^\infty I_{n-k+1}(x)dx + r_0\theta\int_0^\infty P_{n+1}(x)\mu_0(x)\,dx + \theta\int_0^\infty \sum_{i=1}^{M} Q_{n+1}^i(x)\mu_i(x)\,dx \quad n \ge 1 \quad (11)$$

$$Q_n^i(0) = r_i\int_0^\infty P_n(x)\mu_0(x)\,dx, \quad n \ge 0; \quad 1 \le i \le M \quad (12)$$

$$R_1(0) = \bar{\alpha}\lambda I_0 + \bar{\alpha}\int_0^\infty I_1(x)\eta(x)dx \quad (13)$$

$$R_n(0) = \bar{\alpha}\lambda\sum_{k=1}^{n} C_K\int_0^\infty I_{n-k}(x)dx + \bar{\alpha}\int_0^\infty I_n(x)\eta(x)dx, \quad n \ge 2 \quad (14)$$

The normalizing condition is

$$I_0 + \sum_{n=1}^{\infty}\int_0^\infty I_n(x)dx + \sum_{n=0}^{\infty}\int_0^\infty P_n(x)dx + \sum_{n=0}^{\infty}\int_0^\infty \sum_{i=1}^{M} Q_n^i(x)dx + \sum_{n=1}^{\infty}\int_0^\infty R_n(x)dx = 1 \quad (15)$$

Define the probability generating functions

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$$I(x,z)=\sum_{n=1}^{\infty} I_n(x)z^n \,,\; P(x,z)=\sum_{n=0}^{\infty} P_n(x)z^n \,,$$

$$Q_i(x,z)=\sum_{n=0}^{\infty} Q_n^i(x)z^n,\; 1\le i \le M$$

$$R(x,z)=\sum_{n=1}^{\infty} R_n(x)z^n$$

Multiplying equations (1) – (14) by z^n and summing over n, n=0, 1, 2, 3,……we obtain the following partial differential equations.

$$\left(\frac{d}{dx}+\lambda+\eta(x)\right)I(x,z)=0 \quad (16)$$

$$\left(\frac{d}{dx}+p\lambda(1-c(z))+\mu_0(x)\right)P(x,z)=0 \quad (17)$$

$$\left(\frac{d}{dx}+p\lambda(1-c(z))+\mu_i(x)\right)Q_i(x,z)=0,\; 1\le i\le M \quad (18)$$

$$\left(\frac{d}{dx}+p\lambda(1-c(z))+\gamma(x)\right)R(x,z)=0 \quad (19)$$

$$I(0,z) = r_0\bar{\theta}\int_0^{\infty} P(x,z)\mu_0(x)\,dx+\bar{\theta}\int_0^{\infty}\sum_{i=1}^{M} Q_i(x,z)\mu_i(x)\,dx + \int_0^{\infty} R(x,z)\gamma(x)\,dx - \lambda I_0 \quad (20)$$

$$P(0,z) = \frac{\alpha c(z)\lambda I_0}{z} + \frac{\alpha}{z}\int_0^{\infty} I(x,z)\eta(x)\,dx+\frac{\lambda\alpha\bar{q}}{z^2}\int_0^{\infty} I(x,z)c(z)\,dx + \frac{\lambda\alpha q}{z}\int_0^{\infty} I(x,z)c(z)\,dx + \frac{\theta r_0}{z}\int_0^{\infty} P(x,z)\mu_0(x)dx+\frac{\theta}{z}\int_0^{\infty}\sum_{i=1}^{M} Q_i(x,z)\mu_i(x)dx \quad (21)$$

$$Q_i(0,z) = r_i\int_0^{\infty} P(x,z)\mu_0(x)\,dx \,,\quad \text{for } 1\le i\le M \quad (22)$$

$$R(0,z) = \bar{\alpha}\lambda\int_0^{\infty} I(x,z)c(z)\,dx + \bar{\alpha}\int_0^{\infty} I(x,z)\eta(x)\,dx + \bar{\alpha}\lambda I_0 c(z). \quad (23)$$

Solving the partial differential equations (16) - (19), we get

$$I(x,z) = I(0,z)e^{-\lambda x}(1-A(x)) \quad (24)$$

$$P(x,z) = P(0,z)e^{-p\lambda[1-c(z)]x}(1-B_0(x)) \quad (25)$$

$$Q_i(x,z) = Q_i(0,z)e^{-p\lambda[1-c(z)]x}(1-B_i(x)),\quad 1\le i\le M \quad (26)$$

$$R(x,z) = R(0,z)e^{-p\lambda[1-c(z)]x}(1-R(x)) \quad (27)$$

Solving equations (20) - (23) we get,

$$I(0,z) = I_0\lambda\left\{\left(1-\bar{\alpha}c(z)R^*(p\lambda-p\lambda c(z))\right)[z^2-\theta zT_2(z)T_3(z)-\bar{\alpha}z^2 g_2(z)R^*(p\lambda-p\lambda c(z))]-\alpha\bar{\theta}zc(z)T_2(z)T_3(z)\right\}/D(z)T_3(z) \quad (28)$$

$$P(0,z) = \alpha\lambda I_0 T_1(z)/D(z) \quad (29)$$

$$Q_i(0,z) = r_i\,\alpha\lambda I_0 B_0^*(p\lambda-p\lambda c(z))T_1(z)/D(z) \quad 1\le i\le M \quad (30)$$

$$R(0,z) = \bar{\alpha}\,I(0,z)\,[(c(z)+(1-c(z))A^*(\lambda)] + \bar{\alpha}\lambda I_0 c(z). \quad (31)$$

where

$$D(z)=\bar{\theta}\alpha T_2(z)\{zA^*(\lambda)+c(z)(1-A^*(\lambda)(\bar{q}+qz)\} + \bar{\alpha}z^2 g_2(z)R^*(p\lambda-p\lambda c(z)) + \theta zT_2(z)T_3(z) - z^2$$

$$T_1(z) = zA^*(\lambda)+c(z)(1-A^*(\lambda))[1-\bar{\alpha}R^*(p\lambda-p\lambda c(z))\,c(z)]\,(\bar{q}+qz)+zc(z)^2\,\bar{\alpha}R^*(p\lambda-p\lambda c(z))\,(1-A^*(\lambda)) - z\,c(z)$$

$$T_2(z) = B_0^*(p\lambda-p\lambda c(z))[r_0+\sum_{i=1}^{M} r_i\,B_i^*(p\lambda-p\lambda c(z))]$$

$$T_3(z) = [1-\bar{\alpha}\,g_2(z)R^*(p\lambda-p\lambda c(z))]$$

$$g_2(z) = c(z)+(1-c(z))\,A^*(\lambda)$$

By substituting the expressions (28) - (31) in the equations (24) - (27) we get the expressions for I (x, z), P(x, z), Q_i (x, z) and R(x, z). Integrating these with respect to x from 0 to ∞ we obtain I(z), P(z), Q_i (z), R(z), $1 \le i \le M$.

$$I(z) = I_0(1-A^*(\lambda))\left\{\left(1-\bar{\alpha}c(z)R^*(p\lambda-p\lambda c(z))\right)[z^2-\theta zT_2(z)T_3(z)-\bar{\alpha}z^2 g_2(z)R^*(p\lambda-p\lambda c(z))]-\alpha\bar{\theta}zc(z)T_2(z)T_3(z)\right\}/D(z)T_3(z)$$

$$P(z)=\alpha I_0T_1(z)\,\{1-B_0^*(p\lambda-p\lambda c(z))\}\,/\,D(z)(p-pc(z))$$

$$Q_i(z)=\alpha\, I_0T_1(z)\,r_iB_0^*(p\lambda-p\lambda c(z))\{1-B_i^*(p\lambda-p\lambda c(z))\}\,/\,D(z)(p-pc(z)),\quad 1\le i\le M$$

$$R(z) = \bar{\alpha}\,I_0(1-R^*(p\lambda-p\lambda c(z)))\{\,\alpha\bar{\theta}\,c(z)T_1(z)T_2(z) + \alpha\bar{\theta}\,(1-c(z))T_2(z)\,A^*(\lambda)c(z)[\bar{\alpha}\bar{q}R^*(p\lambda-p\lambda c(z))(1-A^*(\lambda))(z\,c(z)-c(z))-z] - (1-c(z))\,A^*(\lambda)\,\{\bar{\alpha}z^2g_2(z)R^*(p\lambda-p\lambda c(z))+\theta zT_2(z)T_3(z)-z^2\}\}\,/D(z)T_3(z)(p-pc(z))$$

Performance measures

The probability that the server is idle during the retrial time is

$$I = I_0(1-A^*(\lambda))\{\alpha[1-(p\lambda m_1\mu_{01}+\sum_{i=1}^{M} r_i p\lambda m_1\mu_{i1})] - p\lambda m_1\gamma_1\bar{\alpha}\theta - \bar{\theta}m_1\} / D_1$$

The probability that the server is busy in essential service is given by

$$P = \alpha\,\lambda\, I_0\,\mu_{01}\,[A^*(\lambda) + (1-A^*(\lambda))(m_1+\alpha q+\bar{\alpha}) - (m_1+1)]\ / D_1$$

The probability that the server is busy in i^{th} multi optional service is given by

$$Q_i = \alpha r_i\lambda I_0\mu_{i1}[A^*(\lambda) + (1-A^*(\lambda))(m_1+\alpha q+\bar{\alpha}) - (m_1+1)]\ / D_1, \qquad 1\le i\le M$$

The probability that the server is under repair is given by

$$R = \lambda\bar{\alpha}\, I_0\gamma_1\bar{\theta}[A^*(\lambda) + (1-A^*(\lambda))(m_1+\alpha q+\bar{\alpha}) - (m_1+1)]\ / D_1$$

where

$$D_1 = \alpha[p\lambda m_1\mu_{01} + \sum_{i=1}^{M} r_i p\lambda m_1\mu_{i1}] + [A^*(\lambda) + (1 - A^*(\lambda))q]\bar{\theta}\,\alpha + p\lambda m_1\,\gamma_1\bar{\alpha}\,\theta + \bar{\theta}\, m_1(1-A^*(\lambda)) - 2\alpha + \theta\alpha$$

Substituting the expressions of I, P, Q_i, and R in the normalizing condition (15) and solving we get the analytical expression for I_0 as

$$I_0 = D_1\ / A^*(\lambda)\{\alpha[p\lambda m_1\mu_{01} + \sum_{i=1}^{M} r_i p\lambda m_1\mu_{i1}] + p\lambda m_1\,\gamma_1\bar{\alpha}\,\bar{\theta}\} + \{\alpha[\lambda\mu_{01} + \sum_{i=1}^{M} r_i\lambda\mu_{i1}] + \lambda\gamma_1\bar{\alpha}\bar{\theta}\}$$
$$[A^*(\lambda) + (1-A^*(\lambda))(m_1+\alpha q+\bar{\alpha}) - (m_1+1)] + \alpha[(\bar{\theta}q+\theta)(1-A^*(\lambda)) - 1]$$

The probability generating function of the number of customers in the orbit is

$$P_q(z) = I_0 + I(z) + P(z) + \sum_{i=1}^{M} Q_i(z) + R(z)$$
$$= I_0\{g_1(z) + (1-A^*(\lambda))(p-pc(z))\{(1-\bar{\alpha}\,c(z)\, R^*(p\lambda - p\lambda c(z)))[z^2 - \theta z T_2(z)T_3(z) - \bar{\alpha}z^2 g_2(z) R^*(p\lambda - p\lambda c(z))] - \alpha\bar{\theta}z\, c(z)T_3(z)T_2(z)\} + \alpha T_1(z)(1-T_2(z))T_3(z) + \bar{\alpha}(1 - R^*(p\lambda - p\lambda c(z)))\{\alpha\bar{\theta}\, c(z)T_1(z)T_2(z) + \alpha\bar{\theta}\, c(z)(1-c(z))A^*(\lambda)\, T_2(z)\,[\bar{\alpha}\bar{q}R^*(p\lambda - p\lambda c(z))(1-A^*(\lambda))(zc(z) - c(z)) - z] - (1-c(z))\ A^*(\lambda)\{\bar{\alpha}\,z^2 g_2(z)R^*(p\lambda - p\lambda c(z)) + \theta z T_2(z)T_3(z) - z^2\}\}\}\ /\ g_1(z)$$

The probability generating function of the number of customers in the system is

$$P_s(z) = I_0 + I(z) + zP(z) + z\sum_{i=1}^{M} Q_i(z) + R(z)$$
$$= I_0\{(1-A^*(\lambda))(p-pc(z))\{(1-\bar{\alpha}\,c(z)\, R^*(p\lambda - p\lambda c(z)))[z^2 - \theta z T_2(z)T_3(z) - \bar{\alpha}z^2 g_2(z) R^*(p\lambda - p\lambda c(z))] - \alpha\bar{\theta}z\, c(z)T_3(z)T_2(z)\} + \alpha T_1(z)T_3(z) - \alpha T_1(z)T_2(z)T_3(z) + \bar{\alpha}(1 - R^*(p\lambda - p\lambda c(z)))\{\alpha\bar{\theta}\, c(z)T_1(z)\, T_2(z) + \alpha\bar{\theta}\, c(z)(1-c(z))A^*(\lambda)\, T_2(z)\,[\bar{\alpha}\bar{q}R^*(p\lambda - p\lambda c(z))(1-A^*(\lambda))(z\,c(z) - c(z)) - z] - (1-c(z))\ A^*(\lambda)\{\bar{\alpha}\,z^2 g_2(z)R^*(p\lambda - p\lambda c(z)) + \theta z T_2(z)T_3(z) - z^2\}\} + g_1(z)\}\ /\ g_1(z)$$

where

$$g_1(z) = D(z)(p - pc(z))\, T_3(z)$$

The mean number of customer in the orbit is

$$L_q = \lim_{z\to 1}\frac{d}{dz}P_q(z) = \frac{N_3D_2 - D_3N_2}{3(D_2)^2}$$

$$N_2 = I_0\{-2pm_1\alpha\bar{\theta}(1-A^*(\lambda))T_4 - 2A^*(\lambda)pm_1\alpha D_1 - 2\alpha^2T_4T_6 - 2\alpha\bar{\alpha}\,\bar{\theta}\,T_4\,p\lambda m_1\gamma_1 + \bar{\alpha}A^*(\lambda)p\lambda m_1\,\gamma_1 D_1\}$$

$$D_2 = -2pm_1\alpha\, D_1$$

$$D_3 = 6pm_1\,\bar{\alpha}\, D_1\,\{m_1(1-A^*(\lambda)) + p\lambda m_1\,\gamma_1\} - 3\alpha[D_1\, pm_2 + D_2\, pm_1]$$

$$N_3 = I_0\{(-3p\alpha\bar{\theta}(1-A^*(\lambda))[2m_1T_4T_6+m_1T_5+m_2T_4]+6A^*(\lambda)pm_1D_1(p\lambda m_1\gamma_1)-3A^*(\lambda)p\alpha\left[2m_1D_1(p\lambda m_1\gamma_1)+m_2D_1+m_1D_2\right]+3\alpha[-2T_4T_6T_8-T_7T_4\alpha-T_5T_6\alpha\right]+\alpha\bar{\alpha}\,\bar{\theta}\{-6(1-A^*(\lambda))p\lambda m_1^2\gamma_1T_4-3p\lambda m_1\gamma_1(2T_6T_4+T_5)-3T_4\,[p^2\lambda^2m_1^2\gamma_2+p\lambda m_2\gamma_1]\}+\bar{\alpha}A^*(\lambda)\{-4p\lambda m_1^2\gamma_1D_1+p\lambda m_1\gamma_1D_2+2D_1(p^2\lambda^2m_1^2\gamma_2+p\lambda m_2\gamma_1)\}\}$$

where

$$T_4 = [A^*(\lambda)+(1-A^*(\lambda))(m_1+\alpha q+\bar{\alpha})-(m_1+1)]$$

$$T_5 = (1-A^*(\lambda))\{2qm_1[1-2\bar{\alpha}-\bar{\alpha}\lambda p\gamma_1]+m_2+4m_1\bar{\alpha}+2m_1\bar{\alpha}\lambda p\gamma_1\}-(m_2+2m_1)$$

$$T_6 = p\lambda m_1\mu_{01}+\sum_{i=1}^{M} r_i p\lambda m_1\mu_{i1}$$

$$T_7 = (p^2\lambda^2m_1^2\mu_{02}+p\lambda m_2\mu_{01})+\sum_{i=1}^{M} r_i\{2p^2\lambda^2m_1^2\mu_{01}\mu_{i1}+p^2\lambda^2m_1^2\mu_{i2}+p\lambda m_2\mu_{i1}\}$$

$$T_8 = -\bar{\alpha}\{m_1(1-A^*(\lambda))+p\lambda m_1\gamma_1\}$$

The Mean number of customer in the system is given by

$$L_s = \lim_{z\to 1}\frac{d}{dz}P_s(z)$$

$$= L_q + P(1) + \sum_{i=1}^{M} Q_i(1).$$

IV. CONCLUSION

In this paper, we analyzed a batch arrival retrial queueing system with starting failures, customer impatience, Multi optional second phase service and orbital search. Various performance measures like the probability that the server is idle, busy, repair in steady state and mean orbit size, mean system size are derived. The research on the present investigation can be further extended by including the concepts of working breakdown, Bernoulli feedback and vacation.

REFERENCES

[1] Ke, J.C and Chang, F.M "$M^x/G_1G_2/1$ Retrial queue with Bernoulli vacation schedules with repeated attempts and starting failures", Applied Mathematical modelling , vol. 33, 3186 - 3196, 2009
[2] Krishnakumar, B. and Suganthi, P. "Analysis of M/G/1 Retrial queue with second optional service and customer balking under two types of Bernoulli vacation schedules", RAIRO - Operation Research, vol. 53, No.2, 415 – 443, 2019.
[3] Madhan, K.C. "On optional deterministic server vacation in a single server queue providing two types of first essential service followed by two types of additional optional service", Appli. Math. Sci. vol. 12(4), 147-159, 2018.
[4] Pavai Madheswari, S., Josephine, S.A. and Suganthi, P. "Retrial queueing system with retention of reneging customers", International journal of pure and applied mathematics, vol. 106, no 5, 11 - 20, 2016.
[5] Rajadurai, P., Varalakshmi, M. and Saravanarajan, M.C. "Analysis of $M^x/G/1$ retrial queue with two phases of service with Bernoulli vacation schedule and random breakdown", International journal of mathematics in operational research vol. 7(1), 19 - 41, 2015.
[6] Suganthi, P. and Pavai Madheswari, S. "Retrial queueing system with customer impatience", Global journal of pure and applied mathematics, vol. 11, no. 5, 3177 - 3188, 2015.
[7] Sumitha, D. and Udaya chandrika, K. "Retrial queueing system with starting failure, single vacation and orbital search", International journal of computer applications, vol. 40, no. 13, 29 - 33, 2012.
[8] Varalakshmi, M., Chandrasekaran, V.M. and Saravanarajan, M.C. "A study on M/G/1 retrial queue with two phases of service, immediate feedback and working vacations", IOP Conference series materials science and engineering, 263, no: 042156, 1 – 16, 2017.
[9] Vanitha,S. "M/G/1 Feedback queue with two stage heterogeneous service and deterministic server vacations", Int.Journal of Applied Engineering Research, vol. 13, no. 22, 15899 - 15907, 2018.