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Back Analysis of Failed Slopes - A Case Study

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***Abstract*—Slope failure has become a common phenomenon around the world, often taking a heavy toll on human lives. Both, natural and man-made slopes created, are susceptible to failure unless sufficient stability is not imparted. Knowledge of all the factors causing failure is an important aspect to understand the mechanical behavior of slope material. The shear strength parameters, cohesion, and angle of internal friction of failed material, determined from laboratory and in-situ tests may be misleading. Back analysis is a much reliable and consistent method to estimate the mobilized parameters in-situ. This paper explores the application of traditional limit equilibrium method (LEM) and Finite Element Method (FEM) to perform back analysis and find out the probability of failure of slopes. It performs probabilistic analysis on a case of failed slope using both the methods. This paper shall discuss certain indispensable drawbacks of limit equilibrium method and how Finite Element method can help to overcome those limitations.**

***Keywords*—Limit Equilibrium Method; Finite Element Method; Probabilistic Analysis; Point Estimate Method; Shear Strength Reduction.**

I. INTRODUCTION

The key challenge in the mining sector is the shortage of land for the disposal of overburden that is generated along with the extraction of precious minerals. With the major shifting of mining activities from underground to open cast because of obvious safety issues, economics and potential to produce huge mineral output, surface mining has become popular. As the valuable deposits are being unearthed at much faster rate due to the growing demand their concentration near the surface is depleting proportionally forcing men to venture into greater depths. This has resulted in the generation of more volume of overburden with less land to stake on. Not only is the unavailability of land a major problem but the huge transportation cost is also an issue as it accounts for approximately 40 % of the mining cost of minerals. This particular problem can be solved by opting for internal dumping. Though internal dumping resolves this pressing issue the increasing number of internal dump failures cannot be neglected. As the internal dumping is done in the immediate vicinity of the working site, the failure of the dump causes major losses as compared to that caused by the failure of external dump slopes. In many cases, it has taken a huge toll on human lives and has buried machinery worth crores. To avoid such losses remedial slope stability measures are required which can only be drawn out by determining the causes that led to slope failures. Hence, a reliable methodology is required to determine the shear strength, pore pressure or other conditions prevailing at the time of failure. Performing laboratory tests on collected field samples of failed dump can provide shear strength parameters of the dump material. But the results always involve uncertainties because the samples collected not usually represent the materials involved in the potential failure surface and also it is difficult to simulate the conditions existing at the time of failure. There are many drawbacks of this conventional method which can be avoided by performing the back analysis. Back Analysis can determine not only the material shear strength parameters but also give a better understanding of the failure which can

Fig. 1 Failed portion of internal dump slope

be used as a basis for selecting remedial measures. The uncertainties involved can be easily dealt with by performing a probabilistic back analysis which quantifies the uncertainty and determines the likelihood of possible occurrence of different outcomes.

This paper will present an overview of probabilistic back analyses of a failed slope using two different methods; Limit Equilibrium method and Finite Element method. It will also include a comparative study of the two methods weighing their advantages and disadvantages.

II. PROJECT BACKGROUND

Western Coalfield Limited (WCL) which is spread over many districts of Maharashtra and Madhya Pradesh is one of the eight subsidiaries of Coal India Limited. It has contributed about 7.02% of the national coal production during 2015-2016 (WCL, Nagpur). A slope failure of a 75 m high dragline dump with an inclination of 43˚ occurred on 3rd June 2009 at one of the opencast mines of WCL. A loading machine was buried while two people working near the failure site died. The dump slipped forward to a distance of 18 m (Figure 1). The surface along which the failure took place is in the shape of a circular arc starting from the slope face little above the toe and extending up to the tension crack located few meters behind the crest. The dump material consists of a mixture of friable sandstone, clay, shale and carbonaceous shale of sizes varying from 4 µm to 1 m (Kainthola et al, 2011). The samples collected from five different zones were tested for the angle of internal friction, cohesion, young's modulus and Poisson's ratio. The geometry of the slope is shown in Figure 2.

III. BACK ANALYSIS METHODS

Factor of safety (FS) has been used commonly to quantitatively assess the stability of a slope. It is used as an index to determine how close or far away a slope is from failure. It is defined as the ratio of resisting forces (and/or moments), to the forces required for equilibrium (driving forces).

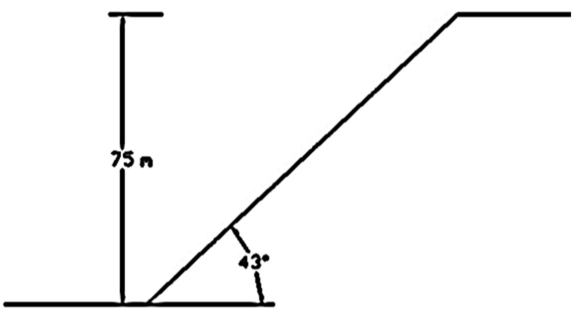

Fig. 2 Geometry of the slope before failure

$$FS = \frac{Resisting\ force}{Driving\ force} \quad (1)$$

The slope is said to be stable when its safety factor is greater than one and if it's less than one then the slope is said to be unstable. FS equal to unity is a critical condition where the shear strength of the slope maybe mobilized.

The slope stability analysis is generally implemented in two distinct ways: Forward analysis and Back analysis. Forward analysis is used to assess if a slope is performing safely as it was intended to, while back analysis is carried out to determine the condition of the slope that existed at the time of its failure i.e.: what was the mobilized strength and the pore-water pressure condition at the time of failure.

There are two methods to back analyze the slope stability parameters: deterministic and probabilistic method (Duncan et al, 2005). The deterministic method aims at finding a single set of parameters that may cause the failure. Back analysis is performed in a probabilistic way when multiple sets of slope stability parameters have to be back-analysed simultaneously under uncertainty (Zhang et al, 2010). The uncertain parameters are modeled as random variables and a probability distribution of the outcome is drawn based on which probability of failure is calculated. The distribution can be improved further by incorporating the slope failure information.

A. Limit equilibrium methods

Traditional limit equilibrium methods are the most widely applied analytical technique in slope stability analysis. They utilize the Mohr-Coulomb criterion to determine the shear strength along the failure surface. The principle of all the LE methods is to reduce the shear strength of the material by a factor of safety to attain the equilibrium against the shear stresses. All the methods are based on the static of equilibrium. There are two ways to satisfy the static of equilibrium. The first approach considers the equilibrium of the entire slope mass and solves for a single free body. The other approach divides the slope into numbers of slices where all the forces acting on each of the slices has to satisfy the equilibrium.

The most popular limit equilibrium technique is a method of slices. In this method, the slope is divided into a number of finite vertical slices and the factor of safety is calculated by taking in consideration the equilibrium of each slice. Several LEMs have been developed since 1936 when Fellenius introduced the first method, Ordinary method of slices. All LEMs are based on different assumptions for inter-slice normal (E) and shear forces (T) acting on the slices into which the slope is divided. Figure 3 illustrates a failure mass divided into a number of slices and the forces acting on one slice. The fundamental difference between the methods is the equilibrium conditions they use to calculate the factor of safety and the shape of the failure plane.

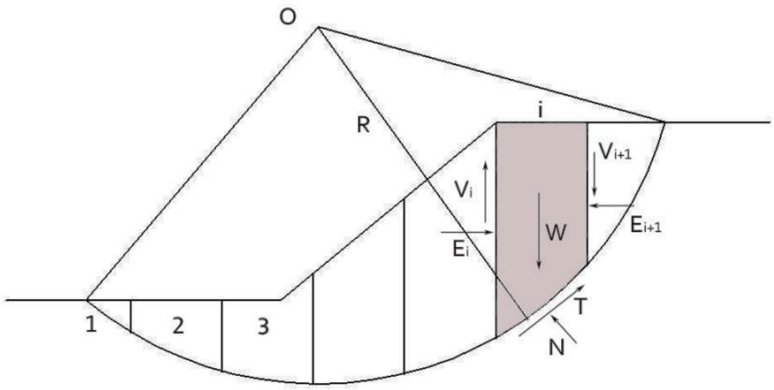

Fig. 3 Forces acting on a slice in the slip circle method.

The inter-slice force function is the ratio of tangential to normal forces acting on a slice and it depends on a number of factors, including stress-strain and deformation characteristics of the material (Aryal, 2008). Most of the LE methods either neglect one or both of the factors since their evaluation complicates the analysis. LEM is very convenient to apply but it has major drawbacks. Most of the slope stability problems are statically indeterminate and complex. LEM oversimplifies the problem by assuming the factor of safety as constant along the entire slip surface which can result in misleading results especially when the slope is heterogeneous and slip surface passes through different materials. Other major drawbacks- not incorporating in-situ stresses and neglecting progressive failure.

B. Finite Element Method

With the significant development in computer technology along with low cost, the application of FEM in slope stability analysis has become increasingly common. The primary advantage of this method over LEM is its flexibility. It requires no assumption about the shape or location of the failure surface in advance or about the inter-slice forces and their directions. This method can be used to analyze slopes with complex configurations. All types of failure mechanism can be virtually modeled using FEM. Most of the general soil material models along with Mohr-Coulomb criterion can be employed. FEM is capable of modeling progressive failure and it can calculate deformations at different slope stress levels.

Shear Strength Reduction technique is one of the approaches FEM uses to perform slope stability analyses which has gained popularity in recent years. In this method, the strength characteristics like cohesion and friction angle are successively reduced by a factor till instability is induced in the system. The factor is termed as Strength Reduction Factor (SRF). An SRF is said to be critical when the finite element model does not converge to a solution or in a simple term, the system becomes unstable (Hammah et al, 2005, 2007).

IV. BACK ANALYSIS OF THE FAILED SLOPE

The main objective of analyzing a failed slope is to determine the mobilized shear strength which can be done by performing laboratory tests on the samples collected from the field. However, the discrepancy between the field and laboratory observations introduces many uncertainties in the analysis which leads to misleading conclusions. These uncertainties can be easily dealt with by using back analysis with a probabilistic approach.

In the deterministic back analysis, it is assumed that the values of all the input parameters are known. But in real world problems, it is not possible to get the exact value of the input parameters. In the probabilistic analysis, statistical distributions are assigned to the input parameters which take cares of the uncertainties in their values. In this study, Rocscience software, Slide and RS^2, are used to perform the probabilistic analysis using LEM and FEM respectively.

The probabilistic analysis can be implemented in following steps:

1. Choose a Sampling method (Point Estimate, Monte Carlo or Latin Hypercube) and a number of samples.
2. Select the type of probabilistic analysis: Global Minimum or Overall Slope.
3. Select the model input parameters (unit weight, cohesion, the angle of internal friction, Poisson’s ratio, etc.) and define them as random variables by assigning them with statistical distributions (mean, standard deviation, variance, etc.).
4. After the random variables have been defined, select compute. The regular deterministic analysis is run at first followed by probabilistic analysis.

According to the statistical distribution, sampling method and numbers of samples (N), the chosen random variables are sampled. This generates N number of samples for each random variable and, hence, N number of sets. Iteration of

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Table 1. Input Parameters for back analysis

Input Parameters	Mean values *(Mohr-Coulomb)*
Cohesion (c)	88.6 kPa
Angle of Friction (ϕ)	24.63˚
Unit weight	24.4 kN/m^3
Young's Modulus	72.3 MPa
Poisson's ratio	0.326

probabilistic analysis is carried for each set of random variables, for example, (c_1, ϕ_1), (c_2, ϕ_2), (c_3, ϕ_3) and so on.

Results obtained from laboratory testing are as shown in Table 1. The cohesion (c) and angle of internal friction (ϕ) are chosen as the random variables for the analysis. Other parameters are considered to be known with more certainty and used as deterministic input variables as they do not affect the SRF and location of the critical failure surface of the slope. Global minimum is selected as the probabilistic analysis type and the number of samples is taken as 1000 for both the analysis method which is further discussed below.

A. Back analysis using LEM

To carry out LEM, Rocscience software Slide is used. Global Minimum (GM) method along with Monte Carlo as the sampling method is used for the probabilistic analysis. The deterministic analysis is carried out first using the mean of all the input parameters to find one critical failure surface. Once the location of the critical failure surface is known probabilistic analysis is performed for this surface using the generated samples of the selected random variables. Among the available LE methods, Bishop Simplified Method (BSM) is used as it is the most suitable for a circular failure and for establishing a common platform for conducting the comparative study between Limit Equilibrium and Finite Element methods. Normal distribution for random variables is selected.

Figure 4 shows the critical failure surface in the slope. The factor of safety (FS) calculated by deterministic analysis is found as 1.077 which is also same as that determined by probabilistic analysis. The probability of failure (PF) is found to be 0.700% which means out of 1000 samples, seven failed. The reliability index (RI) is 2.471 based on normal distribution and 2.550 based on lognormal distribution. A slope is said to be safe if the RI is 3 or greater (Rocscience). This justifies the low value of RI of the failed slope in concern and also the value of FS of the slope which is very close to unity.

B. Back analysis using FEM

FEM – SSR analysis was performed together with Point Estimate Method (PEM) using the *RS*^{2} software. PEM was developed by Arturo Rosenblueth in 1975. Its principle is to run the iterative FEM-SSR analysis for various estimation points and determine the probability of failure by drawing a statistical distribution based on the output result. By incorporating the field observation and slope failure conditions the distribution can be improved further.

For each random variable, cohesion, and angle of internal friction, in this case, two 'point estimates' are determined.

$$Point\ estimates = mean \pm SD \qquad (2)$$

The finite element analysis is carried out for each possible combination of point estimates. This produces 2^m solutions, where m is the number of random variables involved. For example, if the number of random variables is two, as in this

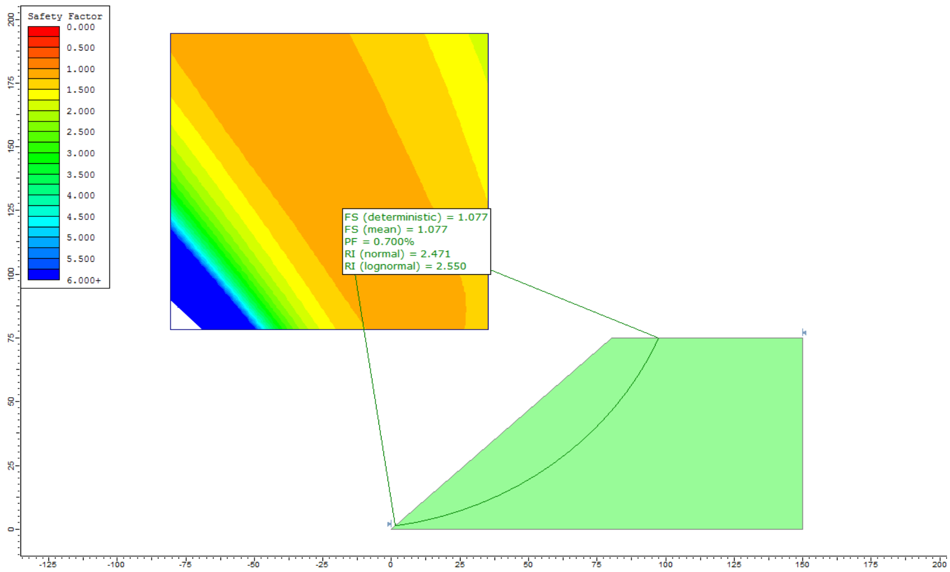

Fig. 4 Back analysis of the failed slope using LEM

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case, there will be four possible combinations, (c_1, ϕ_1), (c_2, ϕ_2), (c_3, ϕ_3) and (c_4, ϕ_4). Since the number of samples was taken as 1000 the FEM-SSR analysis will be performed 4000 times (Number of samples × number of combinations) to determine the factor of safety.

To duplicate the results of LEM analysis few necessary assumptions are made which are as follows:

1). Mohr-Coulomb model was chosen as the constitutive material model.
2). The material was assumed to be elastic perfectly plastic for post-peak behavior.
3). Dilation angle was taken as zero.

If the system is multi-material model Young's modulus and Poisson's ratio should be kept constant for each material. These findings are consistent with assertions made in other reports (Dawson et al, 1999 and Hammah et al, 2012).

The FS determined by FEM-SSR analysis is 1.04 and the PF is calculated as 15%. Figure 5 shows the critical surface of failure and shear strain developed in the slope at the time of failure. The maximum shear strain along the critical failure surface is found to be 7.80×10^{-2}.

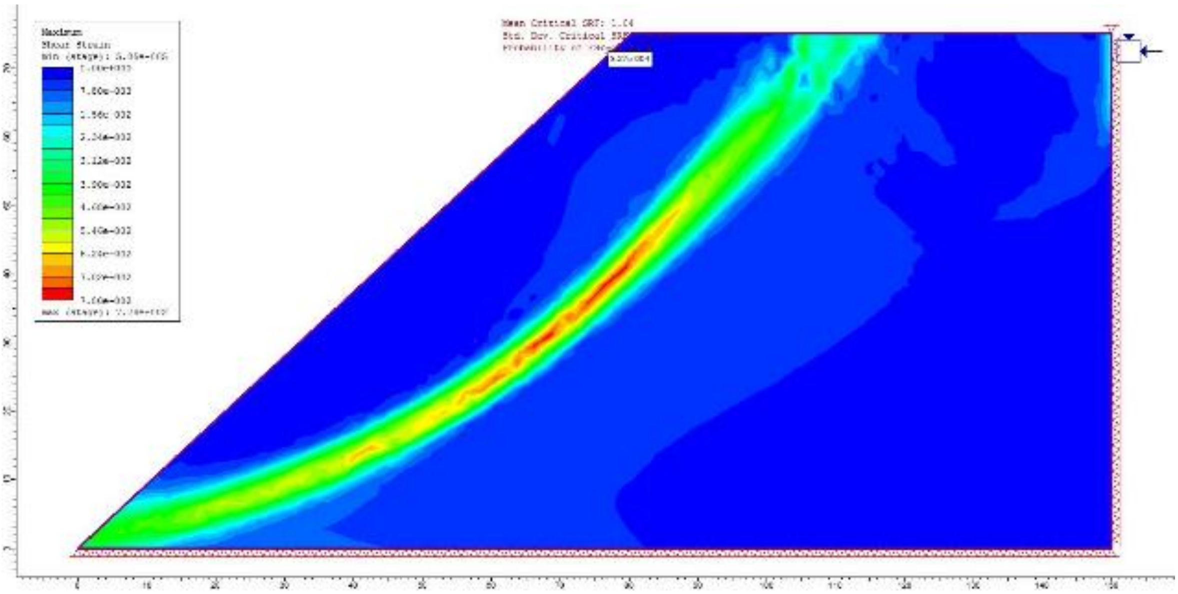

Fig. 5 Maximum shear strain (7.80×10^{-2}) - using FEM-SSR

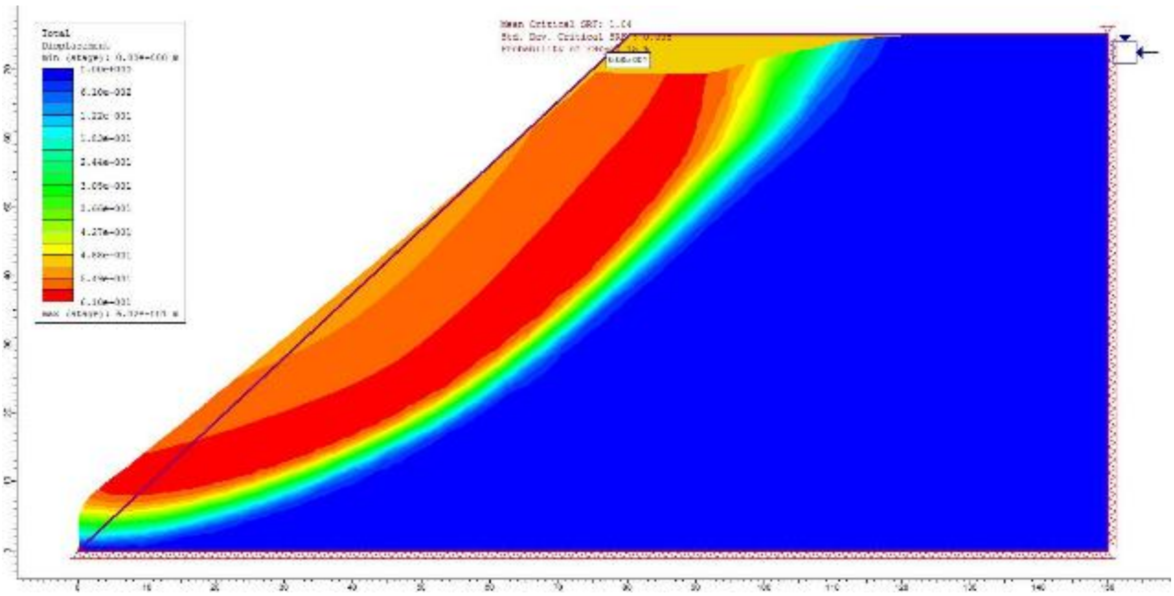

Fig. 6 Total displacement (0.6m) - using FEM-SSR

Figure 6 shows the total displacement occurred when the failure occurred which is equal to 0.6 m. The maximum horizontal and vertical displacement are found to be 0.0002 m (Figure 7) and 0.002 m (Figure 8) respectively. The maximum volumetric strain equal to 2.99×10^{-3} is at the toe of the slope (Figure 9). Figure 10 shows the plasticity of the failed slope. The top 20m thick portion of the slope is filled with tension elements which are responsible for the formation of tension cracks on the crest. Shear elements are noticeable along the circular failure surface originating from the tension zone and continuing till the toe of the slope. The yielded elements of the slope are shown in figure 11. It shows how the crest and the toe along with the circular arc portion of the slope have yielded at the time of failure.

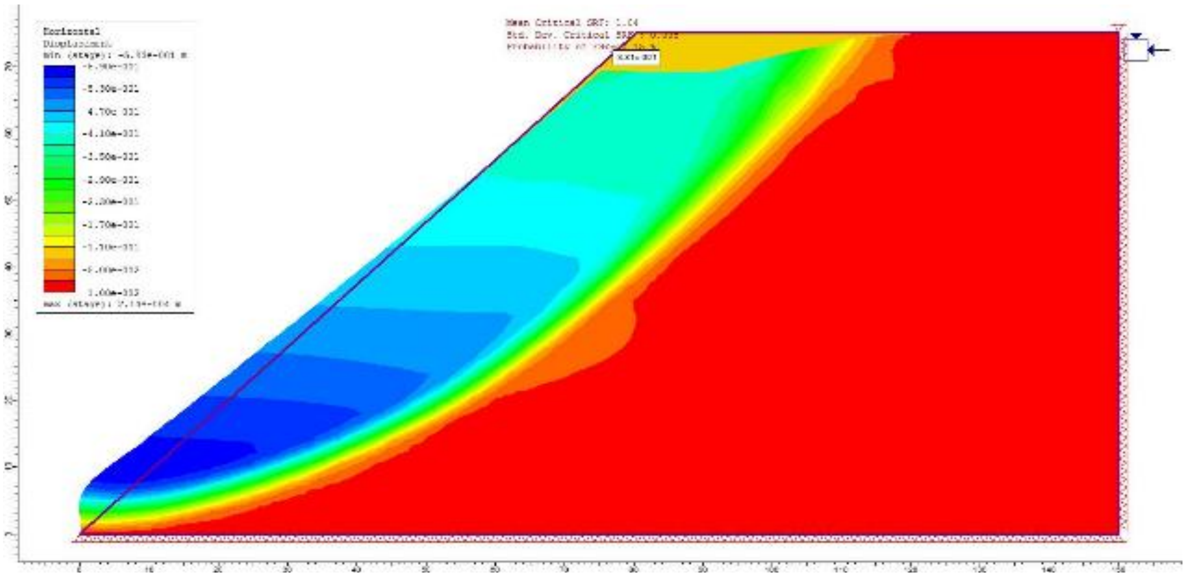

Fig. 7 Horizontal displacement 0.0002m - using FEM-SSR

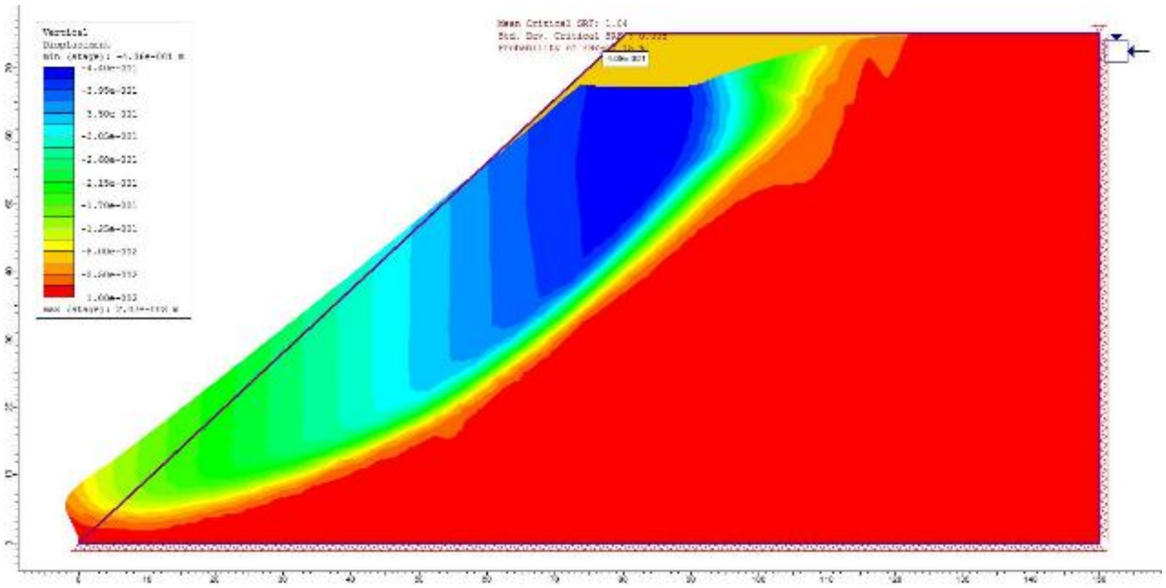

Fig. 8 Vertical displacement 0.002m - using FEM-SSR

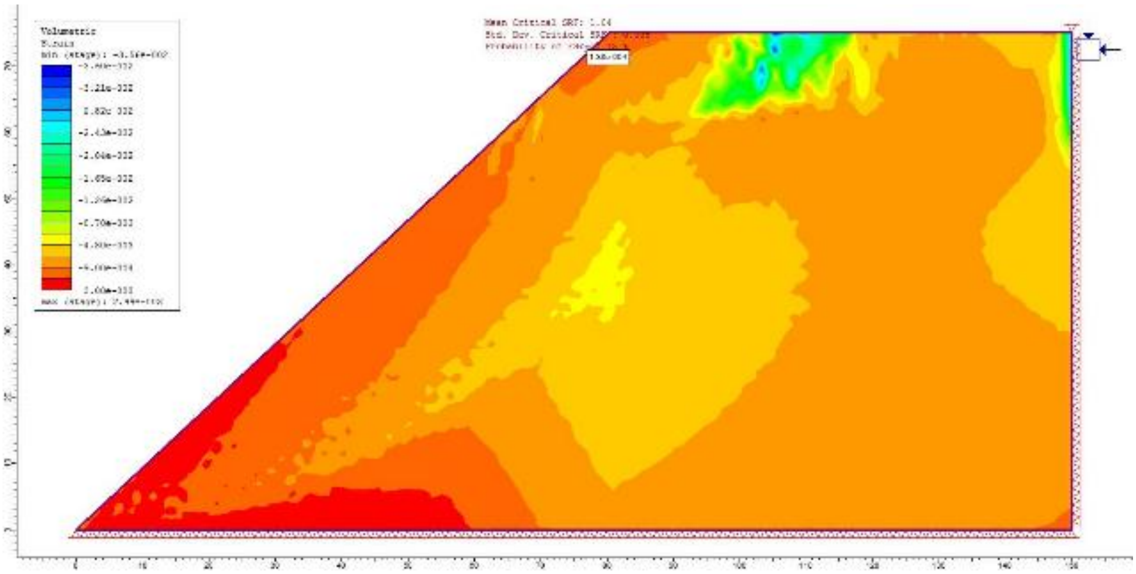

Fig. 9 Volumetric strain (2.99 × 10^{-3}) - using FEM-SSR

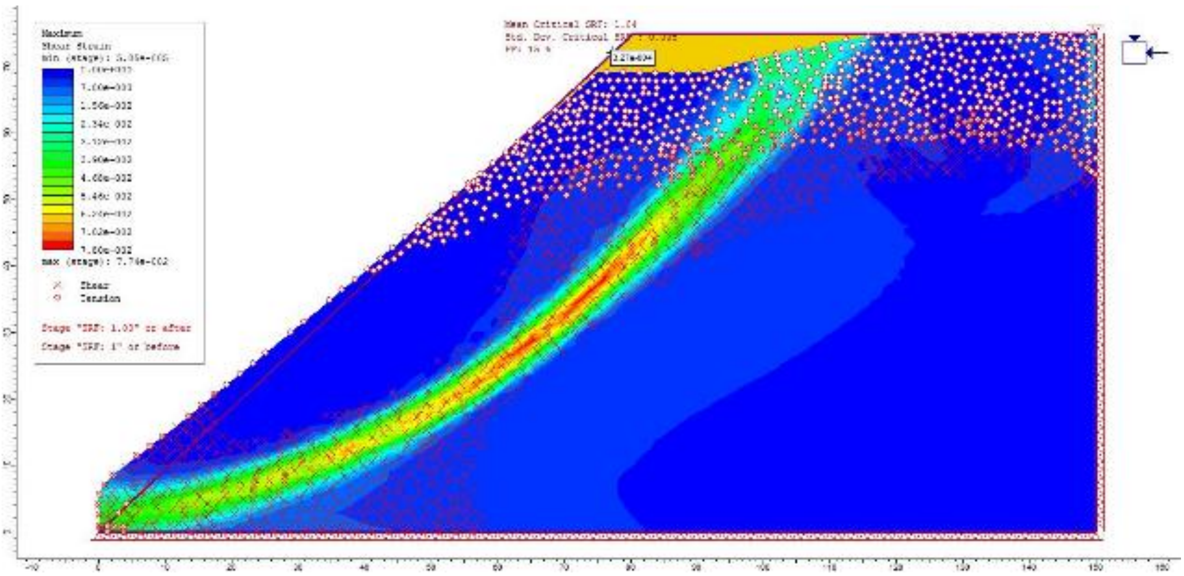

Fig. 10 Distribution of Shear and Tension elements along the failed slope

V. RESULTS AND DISCUSSION

The results obtained by performing probabilistic analysis using both the methods are summarized in Table 2. The results from both the methods are close to each other with the only small difference in decimals. The slightly larger difference in probability of failure is due to the fact that the mean FOS computed by FEM is closer to one and hence its probability of failure is higher than the other.

The results also show that with FOS as unity** (Table 2) the values for the angle of internal friction obtained from the two methods are similar. The difference in the two values for cohesion is insignificant.

Table 2. Results for probabilistic analysis using LEM and FEM

Analysis Method		LEM *(Slope)*	FEM *(RS^2)*
Mean FOS		1.077	1.04
Standard Deviation FOS		0.0311	0.035
Probability of failure		0.007%	15%
At FOS =1**	Cohesion (kPa)	81.94	86.02
	Angle of friction (degree)	23.07	23.96

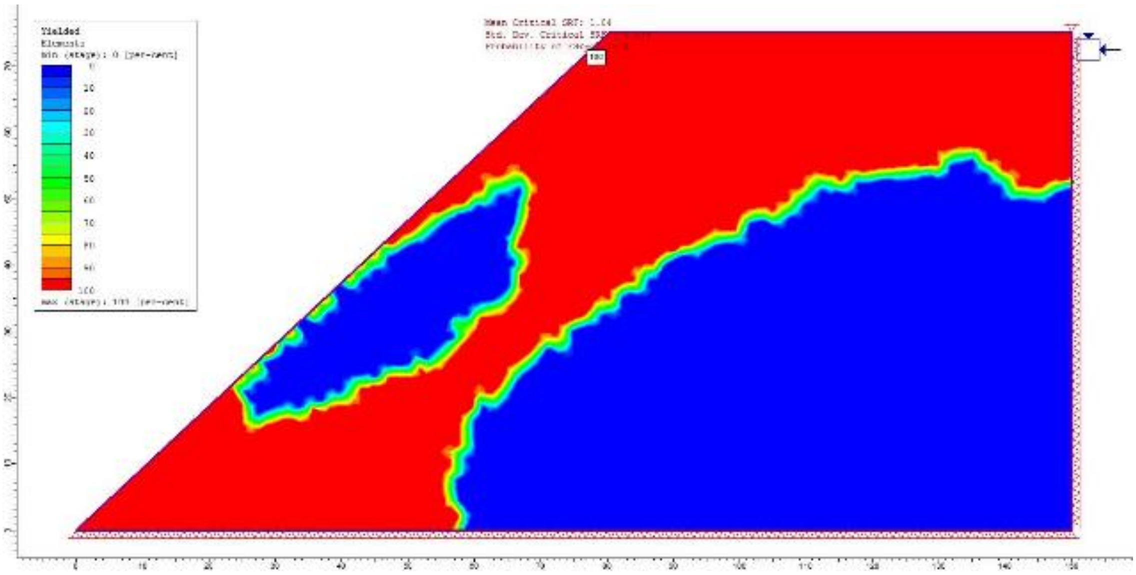

Fig. 11 Yielded elements along the failed slope

VI. CONCLUSION

While analyzing slope stability it is in the best interest to use statistical methods to deal with the uncertainties which are normally involved in all the slope stability problems. Two methods, limit equilibrium method and Finite Element method, based on probability were used in this paper for conducting back analyses of a failed slope. Based on the arrived results the following conclusions were made:

1). The study showed that limit equilibrium method is convenient and more popular but it has some shortcomings.
   a) While LEM is based on some assumptions, FEM-SSR does not require any assumptions for generating failure surface.
   b) LEM does not incorporate in-situ stresses.
   c) It assumes the factor of safety to be constant along the slip surface of the slope. Such assumptions oversimplify the problem when the slip surface passes through many materials of different properties.
   d) LEM does not consider stress evolution or progressive failure which again leads to oversimplification of the problem.

2). The difference in the results obtained from the two analyses is relatively small. The values for cohesion and angle of internal friction obtained from LEM were 81.94 kPa and 23.07˚ respectively. Whereas the FEM produced 86.02 kPa and 23.96˚ as the values for cohesion and angle of internal friction respectively.

3). The probabilities of failure obtained from LEM and FEM analyses are 0.007% and 15% respectively. The huge difference in the values is because the FOS derived from FEM analysis is comparatively closer to unity.

4). Finite element method not only captures the whole failure mechanism in a true sense but it also predicts stresses and deformations.

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