Augmentation of Classical and Adaptive Control for Second Generation Launch Vehicles

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Abstract— Classical control technique is used for attitude control of launch vehicles worldwide, because of its established history of success. Usual method is to model launch vehicle dynamics by linear techniques to achieve adequate stability and tracking performance. Common type of feedback control system for launch vehicles is the Proportional-Integral (PI) controller with appropriate filters to stabilize the lateral bending modes and slosh modes and also ensure sufficient robustness margins for rigid body. This paper presents a Classical Adaptive Augmentation Control (AAC) Algorithm for forward loop gain augmentation in real time, to cater to large dispersion in vehicle parameters beyond the capability of classical control system. The idea is to provide augmentation to a classical control designed autopilot when performance enhancement is required to tackle off-nominal conditions arising out of modeling errors and large dispersion in estimated vehicle parameters (thrust, inertia, slosh, aerodynamics, lateral bending modes). There is high chance that such large dispersion can arise during initial design of a new generation launch vehicle before actual flight. Finally, simulation results for several credible launch vehicle failure scenarios show that the adaptive controller consistently and predictably improves performance and robustness, and achieves stability during extreme off-nominal situations.

Keywords—Adaptive Control; Augmentation; Parameter Uncertainty; Gain Scheduling; Robustness

I. INTRODUCTION

As Launch Vehicles pace up along with the development of new technologies, the core targets of new designs are to enhance payload capability (performance), reliability, and safety at decreasing cost. As computational capability has become advanced, control algorithms play a major role towards achieving this aim. Global control algorithm used in current generation launch vehicles is the Proportional Integral (PI) control with gain scheduling. Although much advancement have taken place over the last few years, PI with gain scheduling control remains dominant due to its strong heritage. Generally, attitude control problem is considerate of the short period dynamics of the vehicle, where the basic aim is to achieve adequate stability and reasonably rapid response to input guidance commands, with average passivity to external disturbances. Launch vehicles are often aerodynamically highly unstable. Despite that, launch vehicle dynamics are readily modeled in literature using linear techniques to arrive at an autopilot configuration that meets the design requirements. For the flight control systems design, the consolidations of Blakelock[1], Greensite[2], and Garner[5] are quite all-inclusive.

Major design problems exist because a launch vehicle is aerodynamically unstable, highly flexible and additional problems due to sloshing of liquid propellants and the inertia effects of engines. These problems are seriously aggravated for certain advanced launch vehicle configurations. Under such a scenario, classical control methods may not be fully effective in meeting the robustness margins for very large dispersions in vehicle parameters mainly because they are not known with sufficient precision before flight. Because a failure of any one of components could mean loss of the vehicle, an extensive ground testing and evaluation program is necessary to provide the maximum confidence for successful flight. This has led to the focus in adaptive control technique.

Greatest benefit of adaptive control that can be exploited is the fact that they don’t require a deductive knowledge of the launch vehicle parameters with great accuracy. A survey of Adaptive Control Systems by Astron, et al., in [6] clarifies the immense potential of the approach. In order to fully extract the benefits of adaptive control for a particular application, the adaptive control system must be designed with knowledge of the complete system to which it is to be applied [7], [8], [9]. This includes general features of the aerospace vehicle, such as control-structure interaction, sloshing of propellants, performance of sensors, and actuator dynamics. Out of the many adaptive control schemes, the direct Model Reference Adaptive Control (MRAC) shows robustness to uncertainties with sometimes improved and more predictable performance, [10], [11], [12].

An algorithm that relies on model reference-driven gain adaptation supplemented by spectral damping is demonstrated by Jeb S Orr, et al., in [13]. The focus is on adaptive control developments that are specifically tuned for application to launch vehicles which maintains consistency with classical control system design.

This paper presents implementation and validation of a classical Adaptive Augmentation Control (AAC) algorithm for a typical 2nd generation launch vehicle, for performance enhancement in the event of large deviations in estimated vehicle parameters. AAC provides minimum augmentation for a typical situation arising out of modeling errors and also ensures sufficient robustness margins for rigid body. This paper presents a Classical Adaptive Augmentation Control (AAC) Algorithm for forward loop gain augmentation in real time, to cater to large dispersion in vehicle parameters beyond the capability of classical control system. The idea is to provide augmentation to a classical control designed autopilot when performance enhancement is required to tackle off-nominal conditions arising out of modeling errors and large dispersion in estimated vehicle parameters (thrust, inertia, slosh, aerodynamics, lateral bending modes). There is high chance that such large dispersion can arise during initial design of a new generation launch vehicle before actual flight. Finally, simulation results for several credible launch vehicle failure scenarios show that the adaptive controller consistently and predictably improves performance and robustness, and achieves stability during extreme off-nominal situations.

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nominal and off nominal performances within bounds and at
the same time ensure best results for unpredicted severe
dispersions thus avoiding vehicle failure possibilities. By
incorporating the AAC technique, the practice of assessing
flight control stability using classical gain and phase margins
is not affected under justifiable assumptions.

The paper is organized in 6 sections. After a brief
introduction of the topic in section I, section II describes
the mathematical model of launch vehicle in pitch plane. Classical
controller design is explained in section III. Section IV deals
with the classical adaptive augmentation (AAC) scheme and
its validation is presented in section V. Finally, the paper is
concluded in section VI.

II. LAUNCH VEHICLE MODELING

A. Equations of short period dynamics

The primary objective of Launch Vehicle Attitude
Control System is to orient the vehicle along the required
trajectory in the presence of external disturbances. First step
towards this is to model the vehicle attitude dynamics taking
into account aerodynamics, control actuator dynamics, vehicle
bending, propellant sloshing, variation in center of gravity (cg)
and moment of inertia etc. as the time progresses. This leads
to a time variant system. Using time slice approach a short
period model is evolved [2] which can be assumed time
invariant for a small duration, so that linear time invariant
control system principles can be used. Further it is assumed to
be decoupled in pitch/yaw/roll and planar analysis is carried out.
Referring to [2],[3], the equations of short period
dynamics can be represented as

1) Rigid body

Consider the geometry of vehicle in pitch plane
represented in Fig 1.

\[
\frac{\ddot{\theta}}{U_0} = \frac{\sum q_{z_i}}{m_0 U_0} + \dot{\theta} \tag{1}
\]

\[
I_{yy} \ddot{\theta} = \sum M_y \tag{2}
\]

Considering the forces and moments acting on the launch
vehicle due to engine inertia, aerodynamics, elasticity, slosh
and actuator effects, effective force and moment equations
may be represented as

\[
\sum F_z = T_\varepsilon \ddot{\varphi} - T_f \ddot{\varphi} - \sum \sigma (\dot{i}^z)(\dot{j}_z) - \int \frac{1}{2} \rho \mu_0 \int A (c_n \sigma^l) + \sum m W_i \ddot{q}_i \tag{3}
\]

\[
\sum M_y = \int \rho (r \ddot{\varphi} - \sum \psi (\dot{i}_z)(\dot{j}_z) = -T_f \ddot{\varphi} - \sum \sigma (\dot{i}^z)(\dot{j}_z) + \int \frac{1}{2} \rho \mu_0 \int A (c_n \sigma^l) + \sum m W_i \ddot{q}_i \tag{4}
\]

2) Slosh mode

Fuel-slosh can be a severe issue in space vehicle stability
and control. Dynamic effects of a sloshing liquid can be
nearly approximated by replacing the liquid mass with a rigid
mass and a harmonic oscillator like pendulum. Using the
pendulum parameters like mass, length, hinge point location
etc., the equation of motion of the \(i^{th}\) pendulum can be
represented as

\[
\ddot{r}_p + \ddot{w}_p \ddot{r}_p + \frac{1}{L_p} \sum \psi^{(i)}(t) \frac{\psi^{(i)}(t)}{2} \tag{5}
\]

3) Bending mode

The elastic deflection at any point along the vehicle
[3] is given by

\[
\ddot{\xi}(t) = \sum \psi^{(i)}(t) \frac{\psi^{(i)}(t)}{2} \tag{6}
\]

where \(\psi^{(i)}(t)\) denotes the normalized mode shape of the \(i^{th}\)
mode in the pitch plane. \(\dot{q}^{(i)}(t)\) is the generalized co-ordinate
due to elasticity for the \(i^{th}\) mode in the pitch plane. It satisfies the
equation

\[
\ddot{q}(t) + 2\zeta_1 w_1 \dot{q}(t) + Q^{(i)} = \frac{M^{(i)}}{M} \tag{7}
\]

where \(Q^{(i)}\) and \(M^{(i)}\) are the generalized force and mass
respectively and are given by

\[
Q^{(i)} = \int f_p (t, \psi^{(i)}(t), t) \psi^{(i)}(t) dt \tag{8}
\]

\[
M^{(i)} = \int m(t) \psi^{(i)}(t)^2 dt \tag{9}
\]

4) Actuator

The second order actuator dynamics may be represented as

\[
\ddot{\delta}^A = -w_A^2 \delta^A_A + w_A^3 \delta_C - 2\zeta_A \dot{w}_A \delta_A \tag{10}
\]

5) Nozzle

The second order nozzle dynamics may be represented as

Fig. 1. Geometry of vehicle in pitch plane
\[
\delta_N = \frac{[I + mL \delta_N]}{I_r} - \frac{1}{I_r} \cdot \frac{\dot{\delta}_N}{\delta_N} + w_N^2(\delta_A - \delta_N) - 2\zeta_N w_N \dot{\delta}_N
\] (11)

**B. State Space Representation**

The complete plant dynamics represented by equations (3) to (11) may be expressed in the state space form given by

\[
K\dot{x} = Ax + Bu
\] (12)

\[
\dot{x} = K^{-1}Ax + K^{-1}Bu
\] (13)

\[
y =Cx + Du
\]

where 15 states are chosen considering rigid body mode, 3 bending modes, 1 slosh mode, actuator and nozzle dynamics given by

\[x=[\theta; \dot{\theta}; \dot{\theta}; q^1; q^2; q^3; \dot{q}^1; \dot{q}^2; \dot{q}^3; \dot{x}; \dot{\delta}_N; \dot{\delta}_N; \dot{\delta}_A; \dot{\delta}_A]^T\] (14)

The control input is the deflection angle:

\[u = [\delta_c]^T\] (15)

The output:

\[y = \begin{bmatrix} \theta & \dot{\theta} & \frac{\dot{\theta}}{ag} & \frac{\dot{\theta}}{rg} \end{bmatrix}^T\] (16)

where \(\theta\) is the pitch angle, \(\dot{\theta}\) and \(\frac{\dot{\theta}}{rg}\) represent the pitch rate sensed by angle and rate gyro respectively.

The launch vehicle model used for analysis consists of two large solid boosters strapped into a liquid core. Initial analysis is done at the atmospheric flight stage at a particular instant where aerodynamic forces are significant. Using the vehicle parameters and by considering the effect of two nozzles at the solid boost phase, the governing equations are modified and rearranged to get required state space matrices.

**C. Simplified System Modeling**

For the purpose of investigating the general features of a highly simplified version of the control system, equations (3)-(11) may be reduced to any simplified degree. Ignoring the effects of bending, sloshing and effects due to actuator dynamics, the plant dynamics may be expressed in the transfer function form as

\[\frac{\theta(s)}{\delta_c(s)} = \frac{\mu_c}{s^2 - \mu_c\alpha}\] (17)

where the control moment coefficient, \(\mu_c = \frac{T_{\alpha}l_c}{I_{yy}}\) (18)

the aerodynamic moment coefficient, \(\mu_\alpha = \frac{T_{\alpha}l_\alpha}{I_{yy}}\) (19)

**III. CLASSICAL CONTROL DESIGN METHOD**

Autopilot design for a launch vehicle is carried out in a conventional way using the classical control techniques [4]. This is well established method when performance criteria for control system are expressed in terms of undamped natural frequency, damping factor, steady state errors gain margins, phase margins, etc. The method of pole placement is used for control system design of a conventional launch vehicle. Here the design/response specifications can be transformed into desired locations of dominant closed loop poles. Using the model developed, the control system gains are selected so as to place the closed loop poles in the above locations. The gains are obtained as function of vehicle parameters and closed loop poles.

Control system design is carried out in two phases, first for a simplified model without slosh and flexibility. Gains are selected for good tracking, rapid response and good damping ensuring the system stability as the time progresses. In the next step a suitable compensator is designed to stabilize the bending modes and sloshing modes as well as improving the rigid body margins in presence of higher order dynamics.

**A. Simplified Autopilot Architecture**

1) **Block diagram of simplified Autopilot**

The block diagram of launch vehicle rigid body model with a simplified autopilot is shown in fig.2. \(K_A\) is the forward loop gain and \(K_R\), the rate gyro gain.

![Block diagram of Simplified Autopilot](Image)

The closed loop transfer function of the simplified rigid body model is given by

\[\frac{\theta(s)}{\delta_c(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{K_A}{s^2 + K_AK_R + (K_A - \mu_c)}\] (20)

2) **Design of simplified Autopilot**

In the transfer function given by equation (20), the values of \(K_A\), the forward loop gain and \(K_R\), the rate gyro gain at a particular time instant have to be determined. The characteristic equation of the simple rigid body model with simplified autopilot is given by

\[s^2 + K_A K_R + (K_A - \mu_c) = 0\] (21)

Equation (21) can be compared with the characteristic equation of a typical second order system

\[s^2 + 2\zeta \omega_n s + \omega_n^2 = 0\] (22)

where \(\omega_n\) and \(\zeta\) represent the undamped natural frequency and damping factor respectively.

Comparing (21) & (22), we get
For a desirable rigid body natural frequency $\omega_n = 3 \text{ rad/sec}$ and damping factor, $\zeta = 0.75$, and substituting the same in (23) & (24) gives

$$K_A = 2 \quad \text{and} \quad K_R = 0.44$$

An increase in value of $K_A$ provides best performance and decrease the steady state error associated, but it affects the stability features. Hence Proportional Integral Control strategy may be introduced to maintain required performance capability without losing stability.

### B. PI Controller Architecture

![Fig.3. Classical Controller Architecture](image)

The block diagram/architecture of PI controlled system is shown in fig. 3. Design of classical controllers like P/PI/PID controllers can be done using the classical root locus technique based on time domain approach where a controller can be designed in cascade with the system to have a pair of dominant closed loop poles which satisfy specified time domain specifications $\omega_n$ and $\zeta$. Here, without changing the position of already placed dominant poles significantly, damping factor is slightly changed so as to eliminate steady state error.

If $G(s_d = D\angle \beta)$ is the open loop transfer function of the system and $G_c(s_d = D\angle \beta)$ is the transfer function of PID controller with respect to the dominant pair pole $(s_d)$ location, satisfying the magnitude condition for dominant pole pair to be on the root locus, the proportional, integral and derivative gains of a PID controller in general can be derived as

$$K_p = \frac{-\sin(\beta + \phi_d)}{A_d \sin \beta} - \frac{2K_i \cos \beta}{D}$$

where $K_i$ is determined such that specified error constant is met. For designing PI controller, the value of $K_d$ is assumed to be zero in (27). Hence the design equations are

$$K_i = \frac{-D \sin \phi_d}{A_d \sin \beta}; \quad K_p = \frac{-\sin(\beta + \phi_d)}{A_d \sin \beta} - \frac{2K_i \cos \beta}{D}$$

The dominant pole for $\zeta = 0.75$ and $\omega_n = 3 \text{ rad/s}$,

$$s_d = D\angle \beta = 2.99\angle 138.6^\circ$$

Referring to Fig. 2.,

$$G(s)H(s) = K_A \mu_c (1 + K_R s)$$

With respect to the dominant pole location,

$$G(s)H(s) s_d = A_d \angle \phi_d = 1.043 \angle 184.03^\circ$$

Integral constant, $K_i$, and proportional constant $K_p$ are obtained from equation (29) as

$$K_i = 0.3 \quad K_p \equiv 1$$

The controller transfer function becomes

$$G_c(s) = K_p + \frac{K_i}{s} = (1 + \frac{0.3}{s})$$

The designed values are applied on simplified system model as well as the complete plant model with a suitable compensator designed (a lag filter) for phase stabilization and tuned until a stable system with satisfactory performance is achieved.

### IV. CLASSICAL ADAPTIVE AUGMENTATION CONTROL ALGORITHM DESIGN

#### A. Control Architecture

![Fig.4. Classical Adaptive Augmentation Control](image)

Fig. 4 represent the block diagram of augmented Adaptive Control. By working as an augmenting controller rather than the primary method of accommodating the changing flight scenarios, the design preserves the strength of classical controller during nominal situations.
B. Zones Of Adaptation

The three main zones where adaptation is expected to work are the following:

- Respond to tracking error
- Respond to undesirable control-structure interaction
- Return to baseline control design during nominal cases

C. Adaptation Law [14]

A multiplicative first-order adaptation law is used,

\[
k_a = \left( \frac{k_{max} - k_a}{k_{max}} \right) a e_r^2 - b k_a^2 y_{sd} - c (k_{FG} - 1)
\]

where \( e_r \) is the model error, \( y_{sd} \) is the damper signal and \( a \), \( b \), \( c \) represents the error gain, damper gain and nominal gain respectively. The error term responds to tracking error, the damper term during control-structure interaction and the nominal term for automatic re-convergence to PI controller, when adaptation is not needed. An upper and lower bound to adaptation is provided given by \( k_{max} \) and \( k_0 \). For at least 6 dB robustness gain margin which corresponds to a magnitude of 2, \( k_{max} \) is chosen as 1.5 and \( k_0 \) is chosen as 0.5.

The output of the adaptive law is the adaptive gain \( k_a \), which is used to adjust the output of the PI controller. The adaptive gain \( k_a \) is used to calculate a total forward loop gain given by

\[
k_{FG} = k_0 + k_a
\]

1) Computation of error term, \( e_r \)

The error term is the part which increases the adaptive gain, when needed. A reference model is designed to attain the desired closed loop performance, similar to second order rigid body system that tracks the guidance commands.

\[
\begin{bmatrix}
\dot{\theta}_r \\
\dot{\theta}_c \\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\left( \frac{\omega_c}{\omega_r} \right)^2 & -\frac{2 \xi \omega_r}{\omega_r} \\
-\frac{\omega_r^2}{\omega_c} & \frac{\omega_r \omega_c}{\omega_c} \\
\end{bmatrix}
\begin{bmatrix}
\theta_r \\
\theta_c \\
\end{bmatrix} + \frac{\omega_r}{\omega_c} \theta_c
\]

where \( d \) is the error mixing constant

2) Computation of damper signal, \( y_{sd} \)

Damper signal is a rectified signal detecting and passing undesirable high-frequency dynamics in the loop. Decay of adaptive gain is proportional to magnitude of signal.

\[
y_{HP} = H_{HP}(s) U_G
\]

\[
y_{sd} = H_{LP}(s) (y_{HP})^2
\]

\( y_{HP} \) and \( y_{sd} \) are outputs of a High pass filter and a Low pass filter being used, with transfer functions \( H_{HP} \) and \( H_{LP} \) respectively.

a) High pass filter

A high pass Chebyschev filter is used with a cut off frequency approximately twice of that of the rigid body frequency.

b) Low pass filter

A maximally flat Butterworth filter of cut off frequency approximately nearing the rigid body control frequency is chosen here. The frequency responses of High pass and Low pass filter chosen are shown in fig 5.

![Frequency response of High Pass & Low Pass Filter](image)

Fig.5. Frequency response of High Pass & Low Pass Filter

V. VALIDATION OF ADAPTIVE AUGMENTATION CONTROL DESIGN

In order to validate the proposed control scheme, simulations of attitude control of launch vehicle are presented. The controller is tested on the typical discretized launch vehicle model to check whether the design criteria are met.

A. Adaptive Controller Parameters

The tuned adaptation gains are given in table 1.

<table>
<thead>
<tr>
<th>Gains</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error gain</td>
<td>a</td>
<td>0.001</td>
</tr>
<tr>
<td>Damper gain</td>
<td>b</td>
<td>100</td>
</tr>
<tr>
<td>Nominal gain</td>
<td>c</td>
<td>0.2</td>
</tr>
<tr>
<td>Error mixing constant</td>
<td>d</td>
<td>25</td>
</tr>
<tr>
<td>Maximum adaptation gain</td>
<td>k_{max}</td>
<td>1.5</td>
</tr>
<tr>
<td>Minimum adaptation gain</td>
<td>k_0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

B. Test Cases

The four credible test failure scenarios selected are:

1. Minimal adaptation during nominal plant situation
2. Low thrust/high inertia (dispersed by 20%)
3. First bending mode frequency decreased by 30%
4. High thrust/Low Inertia

C. Short Period Simulations

The above mentioned test cases are tested on the modeled linear time invariant plant on short period basis initially, subjected to a steering step command, \( \theta_c \).
plant with time varying parameters. The plant parameters, the steering command signal and the reference model vary at each instant. Fig.11 shows the variation in scheduled gains $K_A$ & $K_R$.

**Fig.11: Variation of scheduled gains, $K_A$ & $K_R$**

**Fig.12: Case 1: Attitude Plot**

**Fig.13: Case 3: Attitude Plot**

**Fig.14: Case 4: Attitude Plot**

**D. Planar Simulations**

A long period planar simulation is carried out using vehicle data from 40 to 90 seconds with proper gain scheduling and performance of adaptation is evaluated. Here, the steering command generated by guidance is applied to the
VI. CONCLUSIONS

An adaptive augmenting strategy has been incorporated into the designed pitch axis dynamics of a typical launch vehicle to back support the classical controller (PI) designed, so as to improve performance and handle extreme off-nominal situations. Several credible test cases were selected for validating the performance supremacy of proposed controller. Simulation results show that adaptive augmentation provided sufficient performance improvement, and avoided loss of vehicle for extreme off-nominal cases. By acting as augmenting controller, classical control is maintained as the primary controller thereby preserving the strength and legacy of classical control for nominal as well as bounded dispersion cases. The method is proved to be suitable for ensuring safety of new generation launch vehicles designed for advanced missions during their initial flight testing phase. As forward work, a strict assessment of proposed scheme in a full six-degree of freedom nonlinear environment may be done to validate its use in a relevant flight environment.

REFERENCES


APPENDIX

Notations used

$A$ = reference area; $D$ = drag
$C_n$ = normal force coefficient
\[ \sum F_z = \text{total force acting parallel to vehicle body axis, } Z \]

\[ l_{yy} = \text{moment of inertia of reduced about pitch axis} \]

\[ l_0 = \text{moment of inertia of rocket engine about its c.g.} \]

\[ l_r = \text{moment of inertia of rocket engine about swivel point} \]

\[ K_A = \text{servo amplifier gain} \]

\[ K_I = \text{integrator gain} \]

\[ K_R = \text{rate gyro gain} \]

\[ \ddot{z} = \text{perturbation velocity of vehicle parallel to } Z \text{ axis} \]

\[ \dddot{\alpha} = \text{perturbation angle of attack} \]

\[ \gamma = \text{flight path angle} \]

\[ \tau_{pi} = \text{pendulum angle} \]

\[ \delta = \text{rocket engine deflection angle} \]

\[ \delta_c = \text{command signal to rocket engine} \]

\[ \delta_N = \text{nozzle deflection angle} \]

\[ \delta_A = \text{actuator deflection angle} \]

\[ \zeta_a, \zeta_N = \text{relative damping factor for actuator, nozzle} \]

\[ \zeta(i) = \text{relative damping ratio for } i^{th} \text{ bending mode} \]

\[ \theta_c = \text{attitude command signal} \]

\[ \hat{\theta} = \text{attitude angle} \]

\[ \omega_A, \omega_N = \text{undamped natural frequency for actuator, nozzle} \]

\[ \omega_i, \omega_{pi} = \text{frequency of the } i^{th} \text{ bending mode, pendulum} \]

\[ m = \text{mass of rocket engine} \]

\[ m_0 = \text{reduced mass of vehicle} \]

\[ m_{pi} = \text{mass of } i^{th} \text{ pendulum} \]

\[ m_T = \text{total mass of vehicle} \]

\[ m_R = \text{mass of vehicle} \]

\[ M(i) = \text{generalized mass of } i^{th} \text{ bending mode} \]

\[ \sum M_X = \text{total perturbation moment about pitch axis} \]

\[ q(i) = \text{generalized coordinate of } i^{th} \text{ bending mode} \]

\[ Q(i) = \text{generalized force (moment) of } i^{th} \text{ bending mode} \]

\[ T_c = \text{control thrust} \]

\[ T_t = \text{ungimballed thrust} \]

\[ T_r = \text{total thrust} \]

\[ V = U_0 = \text{forward velocity of vehicle} \]

\[ \dot{U}_0 = \text{acceleration} \]