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Assessment of Power Coefficient of an Offline Wind Turbine Generator System

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Abstract

In this paper, we present the design of an estimator for the assessment of the power coefficient of an offline wind turbine in a variable wind turbine generator system (WTGS) using a direct drive permanent magnet synchronous generator. It is shown that the estimator is capable of supplying accurate estimates of the power coefficient and that it can provides satisfactory convergence qualities. A further advantage of the estimator design presented is that it can be easily connected to WTGS where different types of generators and turbines are employed. The simulation results are presented using graphic user interface (GUI) and MATLAB Simulink.

Keywords- Power Coefficient (C_p), Wind Turbine Generator System, MATLAB Simulink, Permanent Magnet Synchronous Generator (PMSG)

1. Introduction

Use of cost effective and reliable low carbon electricity generation source is becoming an important objective of energy policy in many countries (1). Over the past few years, wind energy shown the fastest growth rate as compared to any other form of electricity generation.

The power coefficient (C_p) is defined as the ratio of the power extracted by the wind turbine relative to the energy available in the wind stream.

$$Cp = \frac{P_t}{P} = \frac{P_t}{\frac{1}{2}\rho \pi R^2 V^3} \qquad (1)$$

The Betz coefficient suggests that a wind turbine system can extract maximum 59.3 percent of the energy in an undisturbed wind stream. Due to the losses attributed to different configurations of rotor blades profiles (blade surface roughness), finite wings, friction, and turbine designs (mechanical imperfections) in wind turbine generator system (WTGS). Due to this only 35 to 40% of the power available in the wind is extractable under practical conditions (2). The Betz Limit is an idealization and a design goal that designers try to reach in a real world turbine. A C_p value of between 0.35 - 0.42 is a realistic design and goal for a workable wind turbine.

By considering the maximum rotor blade pitch angle (equal to zero) the plot of power coefficient (C_p) against tip speed ratio (TSR) (optimum) is shown in figure 1 for two blade system. Study of this curve is very important to maximise the generated electric power. The precise curve is difficult to assess as it depends on air density, temperature, humidity, wind speed and tip speed etc. The difference between the actual TSR (blue curve) and the line defined by a constant TSR is needed to be lowest. This difference represents the power in the wind that is not captured by the wind turbine. Frictional losses, finite wing size, and turbine design due to losses in WECS.

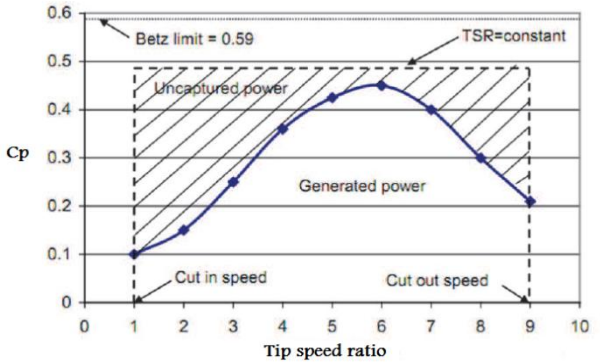


Figure1: C_p variation with TSR for two blade system

In view of the associated losses and inefficiencies, the power coefficient can be expressed as:

$$C_p = C_{pshzmit}\,\eta_{profile}\ \ \eta_{tip\ end}\ \eta_{blades}\,\eta_{friction}\ \eta_{gen}$$

Here $\eta_{profile}$ stand for profile efficiency, Accounting for the drag force which is a function of the slip number 's' and the tip speed ratio 'λ' as:

$$\eta_{profile}\ = \frac{s-\lambda}{s} = 1 - \frac{\lambda}{s}$$

η_{gen} indicate the generator efficiency, η_{blade} indicates the rotor blades efficiency which is depends on number of blades used and their designs.

($C_{p\ Schmitz}$ = In the idealized derivation of the Betz Equation, the wind does not change its direction after the encounter the turbine rotor blades. In fact, it does change its direction after the encounter. This is accounted for by a modified form of the power coefficient known as the Schmitz power coefficient $C_{p\ Schmitz}$ if the same airfoil design is used throughout the rotor blade). For the extraction of maximum power from the rotor, the design of maximum power point controller is necessary which will track the C_p curve for the given rotor structure. So the assessment of power coefficient is very important parameter for the wind generator controller system. Power coefficient (C_p) is a non linear function of mechanical power which depends on the pitch angle and tip speed ratio. By considering the maximum pitch angle, TSR need to be calculated. In this paper we are analyzing C_p value against the variations in TSR. We found that C_p gets maximum value for a particular value of the TSR. For getting C_p, estimator is employed also for designing various control schemes. These estimator based model will have the advantage of being flexible and applicable to different turbines; thus making the controller as independent of the turbine parameters as possible as.

To the best of the authors' knowledge, there is no published work done in this area. From this point of view, the work to be presented has a significant contribution.

I. WTGS MODEL

Block diagram of a WTGS model used in the study is shown in figure 2. The controller is used to assess C_p by which maximum power point tracking point tracking in WTGS is done.

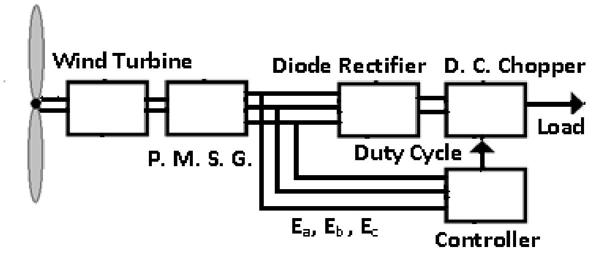


Figure 2: Block Diagram of System Model

II. WIND SPEED MODEL

A model is required that can properly simulate the spatial effects of wind behaviour, including gusting, rapid (ramp) changes, and background noise. A variable wind speed model is obtained by adding repetitive sequence of wind and random number blocks.

$$v_b(t) = rated\ velocity\ of\ turbine$$

$$v_n(t) = noise\ of\ turbine$$

$$v_w(t) = v_b(t) + v_r(t) + v_g(t) + v_n(t) \qquad (1)$$

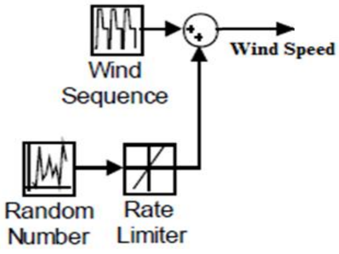


Figure 3: Wind speed model

Rate limiter indicates the rate of change of wind speed. By putting rate limiter in the simulation we are ensuring that wind speed should not be above cut out speed (furling speed) of the turbine and should not be below the cut in speed of the turbine.

III. WIND TURBINE MODEL

The kinetic energy of the wind is given as,

$$KE_w = \frac{1}{2} m\, v^2 \qquad (2)$$

Where v - wind speed, m - air mass = ρ v S Δt

Since S - covered surface of the turbine, Δt - change in time and ρ - air density.

By performing mathematical calculations wind power (P_w) is

$$P_w = \frac{d}{dt} KE_w = \frac{1}{2} \rho\, S\, v^3 \qquad (3)$$

The mechanical power (P_m) extracted by the turbine from the wind is inferior to wind power (P_w). So, the power coefficient (C_p) of the turbine can be defined as

$$C_p = \frac{P_m}{P_w}; \qquad Cp < 1 \qquad (4)$$

Then the mechanical power (P_m) is given by

$$P_m = \frac{1}{2} \rho\, \pi\, R^2\, v^3\, C_p \qquad (5)$$

Where, R is radius of the rotor.

But C_p is a function of the tip speed ratio (λ) of the wind turbine and angle of the blades (β)

$$C_p = (\lambda, \beta) \quad \text{with} \quad \lambda = \frac{\omega\, R}{V} \qquad (6)$$

Where ω - angular rotation speed of the rotor.

$C_p = (\lambda, \beta)$ function is need to have maximum value which is known as Betz limit

$$Cp_{max} = \frac{16}{27} = 0.593 \qquad (7)$$

The wind turbine torque on the shaft can be calculated in terms of mechanical power as:

$$T_m = \frac{P_m}{\omega} = \frac{1}{2}\, \rho\, \pi\, R^2\, \frac{v^3}{\omega}\, C_p \qquad (8)$$

By introducing, $\lambda = \frac{\omega\, R}{V}$

$$T_m = \frac{1}{2}\, \rho\, \pi\, R^3\, \frac{v^2}{\lambda}\, C_p \qquad (9)$$

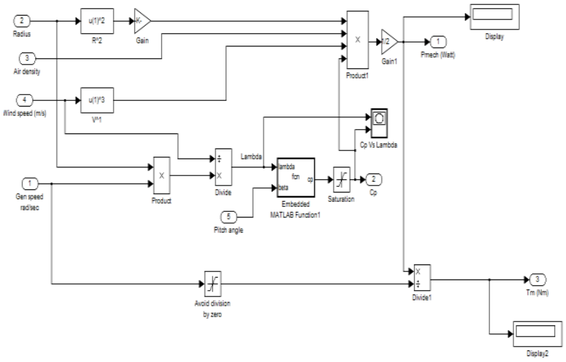


Figure 4: Wind Turbine model

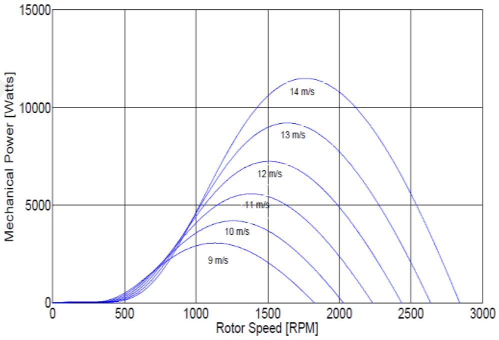


Figure 4: mechanical power output variation with rotor speed characteristics

The power coefficient can be obtained by data fields in the look up tables or by approximating the coefficient using analytical function. The power coefficient analytical function used to model the wind turbine is [4].

$$C_p(\lambda, \theta) = C_1 \left(C_2 \frac{1}{\beta} - C_3\, \theta - C_4\, \theta^x - C_5\right) e^{-C_6 (1/\beta)} \qquad (10)$$

Since C_p function depends on the wind turbine rotor type, the coefficients C_1 to C_6 are $C_1 = 0.5$, $C_2 = 116$, $C_3 = 0.4$, $C_4 = 0$, $C_5 = 5$, $C_6 = 21$ and x is a constant value and can be different for different turbines.

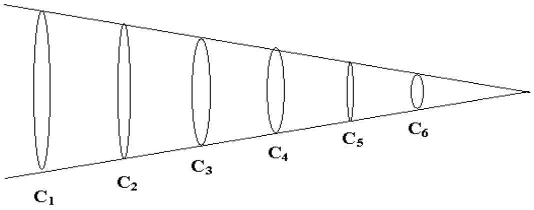


Figure5: Wind Blade Diagram

Additionally, the parameter β is also defined in different ways as [10]

$$\frac{1}{\beta} = \frac{1}{\lambda+0.08\theta} - \frac{0.0035}{1+\theta^3} \qquad (11)$$

Where, θ is pitch angle and $\lambda = \frac{\omega\ R}{V}$

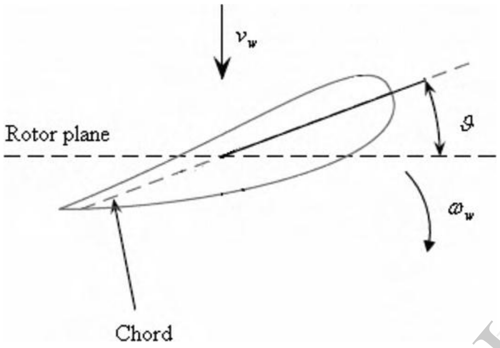


Figure5: Blade Angle, Pitch Angle

IV. Drive Train Model

Figure 6: Spinner

The drive train of a WTGS consists of elements like a blade pitching mechanism with a spinner, a hub with blades, a rotor shaft and a gearbox with breaker and generator. In this paper one mass drive train is considered with multi pole PMSG. At the time of modelling the damping and stiffness coefficients of the shaft are considered. The model of one mass drive train is implemented in MATLAB Simulink as [6]

$$\frac{d\omega}{dt} = \frac{T_m - T_e}{J} - \frac{B\omega}{J} \qquad (12)$$

B - Damping coefficient

ω - Angular velocity of shaft

J - Moment of inertia.

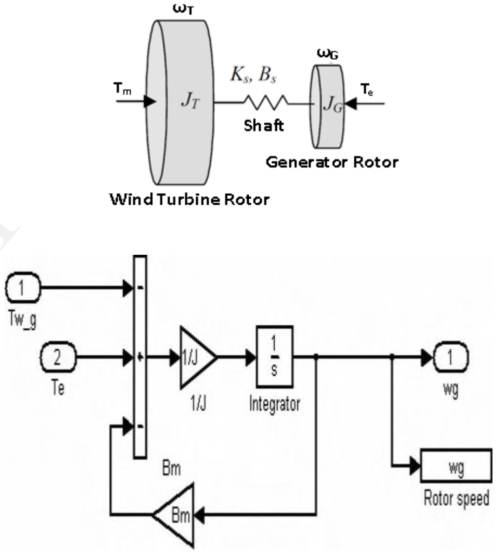


Figure 7: Drive train model

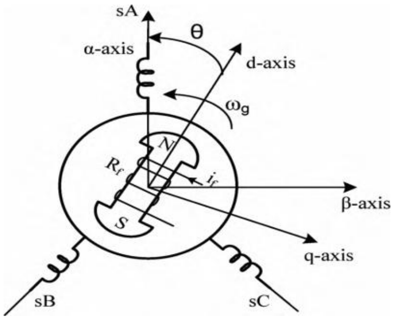
V. PMSG MODEL

Permanent Magnet Synchronous Generator (PMSG) provides an optimal solution for varying-speed wind turbines, of gearless or single stage gear configuration. This eliminates the need for separate base frames, gearboxes, couplings, shaft lines, and pre-assembly of the nacelle. The output of the generator can be fed to the power grid directly so high overall efficiency can be achieved, while keeping the mechanical structure of the turbine simple.

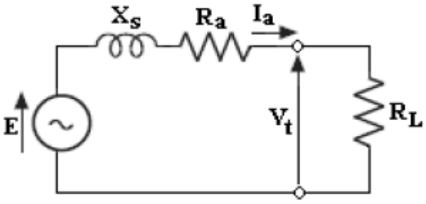
Generated emf per phase,

$$E = v_t + I_a(R_a + jX_s) \qquad (13)$$

The rotor reference frames of the voltages are obtained as,



$$\frac{di_d}{dt} = \frac{1}{L_{ds} + L_{ls}} (-Rsi_d + w_e[L_{ds} + L_{ls}]i_q + v_d) \quad (14)$$



$$\frac{di_q}{dt} = \frac{1}{L_{qs} + L_{ls}} (-Rsi_q - w_e[(L_{ds} + L_{ls})]i_d + \Psi_f] + v_q) \quad (15)$$

The expression for the electromagnetic (EM) torque in the rotor is given by,

$$\tau_e = 1.5p\left((L_{ds} - L_{ls})i_d i_q + i_q \Psi_f\right) \quad (16)$$

Torque developed by the turbine (T_t) and released to the input to the generator (T_m) which is expressed as

$$Tm = \frac{Tt}{G} \quad (17)$$

Where, G – Gear ratio

VI. C_p ESTIMATOR MODEL

Our main objective is to design a power coefficient estimator based upon the WTGS model given below. For this, we take C_p as a state variable instead of a parametric and assume that the power coefficient is unknown and piecewise constant so that on the $\frac{dC_p}{dt} \cong 0$ for time intervals where C_p is constant. As a result, system equations can be augmented by including the dynamics of C_p; yielding a system of order 3 given by:

$$\frac{di_a}{dt} = \frac{\gamma\, KI\, f\, \omega}{L_a} - \frac{V_a}{L_a} - \frac{R_a I_a}{L_a} \quad (18)$$

$$\frac{d\omega}{dt} = \frac{1}{2}\frac{C_p(\lambda)\, \rho\, \pi\, R^2 v^3}{J\omega} - \frac{\gamma\, K_1\, i_f\, i_a}{J} - \frac{Bw}{J} \quad (19)$$

$$\frac{dC_p}{dt} \cong \frac{dC_p}{dw}\,\frac{d\omega}{dt} = 0 \quad (20)$$

The estimation of C_p can be based on these three equations. However, in order to reduce computation complexity we look for a reduced order model from which the estimation of C_p can be made [8]. For this, we shall assume that the values of i_a, ω and u are measured quantities which means that they are accessible outputs of the WTGS. The mechanical power and the wind speed 'u' are inputs to the WTGS. Since i_a, u and ω are measured, they can be injected directly into mechanical equation of the WTGS. As a result, only the mechanical dynamics of the WTGS can be considered for the estimation of C_p. So equations 18, 19 and 20 are reduced as

$$\frac{d\omega}{dt} = \frac{1}{2}\frac{C_p(\lambda)\, \rho\, \pi\, R^2\, v^3}{J\omega} - \frac{\gamma\, K_1\, i_f\, i_a}{J} - \frac{B\omega}{J} \quad (21)$$

$$\frac{dC_p}{dt} \cong \frac{dC_p}{dw}\,\frac{d\omega}{dt} = 0 \quad (22)$$

Turbine output fed to estimator (y) = ω (23)

Where, i_a and u are viewed as inputs to the system and ω as the output. This equation model is called as second order reduced model and can be used for C_p estimation

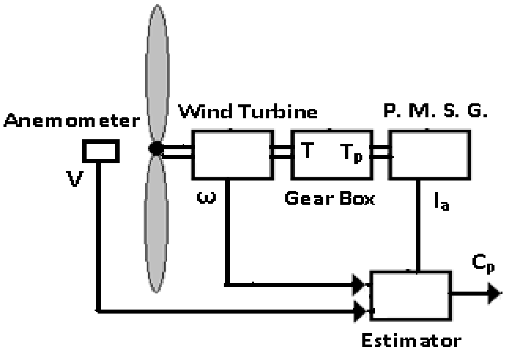


Figure 8: WTGS estimator

VII. C_p for varying wind speed profile

Wind speed profile is measured by anemometer and taken as a constant piecewise function in order to provide more realistic representation of power coefficient of practical system [9]. If we keep rate limiters falling rate between 0 to -1, the values of wind speed fed to turbine are much below the cut in speed of turbine so turbine is not going to operate. For those purpose rate limiters falling rate is kept as a constant and the value is -1.

Wind Speed Raising Rate	C_p	Tip speed Ratio
0	**-0.6958**	**16.75**
0.04	-0.46	15.62
0.08	-0.2674	14.48
0.12	-0.1073	13.5
0.16	0.02336	12.65
0.2	0.129	11.68
0.24	0.2132	11.22
0.28	0.2793	10.62
0.32	0.3301	10.9
0.36	0.3679	9.599
0.4	0.3948	9.158
0.44	0.4127	8.75
0.48	0.423	8.38
0.52	**0.4272**	**8.04**
0.56	0.4263	7.73
0.6	0.4213	7.44
0.64	0.413	7.17
0.68	0.4022	6.92
0.72	0.3894	6.695
0.76	0.3824	6.47
0.8	0.3598	6.27
0.9	0.3191	5.816
1	**0.2861**	**5.495**

VIII. Results And Discussion

In this section, we evaluated the performance of C_p estimator by performing simulation using MATLAB Simulink.

The piecewise constant of Cp is observed maximum i.e. 0.4272 at 8.04 tip speed ratio where $\frac{dC_p}{dt} \cong 0$.

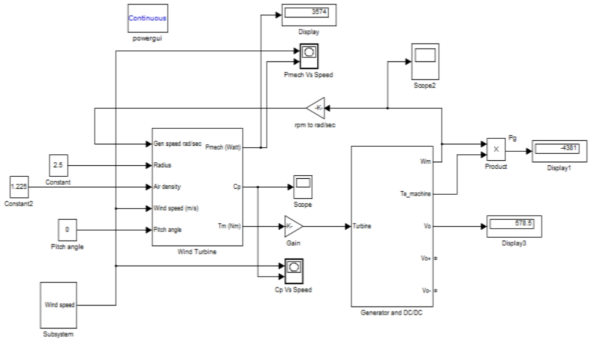


Figure 9: System simulation

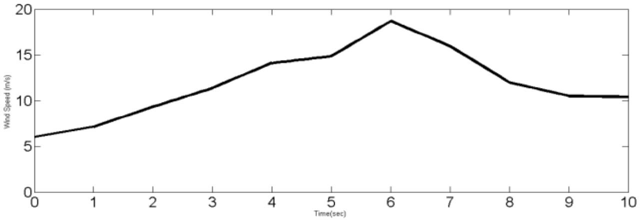


Figure 10: Wind speed profile

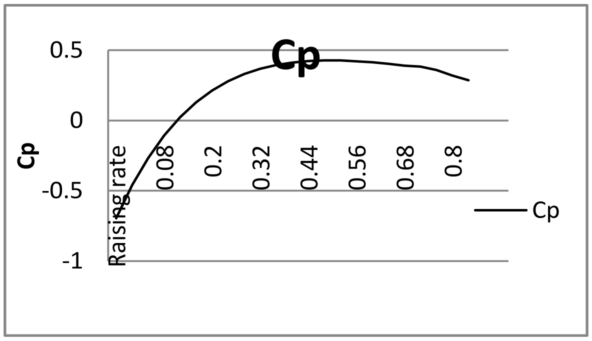


Figure 11: Estimated C_p variation for wind speed profile

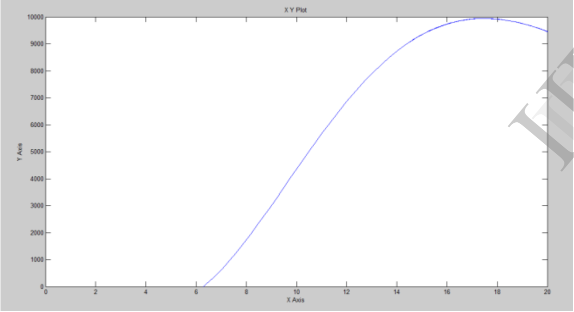


Figure 12: Mechanical power variation for wind speed

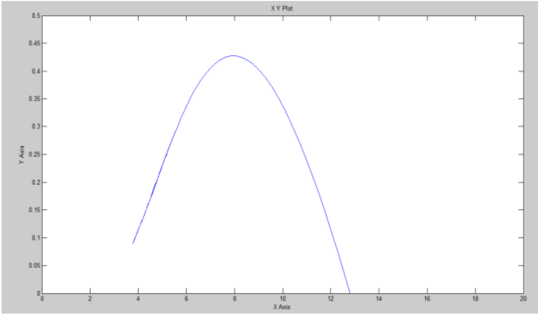


Figure 13: C_p variation for Lambda before implemented

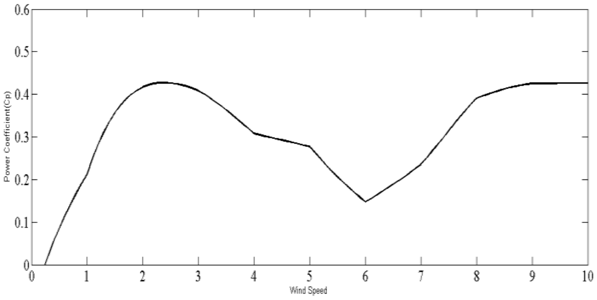


Figure 14: Estimated performance coefficient for given wind speed

For a 20 kW wind turbine, the estimators provide C_p values with good convergence properties and faster response. Fig. 8 shows the power coefficient for various sampled values of wind speed.

It is important to realise that the wind speed (u) is measured by the anemometer at a particular point where the area is swept by the rotor. This is not an ideal representation as it assumes that the wind speed is uniform over that area and does not take into consideration spatial fluctuations which can be eliminated using spatial filtering. Future work would be to consider the case when the power coefficient is not assumed to be a piecewise constant function in order to provide a more realistic representation of the practical system. However, current suitable anemometer technology has a measurement frequency of around 1Hz; subsequently the assumption that C_p is piecewise constant is not too restrictive.

IX. CONCLUSION

In this paper, the C_p estimator is presented for calculation of power coefficient in a WTGS. The results obtained have verified the estimator’s capability of giving good estimates of the power coefficients. This C_p estimator has many important advantages one of them is it allows fewer turbine parameter dependence for associated control system and it can easily be extended to any other WTGS where different generators and / or turbine types are used.

X. ACKNOWLEDGMENT

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Appendix:

Parameters of wind turbine:

Nominal power = P_{mech} = 13 kW

Cut in speed =3 m/s

Rated speed =9 m/s

Rotor radius =2.3 m

Pitch angle = $\beta = 0°$

20 kW WTGS's machine parameters:

Stator resistance, Rs=1.5ohms,

Stator inductances, $L_d = L_q = 0.01$ mH,

Flux induced by magnets, $\Psi = 0.1194$ Wb,

Moment of inertia J = 2 Kg m and

Number of poles p= 4