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Architecture Style Developing through Application of Mathematics:
Concepts of Geometry &Proportion in Architecture

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Abstract

Architecture has its unique relationship with mathematics, incorporating the study of such mathematical concepts as ratio, proportion, scales and symmetry. Put up definitions and explanations of the mathematical concepts of elementary geometry, stating their connection to architecture and ratio and proportion relate to architectural plan with mathematical accuracy in measuring. In this paper showing the connections between geometry and architecture with what appears to be an obvious example from various styles, architectural works which are also derived from basic geometric figures.The aim is to re-search the age old geometrical principles applied in Indian architecture. Deriving ancient principles of inter-relationship between 'Geometry & Architecture' in three major branches of Indian architecture, particularly, Hindu Architecture and Islamic Architecture. Historically, architecture was part of mathematics, and in many periods of the past, the two disciplines were indistinguishable. In the ancient world, mathematicians were architects, whose constructions - The tombs, mosques, temples, pyramids and ziggurats. Geometry was the study of shapes and shapes were determined by numbers.Here geometry becomes the guiding principle. Geometric principles such as those used in triangles (the ratio between base and height, how they are related to the area of the triangle) have been used in many ancient architectural constructions.

Key Words:

Mathematics in architecture, Geometry& proportion, unique relation, Golden proportion and geometric principles.

Summary:

Mathematics and architecture have always enjoyed a close association with each other, not only in the sense that the latter is informed by the former, but also in that both share the search for order and beauty. It is also employed as visual ordering element or as a means to achieve harmony with the universe. Here geometry becomes the guiding principle. Many ancient architectural achievements continue to strike any keen observer with both their grandeur and structural stability. Such structural stability had resulted due to following the principles of mathematics to obtain equilibrium and aesthetics in a balanced proportion. The Great Wall of China, the pyramids of Egypt, The Parthenon, The Colosseum and the TajMahal are all examples of the achievements of ancient architecture. In all these architectural achievements, many fundamental principles of maths have been used.

1. Theory and Principles

a) Golden Mean Ratio
b) Pizza-cutter Theory
c) Egyptian Triangle
d) Greek Geometry & Proportions
e) Cardinal Theory.

1.1 Golden Mean Ratio and Architecture

The golden ratio is also called extreme and mean ratio. According to Euclid, A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less.

Φ = 1/2 + 5 / 2 =**1.618**

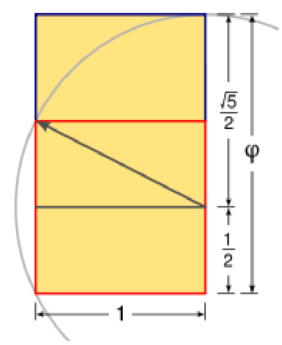
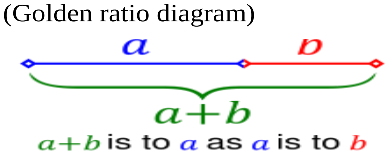


Fig: 1 (Golden ratio diagram)



Derivation of golden rectangle
Step- 1 Construct a unit square.
Step-2 Draw a line from the midpoint of one side to an opposite corner.
Step-3 Use that line as the radius to draw an arc that defines the long Dimension of the rectangle.

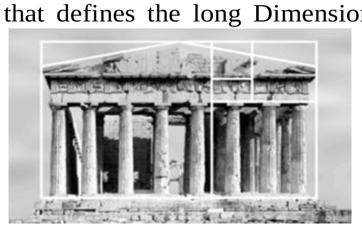


Fig:2 (Parthenon faced proportion ratio)

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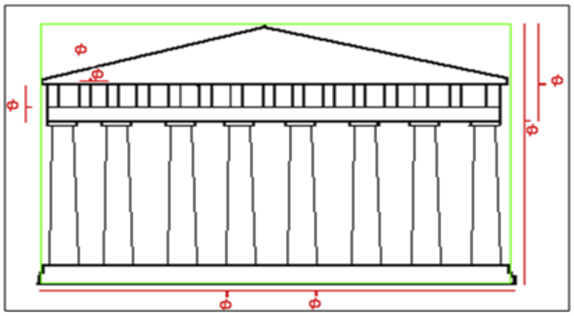


Fig:3 (Parthenon)

Some studies of the Acropolis, including the Parthenon, conclude that many of its proportions approximate the golden ratio. The Parthenon's facade as well as elements of its facade and elsewhere can be circumscribed by golden rectangles.

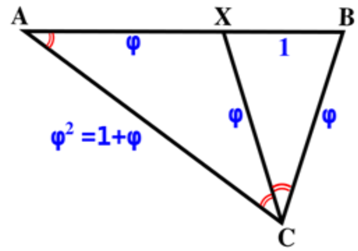
1.2 Pizza-cutter Theory

If angle BCX = α, then XCA = α because of the bisection, and CAB = α because of the similar triangles; ABC = 2α from the original isosceles symmetry, and BXC = 2α by similarity. The angles in a triangle add up to 180°, so 5α = 180, giving α = 36°. So the angles of the golden triangle are thus 36°-72°-72°. The angles of the remaining obtuse isosceles triangle AXC (sometimes called the golden gnomon) are 36°-36°-108.

Fig: 4 (Pizza cut plate)

1.2.1 Derivation of golden triangle

Suppose XB has length 1, and we call BC length φ. Because of the isosceles triangles BC=XC and XC=XA, so these are also length φ. Length AC = AB, therefore equals φ+1. But triangle ABC is similar to triangle CXB, so AC/BC = BC/BX, and so AC also equals φ2. Thus φ2 = φ+1, confirming that φ is indeed the golden ratio.



1.2.2 Derivation of golden pentagon

A pentagram color to distinguish its line segments of different lengths. The four lengths are in golden ratio to one another. The golden ratio plays an important role in regular pentagons and pentagrams. Each intersection of edges sections other edges in the golden ratio. Also, the ratio of the length of the shorter segment to the segment bounded by the 2 intersecting edges (a side of the pentagon in the pentagram's centre) is φ, as the four-color illustration shows.

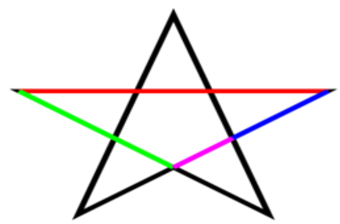


Fig: 5 (Golden pentagon)

1.2.3 Relationship to Fibonacci sequence

It is approximate and true golden spirals. The green spiral is made from quarter-circles tangent to the interior of each square, while the red spiral is a Golden Spiral, a special type of logarithmic spiral. Overlapping portions appear yellow. The length of the side of a larger square to the next smaller square is in the golden ratio.

1.2.4 Golden spiral in nature

Although it is often seen that the golden spiral occurs repeatedly in nature (e.g. the arms of spiral galaxies or sunflower heads), this claim is rarely valid except perhaps in the most contrived of circumstances.

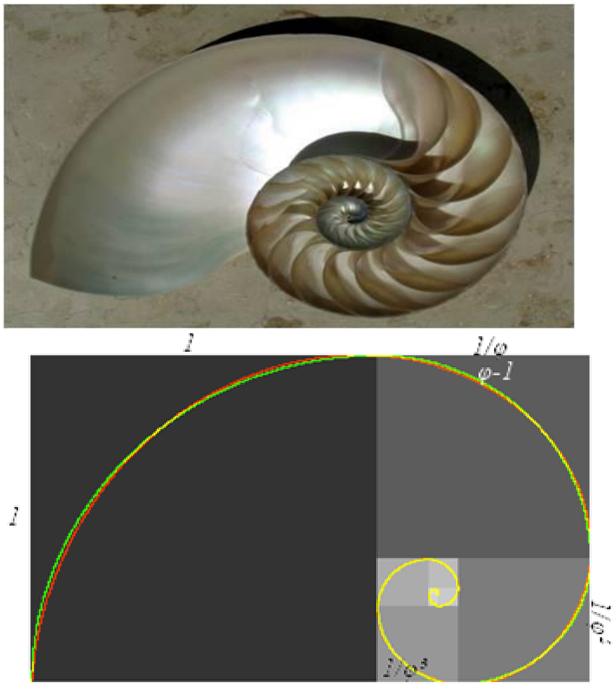


Fig: 6&7 (Golden spiral)

For example, it is commonly believed that nautilus shells get wider in the pattern of a golden spiral, and hence are related to both φ and the Fibonacci series. Nautilus shells exhibit logarithmic spiral growth, but at a rate distinctly different from that of the golden spiral. The reason for this growth pattern is that it allows the organism to grow at a constant rate without having to change shape. Spirals are common features in nature, but there is no evidence that a single number dictates the shape of every one of these spirals.

Fig: 8 (Equation of ratio)

1.3 Egyptian Triangle

This triangle is special because it supposedly contains the golden ratio. In particular, the ratio of the slant height s to half the base b is said to be the golden ratio. To verify this we have to find the slant height.Its height h, by the Pythagorean Theorem, is given by, h2= 2 - 12

Solving for h we get a value of =1.271

1.3.1Computation of Slant Heights

The dimension is to the nearest tenth of a meter, of the Great Pyramid of Cheops, determined by various expeditions. Height = 146.515 m, and base = 230.363 m

Half the base is230.363 ÷ 2 = 115.182 m

So,

S 2 = 146.515 + 115.182 2 = 34,733 m2

S = 18636.9 mm

Does the Great Pyramid contain the Golden Ratio?

Dividing slant height s by half base gives 186.369 ÷ 115.182 = 1.61804

Which differs from (1.61803) by only one unit in the fifth decimal place.

The Egyptian triangle thus has a base of 1 and a hypotenuse equal to. Its height h, by the Pythagorean Theorem, is given by

h2 = φ2 - 12

Solving for h we get a value of √φ.

Project: Compute the value for the height of the Egyptian triangle to verify that it is. Thus the sides of the Egyptian triangle are in the ratio

1: √φ: φ

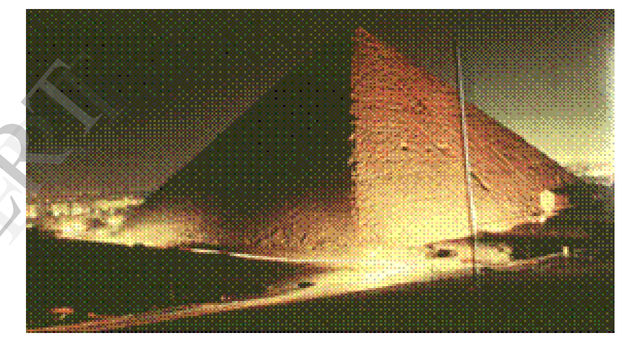


Fig: 9 (Pyramid of Egypt)

1.3.3 Squaring of the Circle in the Great Pyramid

The claim is:

The perimeter of the base of the Great Pyramid equals the circumference of a circle whose radius equal to the height of the pyramid.

Does it? Recall from the last unit that if we let the base of the Great pyramid be 2 units in length, then

Pyramid height = √φ

So:

Perimeter of base = 4 x 2 = 8 units

Then for a circle with radius equal to pyramid height √φ.Circumference of circle = 2 π√φ ~7.992 so the perimeter of the square and the circumference of the circle agree to less than 0.1%.

An Approximate Value for in Terms of π in terms of φ

Since the circumference of the circle (2) nearly equals the

Perimeter of the square (8)
$\pi\sqrt{\varphi} \sim 8$
We can get an approximate value for π,
$\pi \sim 4/\sqrt{\varphi} = 3.1446$
Which agrees with the true value to better than 0.1%.

2.1 Application in Hindu Architecture

The concept of a Hindu temple goes back thousands of years and the building information and the wisdom on which it is based has been orally passed on from generation to generation.

The ancient Marundheeswarar temple in Thiruvanmayur, South Chennai, has a series of pillars with beautiful geometric designs that are quite surprisingly fairly sophisticated mathematical motifs of contemporary scientific interest? The motifs surround the sanctum of the goddess "Tripurasundari", the "belle of the three cities". The number three is crucial in the motifs and the irreducible tripartite nature of the divinity is emphasized through links and knots, which have their usual meanings as well as precise mathematical ones.

The first of the patterns is a set of three identical overlapping equilateral triangles at whose center is a four petalled flower. Unlike the two-dimensional Vantras which typically have several overlapping triangles, this one is sculpted with the third dimension in mind. We can make out when one triangle goes over another. The three triangles overlap in a very specific and remarkable way: no two of the three triangles are linked to each other, but the three are inextricably collectively linked; if any one of the triangles is removed the other two falls apart as well.

2.1 Evaluation of basic geometrical formulas in Indian context-

AryabhataSutrafor'PI'

Mathematics played a vital role in Aryabhata's revolutionary understanding of the solar system. His calculations on pi, the circumference of the earth (62832 miles) and the length of the solar year (within about 13 minutes of the modern calculation)

VaastuShastraprescribes desirable characteristics for sites and buildings based on flow of energy. Many of the rules are attributed to cosmological considerations - the sun's path, the rotation of the earth, magnetic field, etc., the morning sun is considered especially beneficial and purifying and hence the East is a treasured direction. The body is considered a magnet with the head, the heaviest and most important part, being considered the North Pole and the feet the South Pole.

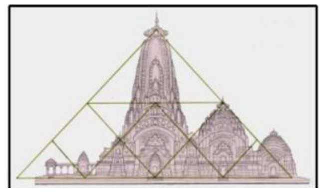


Fig:10 Proportion in Hindu Temple

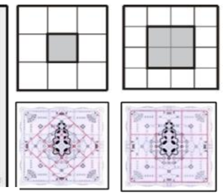


Fig: 11Vaastu Mandala

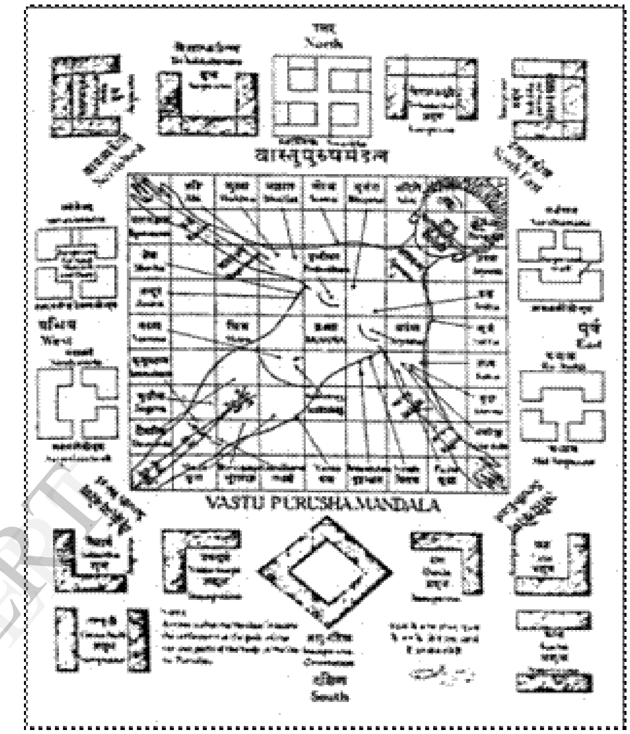


Fig: 12 Vaastumandala

Upapitha(25 squares)
corresponds to Pancha-pada (five divided site)
Ugrapitha(36 squares)
corresponds to Shashtha-pada (six divided site)
Sthandila(49 squares)
corresponds to sapta-pada (seven divided site)
Manduka/ Chandita(64 square)
corresponds to Ashta-pada (eight divided site)
Paramasaayika(81 squares)
corresponds to Nava-pada (nine divided site)
Aasana(100 squares)
corresponds to Dasa-pada (ten divided site)

2.2 Application in Islamic Architecture

Islamic architecture has encompassed a wide range of both secular and religious styles from the foundation of Islam to the present day, influencing the design and construction of buildings and structures within the sphere of Islamic culture. The principle architectural types of Islamic architecture are; the

Mosque, the Tomb, the Palace and the Fort. From these four types, the vocabulary of Islamic architecture is derived and used for buildings of lesser importance such as public baths, fountains and domestic architecture.

2.2.1 Influences and styles

Distinguishing motifs of Islamic architecture have always been ordered repetition, radiating structures, rhythmic and metric patterns. In this respect, fractal geometry has been a key utility, especially for mosques and palaces.

Persian architecture, Moorish architecture
Ottoman Turkish architecture, Fatimid architecture
Mamluk architecture, Indo-Islamic (Mughal) architecture, Sino-Islamic architecture
Afro-Islamic architecture.

The system of measurement based on a constant awareness of the proportions of the human body and on the principles of geometry is the key factors applied in Islamic architecture, in fact in all their creative work. Besides, of less significance in beauty (in aesthetics) in architecture which comes essentially from proportioning and proportioning also results from geometry.

"The square and the circle, and their immediately related shapes are the simplest, most perfect and stable geometrical forms found in nature. These symbolize the perfection of 'God & his Universe'. Therefore in Islamic Architecture, the mathematical system, based on geometry, is established by the application of square and axis, besides the concept of centrality.

The application of square as a generic unit brings the relationship between all parts, from the smallest to the biggest dimension. It regulates not only totality, but also achieves unity in the overall composition. 'In 3-dimensional space, six squares form a cube and become the spaceenclosing elements. A dome resting on the cube is the space covering element.

2.2.2 Mathematical system in the design of FatehpurSikri

The spirit and guiding force of the design of FatehpurSikri was in the tireless efforts of Akbar and his architects to bring to light once more the beauty and purity of Islamic artistic wisdom. The design rationalism was developed on the basis of mathematical system-proportions, rules and measures - in which the use of the axis and the square predominates. It is important to realize presence of the two mosques, the 'SalimChisti's mosque' and the 'Centric Mosque' (a & b respectively in Fig. 18). The first mosque ties constructed by the local stone-cutters around the little cave of SalimChisti, as a grateful gesture to the saint. The second mosque (centeric mosque) was already in existence, not very far away from the days of SalimChisti. This mosque as the focal point of the city (Fig. 19)) together with the SalimChisti's mosque marked the beginning of a rational design approach of FatehpurSikri.

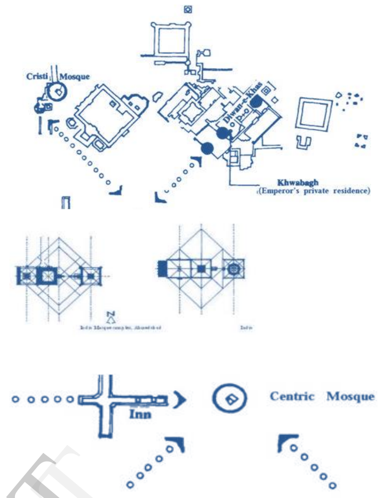


Fig: 14 (Planning pattern in Fatehpur Sikri)

2.2.3 Shahajahanabad (old Delhi)

The planning of Shahajahanabad (old Delhi), built by Akbar's grandson, Shahjahan, about 70 years after the founding of FatehpurSikri, is also based on eight super squares, each comprising nine modular squares . The location of a number of city gates, as in case of FatehpurSikri, is also determined by the super grid.

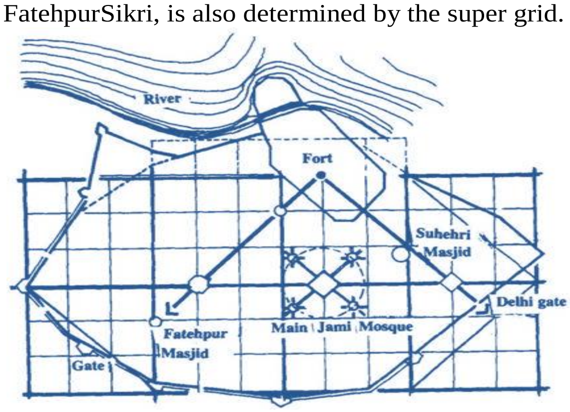


Fig: 15 (Planning pattern in Walled city)

Plan of Shahjabanhad (Old Delhi) based on eight super squares.

Mihrab, 1354; post-Ilkhanid period

The most important interior element in an Islamic religiousbuilding is the mihrab, a wall niche that indicates the directionof Mecca, toward which the faithful must face during the dailyprayers. This mihrab is from the Madrasa Imami, a religious schoolfounded in Isfahan in 1354. It is made of glazed earthenwarecut into small pieces and embedded in plaster. Three kinds ofIslamic designs can be found here —vegetal, calligraphic, andgeometric. The calligraphic inscription in the back of the nichereads: "The Prophet (on him is peace!) Said 'the mosque isthe dwelling place of the pious.'" Calligraphy is the mostrevered art form in Islam because it conveys the word of God.Note the way in which straight-lined geometric shapes havebeen made to fit the curved space. Observe the varied andcomplex decorative elements that cover every visible surfaceof the mihrab. All directly illustrate geometric, calligraphic,or plant forms.

Fig: 16 (Mihrab of Post ilkhanid period)

CONCLUSION

Application of mathematics in architecture from ancient to modern architecture age find new possibilities for contemporary architecture design pattern, form and aesthetics.Mathematics and architecture have always enjoyed a close association with each other, not only in the sense that the latter is informed by the former, but also in that both share the search for order and beauty. It is also employed as visual ordering element or as a means to achieve harmony with the universe. Here geometry becomes the guiding principle.Many ancient architectural achievements continue to strike any keen observer with both their grandeur and structural stability. Such structural stability had resulted due to following the principles of mathematics to obtain equilibrium and aesthetics in a balanced proportionAfter study of geometric pattern, proportion and balance in term of master planning and architecture planning as well as architectural treatment of building blocks, we implicate similar treatment in contemporary architecture as per contemporary need of society .Ancient design and planning based on balance & equality theory ,that reflect in city planning and architectural planning .in present era strongly used of mathematics just reflect in structure & load calculation .in India we have very strong tangible history in architecture ,strong element of architecture and geography ,Indian culture time to time affect architecture .Architects and master craftsmen of that time strongly adopted and recommend mathematical implication in master planning and architecture planning as well as architectural aesthetically treatment in façade and interiors .As an architect and planner we need to be think about adaptive re used mathematical application in contemporary architecture .

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