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Applications of Finite Element Method with Its Usage in Modern Technology

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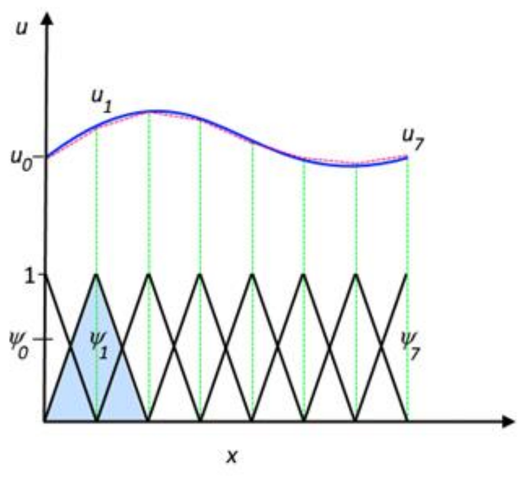
Abstract - The finite elements method (FEM) is a numerical technique for solving problems which are described by partial differential equations and can be formulated as functional minimizations. Here we studied some applications of finite elements methods in various forms along with the study of classical method and numerical method. The finite element analysis is a numerical technique in which all the complexities of the problems, like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. The rapid enhancements in computer hardware technology and lowering of the cost of computers have boosted this method because a computer is the basic need for the application of this method. A number of popular brand of finite element analysis packages are now available commercially. Some of the popular packages are STAAD-PRO, GT-STRUDEL, NASTRAN, NISA and ANSYS. One can analyze several complex structures using these packages.

Key-words: Finite Element Method, Partial Differential Equations, Basis Functions, Mathematical Modeling, Numerical Formulations.

INTRODUCTION

History of the Finite Element Method: The descriptions of the laws of physics for space- and time-dependent problems are usually expressed in terms of partial differential equations (PDEs). For the majority of geometries problems, these PDEs cannot be solved with analytical methods. However, an approximation of the equations can be constructed, typically based upon various types of discretization's. These discretization methods approximate the PDEs with numerical model equations, which can be solved using numerical methods. The solution to the numerical model equations are actually approximations of the real solutions to the PDEs. The finite element method (FEM) is used to compute such approximations.

For example, a function u that may be the dependent variable in a PDE (i.e., temperature, electric potential, pressure, etc. The function u can be approximated by a function u_h using linear combinations of Basis Functions according to the following expressions



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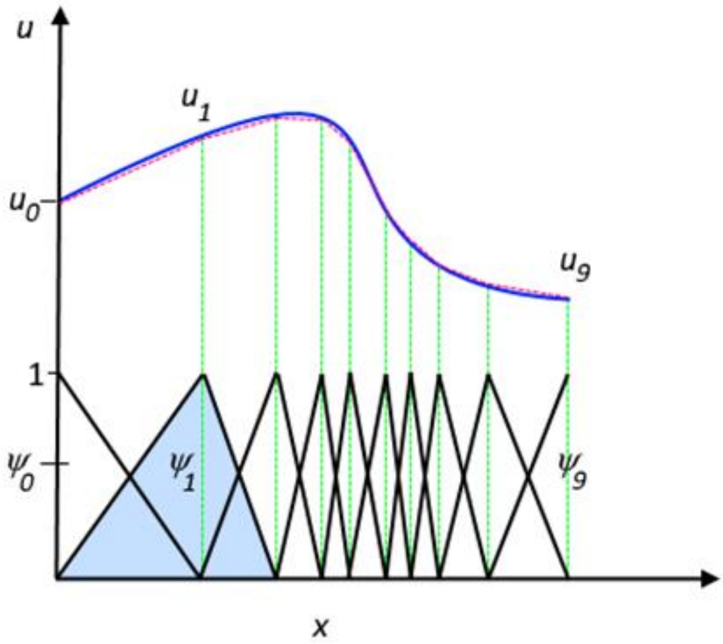
Here, ψ_i denotes the basis function and u_i denotes the coefficients of the functions that approximate u with u_h. The figure below illustrates this principle for a 1D problem. For example u can represent the temperature along the length (x) of a rod that is non-uniformly heated. Here, the linear basis functions have a value of 1 at their respective nodes and 0 at other nodes. In this case, there are seven elements along the portion of the x-axis, where the function u is defined (i.e., the length of the rod). The function u (solid blue line) is approximated with u_h (dashed red line). This is a linear combination of involving linear basis functions (ψ_i is represented by the solid black lines). The coefficients are denoted by u_0 through u_7.

An important benefit to use the finite element method is that we get great freedom in the selection of discretization. This is the case for both the elements that can be used to discretize space and basic functions. In the figure above, for example, the elements are uniformly distributed over the *x*-axis, although this is not necessary. Smaller elements in a region where the gradient of *u* is large can also be applied, as highlighted below.



The function u (solid blue line) is in line with u_h (dashed red line), which is a linear combination of linear basis functions (ψ_i is represented by the solid black lines). The coefficients are denoted by u_0 through u_7.

Both of these figures show that the selected linear basis functions include very limited support (nonzero only over a narrow interval) and overlap along the x-axis. Other functions can also be chosen instead of linear functions depending on the situational problem.

Another important benefit of this method is that the we already have a well-developed theory. This is because of the close relationship between numerical formulation and weak formulation of the PDE problem. For example, the theory provides useful error estimates, or intervals for the error, when the numerical model equations are solved on a computer.

Looking back at the history of FEM, the usefulness of the method was first recognized at the start of the 1940s by Richard Courant, a German-American Mathematician. Actually, Courant highlighted its application to a variety of problems but it took many years before the approach was applied as a general case in fields outside of structural mechanics.

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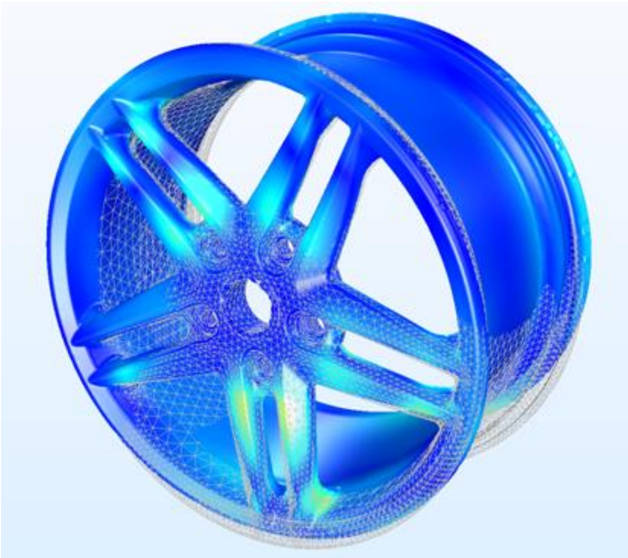

Finite element discretization, stresses, and deformations of a wheel rim in a structural analysis

According to the technical viewpoint, FEM has its historic origins in the Euler's work in the 16th century. But the earliest published mathematical papers on FEM were written by Schellback [1851] and Courant [1943].

Engineers developed FEM independently to deal with structural mechanics problems related to aerospace and civil engineering. The progress began in the mid-1950s with the research papers of Turner, Clough, Martin, and Topp [1956], Argyris [1957], and Babuska and Aziz [1972]. This foundation of FEM was further cemented by the books by Zienkiewicz [1971] and Strang, and Fix [1973]. Oden [1991] gave an interesting review of these historical developments.

The Finite Element Method (FEM) or Finite Element Analysis (FEA) is based on the concept to build a complicated object with simple blocks or to divide a complicated object into small and manageable pieces.

Basic Steps of FEM

1) Discretization of the structure.
2) Identify primary unknown quantity.
3) Selection of displacement function.
4) Formation of the element stiffness matrix and lead vertex.
5) Incorporation of boundary conditions.
6) Solution of Simultaneous Equations
7) Calculation of element strains and stresses.
8) Interpretation of the result obtained.

Methods of Solution

Classical methods

Classical methods show a high degree of knowledge, but the problems are difficult or impossible to solve for anything other than simple geometries and loadings.

Numerical methods

1) Energy: This minimizes the expression for the potential energy of any structure over the whole domain.
2) Boundary element: This approximates functions satisfying the governing differential equations but not the boundary conditions.

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3) Finite difference: This replaces the governing differential equations and boundary conditions using algebraic finite difference equations.
4) Finite element: Approximates the behavior of an irregular, continuous structure under general loadings and constraints with an assembly of discrete elements.

Applications of FEM

FEM is utilized extensively to study and model various applications in different knowledge areas. FEM was originally developed to study the stress in complex aircraft structures. Now they include Computer and Industrial applications, Biomedical, Thermal and Fluid flows, Mechanical engineering disciplines such as aeronautical (design of aircrafts), biomechanical etc. Automotive industries commonly use integrated FEM in design and development of their products. The major achievement of FEM is its applications in any irregular geometry with various boundary conditions. Many engineering phenomena can easily be expressed by governing equations and boundary conditions using FEM. A FE mesh can handle any type of loading conditions such as heat flux, pressure, thermal load, gravity etc, as it can be programmed to define the reaction of structure to specific conditions.

The major benefits of FEM are increased accuracy, enhanced design, a process that is faster and less expensive, higher quality of products, an increased revenue and a reduced chance of field failure. But the successful application of FEM depends on the formulations, appropriate parameters and proper interpretation of the results.

List of some important applications of FEM

1) **Mechanical Engineering:** In mechanical engineering FEM applications include steady and transient thermal analysis in solid and fluids, stress analysis in solids and analysis of manufacturing process simulation.

2) **Geotechnical Engineering:** Some important applications of FEMs are stress analysis, slope stability analysis, interactions of soil structures, fluid-seepage in soils and rocks, dam-analysis, tunnels, bore holes, stress waves propagation and interactions within the dynamic soil structure.

3) **Aerospace Engineering:** FEM is used for several purposes such as structural analysis for natural frequency…modes shapes, response analysis and aerodynamics.

4) **Nuclear Engineering:** FEM application includes steady and dynamic analysis of reactor containment structures, thermo-visco-elastic analysis of reactor components, and steady and transient temperature- distribution analysis of reactor and relates structures.

5) **Electrical and Electronics Engineering:** FEM applications include electrical neutral c analysis, electromagnetic, insulation design analysis in high-voltage equipments, dynamic analysis of motions and heat analysis in electrical and electronic equipments.

6) **Mathematics:** In Mathematics, it is used in Partial Differential Equations. It is important to understand the different genres of PDES and their suitability for use with FEM. Understanding this is particularly important to everyone irrespective of the motivation for using finite element analysis. It is critical to remember that FEM is a tool which is only good for its user. PDES can be categorized as elliptic, hyperbolic and parabolic. When solving these differential equations, boundary and/or initial conditions need to be provided based on the type of PDE, the necessary inputs can be evaluated.
Examples of PDES in each category include:
   - The passion equation (elliptical).
   - The wave equation (hyperbolic).
   - The Fourier law (parabolic).

   These are two main approaches to solve elliptic PDES namely the finite difference methods (FDM) and variational (or energy) methods.

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FEM falls into the second category of variational approaches and are primarily based on the philosophy of energy minimization. Finite element method (FEM) is one of the most popular numerical methods to boundary and initial value problems and also used in computations modeling.

7) **Computers:** The basic concept of FEM can be thought of as splitting the computational domain into individual small patches and finding local solutions that satisfy the nine differential equations within the boundary of this patch.

8) **Animation for Visualization:** Creating realistic images is something that many visual effects companies strive for. For almost half a century, film makers have been using computers to create images of places or people that do not really exist, in order to make their films more immersive. FEM simulation techniques have been revolutionizing special effects in films and video games to create extremely realistic images. FEM with 3D animation applications are very useful and important to create images in visualizing projects. FEM Simulation and animation techniques also help in visualizing events even before they happen and hence understand the events better. For example, FEMs can be used to visualize potential terrorist attacks on buildings, to study the structural and architectural drawings of buildings, plane crash scenario visualization and deformation of vehicles during a natural disaster or attack. FEMs are also used to model Soft body simulation which for studying the deformation of objects under forces. FE simulation techniques have revolutionized special effects in films and video games, creating extremely realistic images. FEM can be applied for computer animation scenes which show real world behavior such as surgery simulations. It can also be used to improve understanding of transport in biological systems (computational bio-fluids), role of geometric parameters of the multi-story buildings and bridges to study the effect during earthquake and other natural calamities.

9) **Biology:** Hanmetal has elegantly used FEMs to account for the complex shape of certain unicellular organisms. FEM has applications in solving PDEs of heat and fluid flows such as the steady state heat transfer problems. Modeling of diverse physical phenomena such as shock flows, traffic flow, acoustic transmission in fog, etc uses the Burgers' equation [13]. Hence to study the flow field it is important that we obtain a solution of this equation. Burgers equations appear often as a simplification of a more complex and sophisticated model. Hence it is usually thought as a toy model, namely, a tool that is used. The human body has a very complex geometry. The shape of the human body parts is irregular and varies with the position, age, gender, and individual. FEM plays a very important role to solve the large and complex problems related to the human anatomy which is characterized by a very irregular geometry. FEM can model for biomedical problems and treatments like Percutaneous valve implantation- a successful alternative to open-heart Surgery, Tissue Modeling, and analysis of formation of various parts of human body. The 3-Dimensional FE models are also effectively used to solve medical problems, internal injuries and diseases that are related to knee, joints, bones, heart, kidney and different other human body parts. As it is difficult to experiment on dentistry, FEM is often used in the areas of dentistry. FEM can also be associated to clinical evaluations as a further tool for diagnosis and treatment planning and provides the clinician the required information about the choice of therapy. FEM is also used to analyze stress generation in the polymerization process of composite materials. It is also used in stress analysis related to different restorative protocols such as tooth implants, root canal and other orthodontic approaches.

FEM plays a major role in the following problem areas of dentistry:

i) **Non-Carious Cervical Lesions (NCCL):** FEM is used in the investigation of NCCL where stress is one of the factors. FEM along with clinical observations help the dentist find stress factors in initiation and development of NCCL.

ii) **Root Canal Treatment:** While preparing for the root canal procedures, many factors are outside the control of the dentist (natural root morphology, canal shape, canal size and dentine thickness). These factors can be addressed during the treatment to reduce fracture susceptibility.

iii) **Restoration of root filled teeth:** Endodontically treated teeth are generally compromised by coronal destruction. This can involve dental caries, fractures, previous restorations as well as endodontic access. We need to fully understand the effect of restoration type and post application for reconstructing endodontically treated teeth. FEM is used in investigating the stress distribution in an endodontically treated premolar restored with compositeres in with or without a glass fiber post system. FEM helps to evaluate the influence of the type of the material (carbon and glass fiber) and the

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external configuration of the dowel (smooth and serrated) on the stress distribution of teeth that have been restored with dynamic dowel systems.

iv) **Composite and resin cement shrinkage:** Resin-composite materials are increasingly used today in adhesive dental restorative procedures due to the possibility of bonding to dental tissues, but one of the disadvantages of these materials is polymerization shrinkage. Shrinkage stress, which is associated with various clinical symptoms, like fracture propagation, micro-leakage, etc, cannot be measured directly as it is a transient and nonlinear process. Only FEM, because of its peculiar property are suitable for studying and analyzing residual shrinkage stress of resin cement used to cement a ceramic inlay in dental systems. It can be proved with experimental results that, resin cement polymerized immediately after cementation, produce more residual stress than when is delayed for 5 minutes after setting ceramic inlay and polymerization.

10) **Plate Dynamics:** Plate elements are specifically used to analyze the bending deformation of plate structures and the resultant forces like shear forces and moments. For example, natural frequencies and mode shapes of the micro-motor. In [7] the authors have presented an FEM to predict the probable shape of the cut side. The model investigates the effect of potential parameters influencing the blanking process and their interactions. This helped in choosing the process leading parameters for two identical products manufactured from two different materials blanked with a reasonable quality on the same mold intended model objectives. A combination of both FEM and Design of Experiments (DOE) techniques is used to optimize the process. The combination used here reduces the experimental cost and hence provides a good contribution towards the optimization of sheet metal blanking process.

11) **Industrial and Business Management:** With technological advances in business, the creation and design of products are also changing rapidly. In the era of global competition, providing a high quality, innovative products, at the best manufacturing cost is a challenging task. The manufacturing system of business can be supported with computer aided design, while an ordered or developed product can be quickly designed and shown to the customers. We can approximately calculate the manufacturing time, cost and waste rates without any additional manufacturing cost. One of the industrial applications we consider here is to create virtual resistance values for the products like furniture, wooden sports and musical instruments. A computational FE model may predict a better accuracy for the behavior and modes of vibration of wooden instruments, so that the builder can consider certain features before the production of the instrument. By developing the virtual resistance values and applying on the real products, we compare software values to improve the manufacturing of the products. However, these programs require significant amount knowledge of the finite element theory and the features of the finite element software. Therefore, the user of the finite element package should have sufficient information on the nature of the design problem in order to check the validity of the results. The challenge here is to predict the responses of structures and materials to environmental factors such as force, heat, vibration, etc.

12) **Manufacturing and Design of Sports Equipment's:** The introduction of composite materials in the production of sporting equipment has made participation in sports relatively safer. It has also made the equipment associated with them more enduring and durable. FEM is also used to design sports equipment's like, tennis racket, ice hockey stick, golf ball etc, and also building golf clubs and stadiums. It plays a vital role in construction and design of equipment for sports injuries such as nose protector, sports shoes and many others.

A specific FEM model has been developed to design and fine-tune a tennis racket to players' styles. This models the five milliseconds in which the ball is in contact with the racket, during which time, it compresses, deforms and rebounds. This design of the racket featuring 35 tensioned strings allows the model to simulate the oblique spinning impacts of tennis balls in real games. The results of the model were compared to high-speed video analysis of real shots for the validation. The impact of shoes on the ground during sports activities is unavoidable. Wearing proper sport shoes is absolutely imperative for the appropriate execution of sport activity and to reduce the chance of fatigue and injury generally associated with sports. A 3D FE mesh is hence to construct models of foot and sport shoes with different designs. This helps in better understanding of shoe-foot interaction, the stress, and plantar pressure. The model also helps for proper selection of shoe materials and structures that reduces the stress factor.

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Wooden Hockey Sticks are one piece uniform construction that consists of compressed strips bound with resin. The thickness and the type of resin used to bind the strips directly affect the properties of the shaft. Bending and torsion forces along the shaft are the most common problems with an Ice wooden hockey stick. Construction of a lightweight yet durable shaft has been a challenging task before the manufacturers. FEM help in creating an appropriate model for ice-hockey sticks made of wood or other composite materials. As FEM Modeling of a layered composite is a very difficult task, one has to find simple procedure for optimization of the transverse and longitudinal layer thickness after finding a proper composite meshing element.

13) **Design of Musical Instruments:** Musical instruments are complex structures, which do not have a direct relationship between the features and materials with the sounds produced by the instrument. The characteristics of musical instruments such as guitar and violin have some major features which determine instrument quality and character which are Ring (sound quality), Attack behavior, Overall loudness and Degree of possible ring variation. FEM is a specifically powerful tool for dynamic and vibration analysis of many musical instruments such as guitar design process and carving process of xylophones. It helps in adjusting frequencies of modes when assembling and fine tuning stringed musical instruments and hence developing the improved musical instruments. FEM can be also be used to design of a guitar-neck system to solve problems like bending and twisting of the neck due to string forces, moisture content, expansion forces, cylindrical orthotropic nature of wood and many other related problems. Linear static analysis is used to create stability under string forces and the forces of moisture content expansion in wood. Nonlinear statics can be used for developing the truss-rod and slot for optimum adjustability. With modal analysis, one can minimize dead spots and design the improved, integrated guitar neck system. The effect of the vibration modes of musical instruments has been quite instrumental in their manufacture process. The signal processing is one of the essential techniques for understanding and modeling the structure of musical instruments as well as the quality of the sound. In, the authors have proposed a modal analysis in the different stages of construction of an acoustic guitar and studied modal vibrations analysis. In this case, a set of certain specific parameters of design are used to improve sound performance. The researchers can extend these studies to understanding the analysis and design of building classical guitars and other musical instruments like flute, violin, Indian Tabla etc

Algebraic Equations, Ordinary Differential Equations, Partial Differential Equations, and the Laws of Physics

The laws of physics are usually expressed in mathematical language. For example, conservation laws such as the law of conservation of energy, conservation of mass, and conservation of momentum can all be expressed as partial differential equations (PDEs). Constitutive relations can also be utilized to express these laws in terms of variables such as temperature, density, velocity, electric potential and many other dependent variables.

Differential equations represent expressions that determine a small change in a dependent variable w.r.t. a change in an independent variable (x, y, z, t). This small change is also referred to as the derivative of the dependent variable w.r.t. the independent variable.

For example, consider a solid with time-varying temperature but very negligible variations in space. In such a case, the equation for conservation of internal (thermal) energy will result in an equation for the change of temperature, with a very small change in time, due to a heat source *g*:

$$U \approx U_n \tag{1}$$

$$U_n = \sum u_1 \psi_1 \tag{2}$$

In some situations, knowing the temperature at a time t_0, called an *initial condition*, allows for an analytical solution of Eq. 3that is expressed as:

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$$ec_p \frac{dT}{dt} = g(T,t) \tag{3}$$

Temperature, *T*, is the dependent variable and time, *t*, is the independent variable. The function may describe a heat source that varies with temperature and time. Eq. 3 states that if there is a change in temperature in time, then this has to be balanced (or caused) by the heat source. The equation is a differential equation expressed in terms of the derivatives of one independent variable (*t*). Such differential equations are known as *ordinary differential equations* (ODEs) error

$$e = U - U_n$$

The temperature in the solid is therefore expressed through an *algebraic equation* (4), where giving a value of time, t_1, returns the value of the temperature, T_1, at that time.

Often, there are variations in time and space. The temperature in the solid at the positions closer to a heat source maybe slightly higher than elsewhere. Such variations further can give rise to a heat flux between the different parts within the s olid. In such cases, the conservation of energy can result in a heat transfer equation that expresses the changes in both time and spatial variables (x), such as:

$$T = f(t) \tag{4}$$

As before, *T* is the dependent variable, while **x** (**x** = (*x, y, z*)) and *t* are the independent variables. The heat flux vector in the solid can be denoted by **q** = (q_x, q_y, q_z) while the *divergence* of **q** is used to describe the change in heat flux along the spatial coordinates. For a Cartesian coordinate system, the divergence of **q** can be defined as

$$ec_p + \nabla . q = g(T_1 t_1 x) \tag{5}$$

Eq. 5 states that if there is a change in net flux when changes are added in all the directions so that the divergence (sum o f the changes) of **q** is not zero, then this has to be balanced (or caused) by a heat source and/or a change in temperature in time (accumulation of thermal energy).
The heat flux in a solid is described by the constitutive relation for heat flux by conduction. This is also referred to as *Fourier's law*:

$$\nabla . q = \frac{\delta q_x}{\delta x} + \frac{\delta q_y}{\delta y} + \frac{\delta q_z}{\delta z} \tag{6}$$

In the above equation, *k* denotes the thermal conductivity. Eq. 7 states that the heat flux is proportional to the gradient in temperature, with the thermal conductivity as proportionality constant. Eq. 7 and Eq. 5 give us the following differential expression:

$$q = -k\nabla T \rightarrow q = (-\frac{k\delta T}{\delta x} - \frac{k\delta T}{\delta y} - \frac{k\delta T}{\delta z}) \tag{7}$$

Here, the derivatives are expressed in terms of *t, x, y*, and *z*. When a differential equation is expressed in the terms of the derivatives of more than one independent variable, it is known as a partial differential equation (PDE), since each derivativ e represents a change in one direction out of several possible directions. The derivatives in ODEs are expressed using *d*, while partial derivatives and are expressed using the curlier ∂.

In addition to Eq. 8, the temperature at a time t_0 and temperature or heat flux at some position $\mathbf{x}_0$ could be known as well. Such knowledge can be applied in the initial condition and boundary conditions for Eq. 8 In many situations, PDEs cannot be resolved with analytical methods to give the value of the dependent variables at different times and positions. It may, for example, be very difficult or impossible to obtain an analytic expression such as:

$$ec_p \frac{\delta T}{\delta t} + \nabla . (-k\nabla T) = g(T,t,x) \tag{8}$$

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From Eq. 8, we get

$$T = f(t, x) \tag{9}$$

Rather than solving PDEs analytically, an alternative option is to search for approximate *numerical solutions* to solve the numerical model equations. The finite element method is exactly this type of method – a numerical method for the solution of PDEs.

Similar to the thermal energy conservation referred above, we derive the equations for conservation of momentum and mass that form the basis for fluid dynamics. Further to that, we can also derive the equations for electromagnetic fields and fluxes for space- and time-dependent problems, forming systems of PDEs.

Continuing this discussion, let's see how the so-called weak formulation can be derived from the PDEs.

CONCLUSION

The primary purpose of this paper is to highlight the importance and trends of FEM in various areas of science and engineering. Pertaining to its applications to the objects with irregular geometrical shapes, FEM has comprehensive effects compared to other numerical methods like Finite Difference Method, Finite Volume Method etc. An attempt is made in this paper to bring into focus the applicability and relative importance of FEM, without getting into a detailed theoretical account of the topics. Mathematics Community and the communities of varied disciplines viz. doctors, engineers, programmers, manufacturers, sportsmen, musicians and other professionals should come together for successful application of FEM.

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