

# Application of the Power Spectral Density for Damage Identification and Location in Beam Structures

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**Abstract-** The objective of current work is to show by the effectiveness of using root mean square value of power spectral density function and to establish its capability to detect and localize damage. The beam of rectangular cross section of dimension 760 x 30 x 20 mm with cantilever and fixed – fixed support condition has been used. In the present study damage is simulate by reducing the area of moment of inertia of a desired element for different damage cases. The power spectral density and random signal response data of the beam with damage of different sizes are obtained by forced vibration analysis using MATLAB 7.0. RMS values of power spectral density and random response signals collected for undamaged and damaged structures, respectively, and can be utilized to determine whether or not the structure is damaged. Severity of damage is found out by damage index method. This method is very useful in real time damage identification in structures.

**Keywords-** Damage detection, Power Spectral density, Condition monitoring .

## I. INTRODUCTION

Significant work has been done in the area of detecting damage in structures using changes in dynamic response of the structure. Because the natural frequencies and mode shapes of a structure are dependent on the mass and stiffness distribution, any subsequent changes in them should, theoretically, be reflected in changes in the frequencies and mode shapes and their sensitivities to damage level. Consequently, structural safety and functionally will be significantly improved and a condition-based maintenance procedure can be developed. More recently resonant methods based on modal data have been used both to identify that damage exists, and to locate it. Some techniques, such as those Cawley and Adams [1], treat frame work as discrete systems, and compare the modal behavior of the damaged structure with that of the undamaged structure. Mannan and Richardson [2] have presented an approach for not only detecting but also locating a structural fault. And also they presented a method to determine mass, stiffness and damping properties from measured frequencies response functions and showed how changes in those parameters could be used to locate the fault. Presented a method based on the decreases in modal strain energy between two structural degrees of freedom as defined by the curvature of the measured mode shapes. This method as been successful applied to data from a damaged bridge Stubbs et al. [3].

Pandey et al. [4] demonstrated that absolute changes in mode shape curvature

can be a good indicator of damage for the FEM beam structure they considered. A comprehensive literature review of damage detection methods using vibration signals for structural and mechanical systems is provided by Doebling, et al [5]. The displacement functions converted into curvature function, which are further processed to yield a damage index Maia et al. [6] have found that in place of using the displacement mode shapes. Strain or curvature shapes (surface strain in a beam is proportional to curvature) are more effective at identifying the location of damage. Cornwell et al. [7] presented a method which requires that the mode shapes before and after damage be known, but the modes do not need to be mass normalized and only a limited to structures that are characterized by one dimensional curvature (i.e. curvature that is uniquely a function of one independent spatial variable). In other words, the 1-D strain energy method has been successfully applied to 2-D and 3-D structures but only decomposing into beam like structures. Ratcliffe [8] has reported method for locating structural damage using experimental vibration data. This method uses measured frequency response functions to obtain displacement as function of frequency. Liberatore and Carman [9] presented a method estimates power spectral density analysis and quantified by method means of its root mean square value. These values are combined with mode shapes to locate the damage. Mallikarjuna D Reddy et. al[10] showed the effectiveness of using wavelet transform for detection and localization of small damages. The spatial data used here are the rotational mode shapes of the damaged and undamaged plate-like structures. Mallikarjuna D Reddy et. al[11] The objective of the current work is to show the effectiveness of using wavelet transform for detection and localization of small damages. The spatial data used here are the rotational mode shapes of the damaged and undamaged plate-like structures.

Despite of the extensive studies of vibration analysis on damaged structures, only few effective and practical techniques are found for very small damage identification. This paper, therefore, focuses on the study of Localized changes in stiffness results in a RMS of PSD's

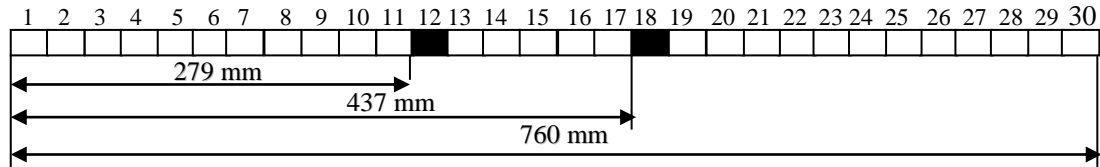
changes therefore this feature will be studied as a possible

II. CANTILEVER BEAM MODEL AND DAMAGE SCENARIOS:

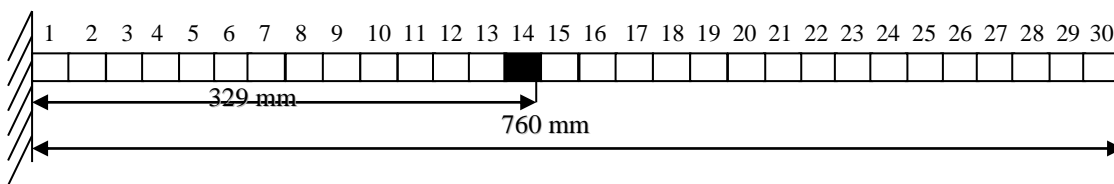
A cantilever beam of rectangular cross section of dimension 760 x 30 x 20 mm with Young's Modulus of 200 GPa and density of 7860 Kg/m<sup>3</sup> is used for numerical

method for damage detection purposes.

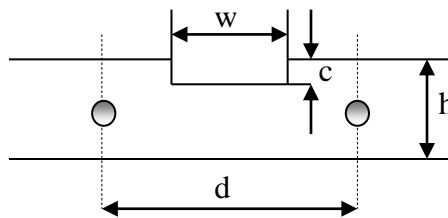
simulation to evaluate power spectral density for damage identification and location. For finite element purpose, the beam is divided into 30 two node one-dimensional elements as shown in Figure 1(a). Damage is defined as c/h ratio, where c gives the depth of damage and h is the height of beam as shown in Figure 1(b).



(a)



(b)



(c)

Figure1. Cantilever beam with damage simulation (a) damage at Single location (b) damage at two locations (c) Damage geometry

The parameter d represents the spatial sampling distance which is the distance between successive measurement to obtain mode shape and w is the width of cut which is equal to the width of an element. Five simulated damage scenarios are given in Table 1 where first three cases corresponds to damage at single location and last two corresponds to damage at two location. In the present study damage is simulated by reducing the area moment of inertia of a desired element. Reduction in stiffness is calculated as the ratio of difference between the area moment of inertia of undamaged and damaged element to undamaged area moment of inertia.

Table 1 Simulated damage scenarios and reduction in stiffness.

Damage scenario (CD: Cantilever damaged)	Damaged element Number	c/h	Reduction in stiffness (Damage severity)
CD 1	14	0.2	0.48
CD 2	14	0.15	0.38
CD 3	14	0.1	0.27
CD 4	12 and 18	0.15	0.38
CD 5	14 and 15	0.1	0.27

III. RESULTS AND DISCUSSIONS:

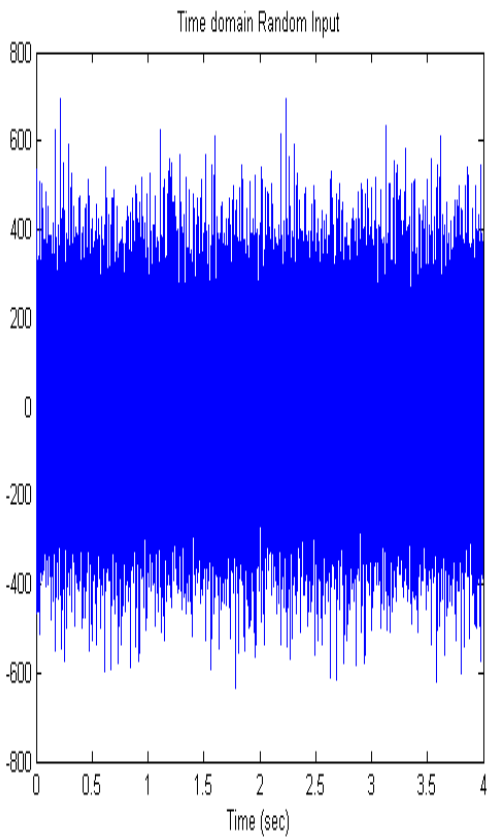
For each damage scenario, the dynamic characteristics (frequencies and mode shapes) before and after the damage were numerically evaluated, with programs coded in MATLAB 7.0. The first five frequencies are listed in Table 2.

Table 2. First five natural frequencies for all damage scenarios

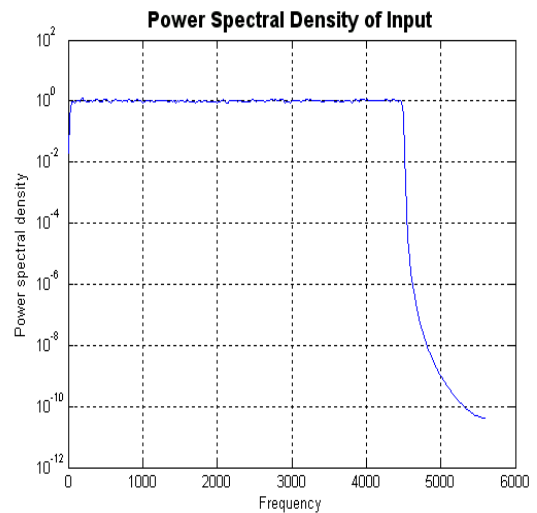
Damage Scenario	Natural frequency (Hz)				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Undamaged	28.426	178.14	498.81	977.48	1615.9
CD1	28.082	174.02	493.95	966.55	1578.1
CD2	28.198	175.38	495.52	970.08	1589.8
CD3	28.291	176.48	496.82	973.01	1599.9
CD4	27.909	170.02	483.85	964.83	1533.2
CD5	28.181	174.62	496.12	964.93	1592.9

For the cantilever beam vibration test a random excitation is applied to the structure. Figure 2a shows the general random signal representing input force. The random signal is specified so that it would have a uniform power spectral density (PSD) in the range of frequency 0 to 4500 Hz as shown in Figure 2b. The output response of the PSD in range of 0 to 4500 Hz

shown in Figure 3a.

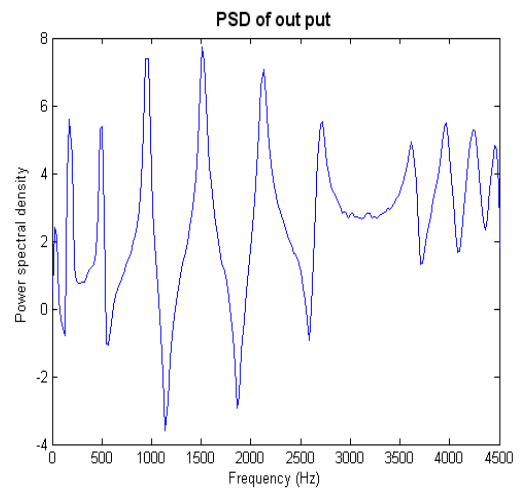


(2a)

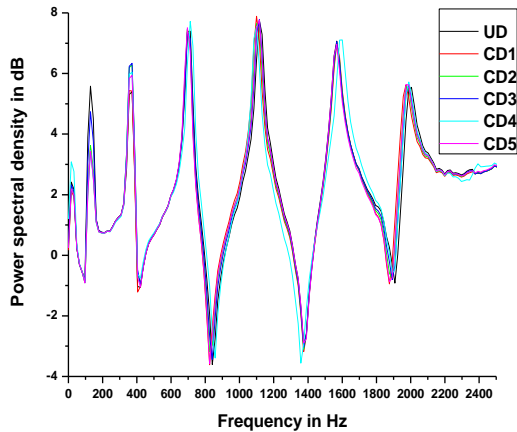


(2b)

Figure (2a). General random signal representing input force. (2b) Power spectral density of a random input force in the frequency range 0 to 4500 Hz.



(3a)



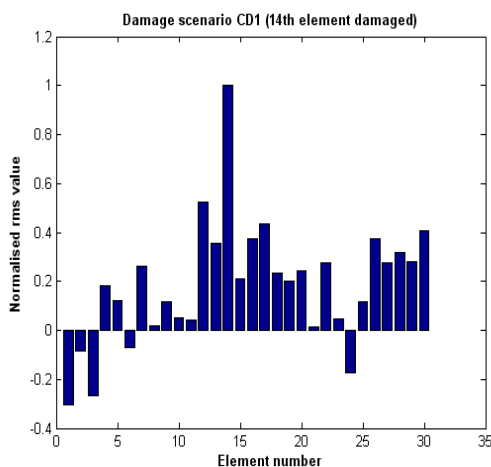
(3b)

Figure. (3a) Out put response of power spectral density in the frequency range 0 to 4500 Hz. (3b) Power spectral density for undamaged and damaged cases.

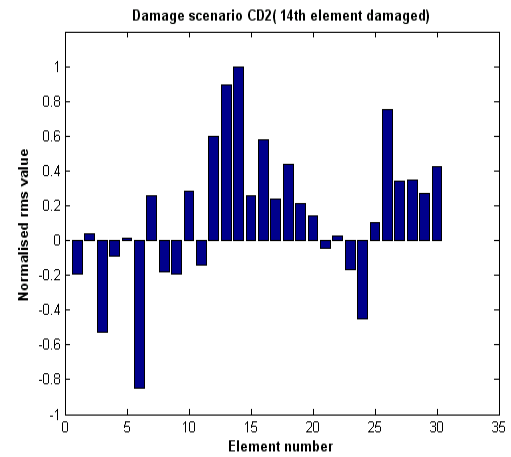
The PSD's for the typical location before and after damages are plotted as shown in Figure 3b with in the range of frequency interest. It shows a clear shift in frequencies i.e. decrease in frequencies with respect to increase in damage. In the analytical section specified that RMS values of the PSD could be used to identify damage. So the RMS value of PSD's for undamaged and damaged structures from all elements of the beam collected and the selection of difference in the RMS values as feature of damage detection and localization.

3.1. Single damage scenarios:

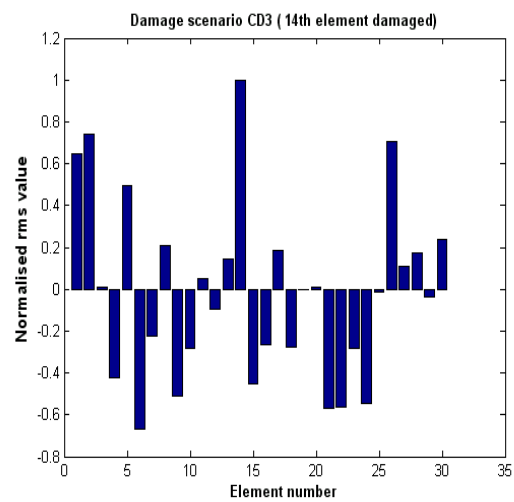
Figure 4a shows the absolute difference in RMS values of power spectral density for damage scenario (CD1) between the intact beam and the damaged beam. This feature also is normalized by setting the largest value equal to 1. The maximum difference value for each RMS values of power spectral density occurs in the damage location. In other areas of the beam this characteristic is much smaller.



(4a)



(4b)



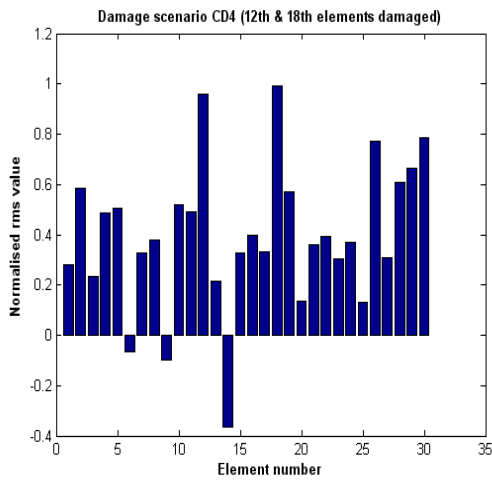
(4c)

Figure. 4. Variation of normalized RMS value Vs element number for damage scenario (a) CD1. (b) CD2. (c) CD3.

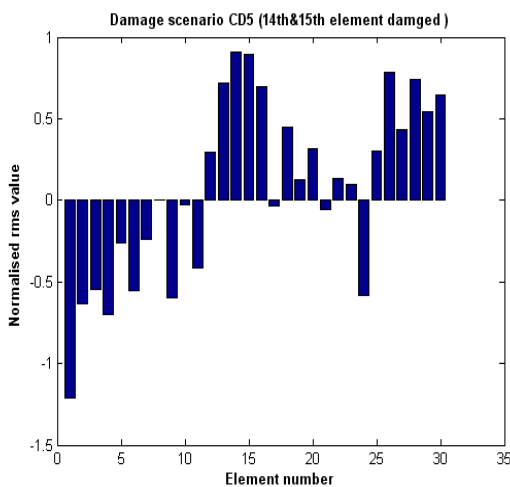
The same analysis is performed to for the other damage scenarios (CD2) and (CD3) and analogous results were obtained. Each plot shows peak at the damage location as observed in Figure 4b and 4c. Therefore this feature successfully identified the position of the damage along the length of the beam.

3.2. Multiple damages scenarios:

In order to investigate the behavior of RMS values when multiple damages are present in the beam, the same analysis is performed for a beam containing damage two locations.



(5a)



(5b)

Figure 5. Variation of normalized RMS value Vs element number for damage scenario (a) CD4. (b) CD5.

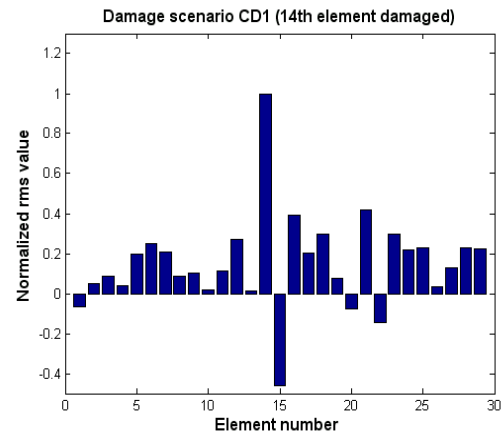
The first of the multiple damage scenarios is 12<sup>th</sup> and 18<sup>th</sup> elements get reducing the area moment of inertia by  $c/h=0.15$  i.e. (CD4) as shown in Figure 1b. RMS values of PSD's were calculated for this multiple damaged and intact beam and then result is plotted as shown in Figure 5a. The second multiple damage scenario is side by side elements (14<sup>th</sup> & 15<sup>th</sup>) get reducing the area moment of inertia by  $c/h=0.1$  i.e. (CD5) Then the results are plotted as shown in Figure 5b.

It clearly showed that procedure suitable for real application where, in most cases, structures may contain several defects at the same time.

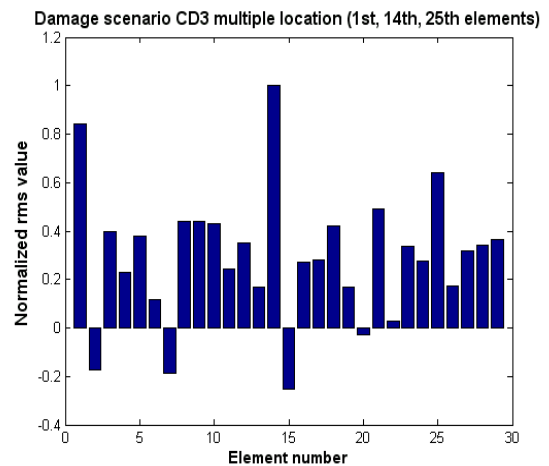
### 3.3 Fixed - Fixed beam case:

Similar analysis repeated for the same beam with fixed-fixed boundary condition. In this section two damage cases have been studied. Single damage case (CD1) for 14<sup>th</sup> element and multiple damages case (CD1) for 1<sup>st</sup>, 14<sup>th</sup>, 25<sup>th</sup> elements. For single damage case the normalized RMS

value has been plotted versus element number as shown in Figure 6a which clearly show the maximum normalized RMS value at the damage location.



(6a)



(6b)

Figure 6. Fixed-fixed beam with damage locations (a) Variation of normalized RMS difference Vs element number for damage scenario, CD1. (b) Variation of normalized RMS difference Vs element number for damage scenario, CD1 for 1<sup>st</sup>, 14<sup>th</sup>, 25<sup>th</sup> elements.

Similarly for multiple damage case Figure 6b shows the maximum normalized RMS values at 1<sup>st</sup>, 14<sup>th</sup> and 25<sup>th</sup> elements. This method is more sensitive to locate damage even at the boundaries. From this analysis we can say that the method is very useful in real time damage identification for bridge like structures.

#### IV. DAMAGE QUANTIFICATION:

Power spectral density of RMS values are defined as  $(PSD\_RMS)_U$  and  $(PSD\_RMS)_D$

for undamaged and damaged structure, respectively, and can be utilized to determine the severity of damage by Damage Index (DI).

$$DI = \sum_{i=1}^N \left| \frac{(PSD\_RMS_i)_D - (PSD\_RMS_i)_U}{(PSD\_RMS_i)_U} \right| * 100$$

Where

$(PSD\_RMS)_U$  - Power spectral density root mean square value for undamaged case.

$(PSD\_RMS)_D$  - Power spectral density root mean square value for damaged case.

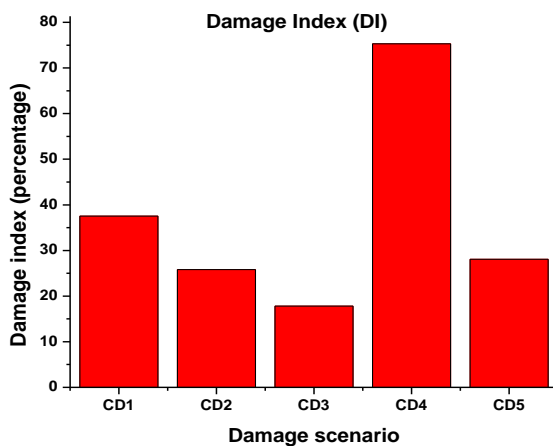


Figure 7. Damage index for different damage scenarios

Figure 7 shows the quantification of damage in percentage for different damage scenarios for cantilever beam case as the damage index (DI) increases with increases in damage severity.

#### V. CONCLUSIONS:

A method for identification and localization of structural damage is implemented analytically for cantilever beam. The method is focused on RMS values of power spectral density is most sensitive to damage. The proposed method only requires the RMS values of PSD of the structure before and after damage.

According to the results, the proposed approach not only successfully located the single damage and multiple damage scenarios. It is proved that Damage Index value (DI) calculated using RMS values of PSD which will be effectively used for quantification of damage seniority.

It is proved that, the proposed method is suitable for real time condition monitoring applications like bridge structures, aircraft, Wind turbines etc.

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