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Application of the Affine Theorem to an Orthotropic Rectangular Reinforced Concrete Slab Having a interior Corner Opening

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***Abstract* - An attempt has been made to applythe affinity theorem to determine collapse load of two-way orthotropic slab withainterior corner openingwith *Onelong side continuous and other three sides simply supported slab (OLC) and One long sidesimply supported and other three sides continuous slab (OLD)*.Keeping in view the basic principles of yield line theory, all possible admissible yield line patterns are considered for the given configuration of the slab subjected to uniformly distributed load (udl). A computer program has been developed to solve the virtual work equations derived in this paper. Illustration of above methodology has been brought out with numerical examples. Relevant tables and charts for given data and the governing admissible failure patterns of the slab for different sizes of openings are presented using the affine theorem. In this paper, authors also present the transformation of orthotropic slab into an equivalent isotropic slab using the affine theorem. The analysis is carried out with aspect ratio of opening quite different to that the slab.**

Key Words:aspect ratio, interior corneropening, configuration, affinity theorem, orthotropic slab, uniformly distributed load, ultimate load, ultimatemoment and virtual work equations.

INTRODUCTION

Openings in slabs are usually required for plumbing, fire protection pipes, heat and ventilation ducts and air conditioning. Larger openings that could amount to the elimination of a large area within a slab panel are sometimes required for stairs and elevators shafts. For newly constructed slabs, the locations and sizes of the required openings are usually predetermined in the early stages of design and are accommodated accordingly. Such two way slabs subjected to uniformly distributed load and supported on various edge conditions are being analyzed by using yield-line method as suggested by Johansen, K.W[1]. Many researchers (Goli,H.B. et al[2], Rambabu,K. et al[3],Islam,S. et al[4], Zaslavsky.Aron[5], Siva Rama Prasad et al[6], Sudhakar,K.J. et al[7], Veerendra Kumaret al[8]) adopted the yield-line analysis and virtual work method in deriving the virtual work equations of the rectangular reinforced concrete solid slabs subjected to uniformly distributed load and supported on various edge conditions. Johansen, K.W[1],also presented the analysis of orthogonal solid slabs implicitly to that of an equivalentisotropic slab by using “Affine Theorem” provided the ratio of negative to positive moments is same in the orthogonal directions. Various design charts are presented byIslam,S. et al[4] for continuous slab(CS) and simply supported(SS) slabs with equalopenings,i.e. ratio of openings and aspect ratio are same. *Whereas in this paper similar charts are presented for twoedge conditions with aspect ratio of slab and opening different.*

Methodology

The method of determining collapse loads based on principle of virtual work has proved to be a powerful tool for a structural engineer, despite it gives an upper bound value. The work equations are formed by equating the energy absorbed by yield lines and the work done by the external load of the orthogonal rectangular slab with interior corneropenings where a small virtual displacement is given to the slab. The same principle was also used by Islam,S. et al[4]in their paper. In other words, the work equation is given by

$$\iint W_{ult}\delta(x,y)dxdy = \sum\left(m_{ult,x}\theta_x y_0 + m_{ult,y}\theta_y x_0\right) \text{-------------------} \quad (1)$$

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where $W_{ult}$is the ultimate load per unit area of slab, $\delta(x,y)$ is the virtual displacement in the direction of the loading at the element of area of dimensions dx,dy, $m_{ult,x}$ and $m_{ult,y}$are the yield moments per unit width in the x and y directions, θ_x and θ_y are the components of the virtual rotation of the slab segments in the x and y directions and x_0 and $y_0$are the projected length of the yield lines in x and y directions of slab. The equation (1) contains terms C_1, C_2, $C_3$and C_4 which define the positions of the node points of the yield lines. The values of C_1, C_2, C_3 and C_4 to be used in the equation are those which give the minimum load to cause failure. A computer program has been written to find the values of C_1, C_2, C_3 and C_4 (in terms of r_1, r_2, r_3 and r_4) corresponding to minimum load carrying capacity of the slab.For definitions of various parameters refer notations. Johansen[1] has proved that the yield line theory is an upper bound method, so care has been taken to examine all the possible yield line patterns for TAC slab to ensure that the most critical collapse mode is considered otherwise the load carrying capacity of the slab will be overestimated.

Formulation of Virtual Work Equations

There is several possible yield line patterns associated with different edge conditions of the slab. *For the OLC edge conditionof slab, the possible admissible failure yield line patterns aretwenty,For the OLD edge conditions of slab, the possible admissible failure yield line patterns aretwenty.* These admissible failure yield line patterns are obtained basing on the yield line principleof Johansen.K.W[1]. For the given configuration of the slab, these twenty failure patterns and corresponding equations have been investigated depending upon the support condition of the slab using a computer program.

The orthogonal reinforced rectangular slab having interior corner opening with the given configuration and the yield criteria are shown in notation Fig.5. The slab is subjected to uniformly distributed load (W_{ult}). Note that the slab is not carrying any load over the area of the opening.

The generalized virtual work equations for continuous edge (CS) condition of slab are derived for the predicted possible admissible failure yield line patterns using the virtual work equation. To get the equations for other edge conditions of the slab, modification should be carried out in the numerators of the equations of each failure patterns.*ForOLCslab I_1=I_2=I_3=0 or I_1= I_3=I_4= 0, for OLD slab I_2=0 or I_4=0.*

Virtual Work Equations for Two Adjacent sides Continuous (TAC) Slab

*Twentypossible failure patterns are predicted for two edge conditions of the slab.*The governing failure pattern for different edge conditions and for different data is presented in Table-1. Let δ be the virtual displacement at a & b (Fig. 1), for theconsidered failure Pattern-1of aslab. Three unknown dimensions C_1, C_2, &C_3 are necessary to define the yield line propagation completely.All otheradmissible failure patterns are as shown in APPENDIX-A.

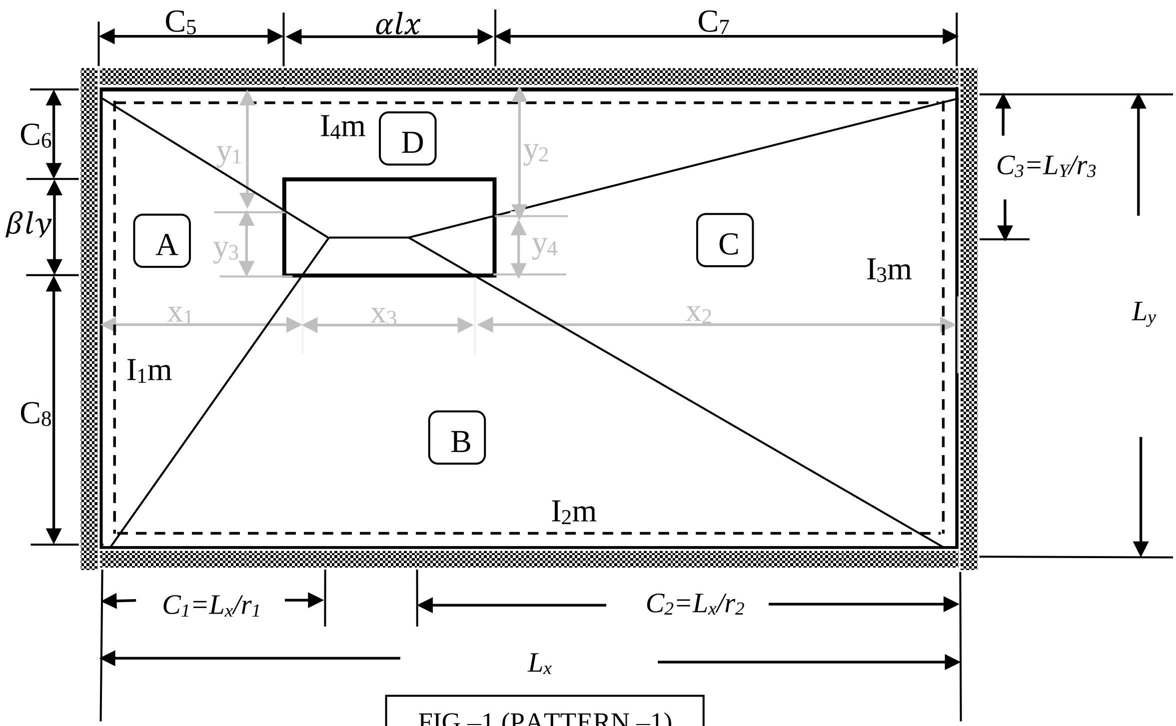

FIG –1 (PATTERN –1)

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The derived equations below are for the continuously supported rectangular slab with interior corner opening.

PATTERN 1:

The external work done by segment A:-

$$=\left[W_{ult}\frac{1}{2}C_5y_1\frac{C_5}{3C_1}+W_{ult}C_5y_3\frac{C_5}{2C_1}+W_{ult}\frac{1}{2}C_8x_1\frac{x_1}{3C_1}\right]$$

$$=W_{ult}L_y{}^2r\left[\frac{r_1r_5^3}{6r_3}+\frac{r_1r_5^2}{2}\left(1-r_8-\frac{r_1r_5}{r_3}\right)+\frac{r_3^2r_8^3}{6r_1(r_3-1)^2}\right]$$

The external work done by segment B:-

$$=\left[W_{ult}\frac{1}{2}x_1C_8\frac{C_8}{3(L_y-C_3)}+W_{ult}x_3C_8\frac{C_8}{2(L_y-C_3)}+W_{ult}\frac{1}{2}x_2C_8\frac{C_8}{3(L_y-C_3)}\right]$$

$$=W_{ult}L_y{}^2r\left[\frac{r_3^2r_8^3}{6r_1(r_3-1)^2}+\frac{r_3^2r_8^3}{2(r_3-1)}\left(1-\frac{r_3r_8}{(r_3-1)}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)+\frac{r_3^2r_8^3}{6r_2(r_3-1)^2}\right]$$

The external work done by segment C:-

$$=\left[W_{ult}\frac{1}{2}C_8x_2\frac{x_2}{3C_2}+W_{ult}y_4C_7\frac{C_7}{2C_2}+W_{ult}\frac{1}{2}y_2C_7\frac{C_7}{3C_2}\right]$$

$$=W_{ult}L_y{}^2r\left[\frac{r_3^2r_8^3}{6r_2(r_3-1)^2}+\left(1-r_8-\frac{r_2r_7}{r_3}\right)\frac{r_2r_7^2}{2}+\frac{r_2^2r_7^3}{6r_3}\right]$$

The external work done by segment D:-

$$=\left[W_{ult}\frac{1}{2}C_7y_2\frac{y_2}{3C_3}+W_{ult}\alpha L_xC_6\frac{C_6}{2C_3}+W_{ult}\frac{1}{2}C_5y_1\frac{y_1}{3C_3}\right]$$

$$=W_{ult}L_y{}^2r\left[\frac{r_2^2r_7^3}{6r_3}+\frac{r_3r_6^2(1-r_5-r_7)}{2}+\frac{r_1^2r_5^3}{6r_3}\right]$$

Total work done = work done by segment's (A+B+C+D)

$$=W_{ult}L_y{}^2r\left[\begin{array}{l}\left[\frac{r_1r_5^3}{6r_3}+\frac{r_1r_5^2}{2}\left(1-r_8-\frac{r_1r_5}{r_3}\right)+\frac{r_3^2r_8^3}{6r_1(r_3-1)^2}\right]+\left[\frac{r_3^2r_8^3}{6r_1(r_3-1)^2}+\frac{r_3^2r_8^3}{2(r_3-1)}\left(1-\frac{r_3r_8}{(r_3-1)}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)+\frac{r_3^2r_8^3}{6r_2(r_3-1)^2}\right]\\+\left[\frac{r_3^2r_8^3}{6r_2(r_3-1)^2}+\left(1-r_8-\frac{r_2r_7}{r_3}\right)\frac{r_2r_7^2}{2}+\frac{r_2^2r_7^3}{6r_3}\right]+\left[\frac{r_2^2r_7^3}{6r_3}+\frac{r_3r_6^2(1-r_5-r_7)}{2}+\frac{r_1^2r_5^3}{6r_3}\right]\end{array}\right]$$

Energy absorbed yield lines: -

$$=\left[\begin{array}{l}\left[m_{ult}K'_x(y_1+C_8)\frac{1}{C_1}+m_{ult}I_1L_y\frac{1}{C_1}\right]+\left[m_{ult}K'_y(x_1+x_2)\frac{1}{(L_y-C_3)}+m_{ult}I_2L_x\frac{1}{(L_y-C_3)}\right]\\+\left[m_{ult}K'_x(y_2+C_8)\frac{1}{C_2}+m_{ult}I_3L_y\frac{1}{C_2}\right]+\left[m_{ult}K'_y(C_5+C_7)\frac{1}{C_3}+m_{ult}I_4L_x\frac{1}{C_3}\right]\end{array}\right]$$

$$=m_{ult}\left[\begin{array}{l}\left[\frac{K_x^1r_1}{r}\left(\frac{r_1r_5}{r_3}+r_8\right)+\frac{I_1r_1}{r}\right]+\left[\frac{K_y^1rr_3^2r_8}{(r_3-1)^2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)+\frac{I_2rr_3}{(r_3-1)}\right]+\left[\frac{K_x^1r_2}{r}\left(\frac{r_2r_7}{r_3}+r_8\right)+\frac{I_3r_2}{r}\right]+\\\left[K_y^1rr_3(r_5+r_7)+I_4rr_3\right]\end{array}\right]$$

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Equating total work done by the segments to energy absorbed by yield lines we get

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3}+r_8\right)+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3^2 r_8}{(r_3-1)^2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[\frac{K_x^1 r_2}{r}\left(\frac{r_2 r_7}{r_3}+r_8\right)+\frac{I_3 r_2}{r}\right]+ \\ \left[K_y^1 r r_3 (r_5+r_7)+I_4 r r_3\right] \end{array}\right]}{r\left[\begin{array}{l} \left[\frac{r_1 r_5^3}{6 r_3}+\frac{r_1 r_5^2}{2}\left(1-r_8-\frac{r_1 r_5}{r_3}\right)+\frac{r_3^2 r_8^3}{6 r_1 (r_3-1)^2}\right]+\left[\frac{r_3^2 r_8^3}{6 r_1 (r_3-1)^2}+\frac{r_3^2 r_8^3}{2(r_3-1)}\left(1-\frac{r_3 r_8}{(r_3-1)}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)+\frac{r_3^2 r_8^3}{6 r_2 (r_3-1)^2}\right] \\ +\left[\frac{r_3^2 r_8^3}{6 r_2 (r_3-1)^2}+\left(1-r_8-\frac{r_2 r_7}{r_3}\right)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\frac{r_3 r_6^2 (1-r_5-r_7)}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right] \end{array}\right]}$$

Equation 2 for Failure Pattern – 2

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_5 r_1^2}{r}+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3}{(r_3-1)}(r_5+r_7)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[\frac{K_x^1 r_2^2 r_7}{r}+\frac{I_3 r_2}{r}\right]+\left[\frac{K_y^1 r_3}{r}(r_5+r_7)+\frac{I_4 r_3}{r}\right]\right]}{R*\left[\begin{array}{l} \left[\frac{r_1^2 r_5^3}{6 r_3}+(1-r_1 r_5)\frac{r_5^2 r_1}{2}+\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}\right]+\left[\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}+\frac{\alpha r_8^3 r_3}{2(r_3-1)}+\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}\right]+ \\ \left[\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}+\frac{(1-r_7 r_2) r_2 r_7^3}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\frac{\alpha r_6^2 r_3}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right] \end{array}\right]}$$

Equation 3 for Failure Pattern –3

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3}+r_8\right)+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3}{(r_3-1)}\left(\frac{r_8 r_3}{r_1 (r_3-1)}+r_7\right)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[K_y^1 r r_3 (r_7+r_5)+I_4 r r_3\right]+ \\ \left[\frac{K_x^1 r_2^2 r_7}{r}+\frac{I_3 r_2}{r}\right] \end{array}\right]}{R*\left[\begin{array}{l} \left[\frac{r_1^2 r_5^3}{6 r_3}+\left(1-r_8-\frac{r_1 r_5}{r_3}\right)\frac{r_1 r_5^2}{2}+\frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2}\right]+\left[\frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2}+\left(1-r_7-\frac{r_8 r_3}{r_1 (r_3-1)}\right)\frac{r_3 r_8^2}{2(r_3-1)}+\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}\right]+ \\ \left[\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}+(1-r_2 r_7)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\frac{\alpha r_3 r_6^2}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right] \end{array}\right]}$$

Equation 4 for Failure Pattern – 4

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{K_x^1 r_1}{r}\left(r_6+\frac{r_1 r_5 (r_3-1)}{r_3}\right)+\frac{I_2 r_1}{r}\right]+\left[\frac{K_y^1 r r_3}{(r_3-1)}(r_5+r_7)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[\frac{K_x^1 r_2^2 r_7}{r}+\frac{I_3 r_2}{r}\right]+ \\ \left[K_y^1 r r_3\left(\frac{r_6 r_3}{r_1}+r_7\right)+I_3 r r_3\right] \end{array}\right]}{R*\left[\begin{array}{l} \left[\frac{r_3^2 r_6^2}{6 r_1}+\left(1-r_6-\frac{r_1 r_5 (r_3-1)}{r_3}\right)\frac{r_1 r_5^2}{2}+\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}\right]+\left[\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}+\frac{\alpha r_3 r_8^2}{2(r_3-1)}+\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}\right]+ \\ \left[\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}+(1-r_2 r_7)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\left(1-r_7-\frac{r_6 r_3}{r_1}\right)\frac{r_3 r_6^2}{2}+\frac{r_3^2 r_6^2}{6 r_1}\right] \end{array}\right]}$$

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Equation 5 for Failure Pattern – 5

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}\left(r_6 + r_8\right) + \frac{I_1 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{\left(r_3 - 1\right)}\left(\frac{r_8 r_3}{r_1\left(r_3 - 1\right)} + r_7\right) + \frac{I_2 r r_3}{\left(r_3 - 1\right)}\right] + \left[\frac{K_x^1}{r}\left(r_2^2 r_7\right) + \frac{I_3 r_2}{r}\right] + \\ \left[K_y^1 r r_3\left(r_7 + \frac{r_3 r_6}{r_1}\right) + I_4 r r_3\right] \end{bmatrix}}{R * \begin{bmatrix} \left[\frac{r_3^2 r_6^3}{6r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_8^3 r_3^2}{6r_1\left(r_3 - 1\right)^2}\right] + \left[\frac{r_8^3 r_3^2}{6r_1\left(r_3 - 1\right)^2} + \left(1 - r_7 - \frac{r_3 r_8}{r_1\left(r_3 - 1\right)}\right)\frac{r_6^2 r_3}{2\left(r_3 - 1\right)} + \frac{r_2^2 r_7^3\left(r_3 - 1\right)}{6r_3}\right] \\ + \left[\frac{r_2^2 r_7^3\left(r_3 - 1\right)}{6r_3} + \frac{r_2 r_7^2}{2}\left(1 - r_2 r_7\right) + \frac{r_2^2 r_7^3}{6r_3}\right] + \left[\frac{r_2^2 r_7^3}{6r_3} + \left(1 - r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6r_1}\right] \end{bmatrix}}$$

Equation 6 for Failure Pattern – 6

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}\left(r_6 + \frac{r_1 r_5\left(r_3 - 1\right)}{r_3}\right) + \frac{I_1 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{\left(r_3 - 1\right)}\left(r_7 + r_5\right) + \frac{I_2 r r_3}{\left(r_3 - 1\right)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \\ \left[K_y^1 r_3 r\left(\frac{r_3 r_6}{r_1} + r_7\right) + I_4 r_3 r\right] \end{bmatrix}}{R * \begin{bmatrix} \left[\frac{r_3^2 r_6^3}{6r_1} + \left(1 - r_6 - \frac{\left(r_3 - 1\right) r_5 r_1}{r_3}\right)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3\left(r_3 - 1\right)}{6r_3}\right] + \left[\frac{r_1^2 r_5^3\left(r_3 - 1\right)}{6r_3} + \frac{\alpha r_3 r_8^2}{2\left(r_3 - 1\right)} + \frac{\left(r_3 - 1\right)\left(r_2 r_7 - 1\right)}{2r_2 r_3} + \frac{\left(r_3 - 1\right)}{6r_2 r_3}\right] \\ + \left[\frac{\left(r_3 - 1\right)}{6r_2 r_3} + \frac{1}{6r_2 r_3}\right] + \left[\frac{1}{6r_2 r_3} + \frac{\left(r_7 r_2 - 1\right)}{2r_3 r_2} + \left(1 - r_7 - \frac{r_6 r_3}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6r_1}\right] \end{bmatrix}}$$

Equation 7 for Failure Pattern – 7

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}\left(r_8 + \frac{r_1 r_5}{r_3}\right) + \frac{I_1 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{\left(r_3 - 1\right)}\left(r_7 + \frac{r_8 r_3}{r_1\left(r_3 - 1\right)}\right) + \frac{I_2 r_3 r}{\left(r_3 - 1\right)}\right] + \\ \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \left[K_y^1 r r_3\left(r_5 + r_7\right) + I_4 r r_3\right] \end{bmatrix}}{R * \begin{bmatrix} \left[\frac{r_1^2 r_5^3}{6r_3} + \left(1 - r_8 - \frac{r_1 r_5}{r_3}\right)\frac{r_1 r_5^2}{2} + \frac{r_3^2 r_8^3}{6r_1\left(r_3 - 1\right)^2}\right] + \\ \left[\frac{r_3^2 r_8^3}{6r_1\left(r_3 - 1\right)^2} + \left(1 - r_7 - \frac{r_3 r_8}{r_1\left(r_3 - 1\right)}\right)\frac{r_3 r_8^2}{2\left(r_3 - 1\right)} + \frac{\left(r_2 r_7 - 1\right)\left(r_3 - 1\right)}{2r_3 r_2} + \frac{\left(r_3 - 1\right)}{6r_2 r_3}\right] + \\ \left[\frac{\left(r_3 - 1\right)}{6r_2 r_3} + \frac{1}{6r_3 r_2}\right] + \left[\frac{1}{6r_3 r_2} + \frac{\left(r_2 r_7 - 1\right)}{2r_3 r_2} + \frac{\alpha r_3 r_6^2}{2} + \frac{r_1^2 r_5^3}{6r_3}\right] \end{bmatrix}}$$

Equation 8 for Failure Pattern –8

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left(\frac{K_x^1 r_1}{r}(r_6 + r_8) + \frac{I_1 r_1}{r}\right) + \left[\frac{K_y^1 r r_3}{(r_3-1)}\left(\frac{r_8 r_3}{r_1(r_3-1)} + r_7\right) + \frac{I_2 r r_3}{(r_3-1)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \\ \left[K_y^1 r r_3\left(r_7 + \frac{r_3 r_6}{r_1}\right) + I_4 r r_3\right] \end{bmatrix}}{R * \begin{bmatrix} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2}\right] + \left[\frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2} + \left(1 - r_7 - \frac{r_3 r_8}{r_1(r_3-1)}\right)\frac{r_8^2 r_3}{2(r_3-1)} + \frac{(r_2 r_7 - 1)(r_3-1)}{2 r_3 r_2} + \frac{(r_3-1)}{6 r_3 r_2}\right] + \\ \left[\frac{(r_3-1)}{6 r_2 r_3} + \frac{1}{6 r_2 r_3}\right] + \left[\frac{1}{6 r_2 r_3} + \frac{(r_2 r_7 - 1)}{2 r_3 r_2} + \left(1 - r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{bmatrix}}$$

Equation 9 for Failure Pattern – 9

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3} + \frac{r_1 r_5 (r_3-1)}{r_3}\right) + \frac{I r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{(r_3-1)}(r_7 + r_5) + \frac{I_2 r r_3}{(r_3-1)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \left[K_y^1 r r_3 (r_7 + r_5) + I_4 r r_3\right]\right]}{R * \begin{bmatrix} \left[\frac{r_1^2 r_5^3}{6 r_3} + (1 - r_1 r_5)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}\right] + \left[\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3} + \frac{\alpha r_8^2 r_3}{2(r_3-1)} + \frac{(r_2 r_7 - 1)(r_3-1)}{2 r_3 r_2} + \frac{(r_3-1)}{6 r_2 r_3}\right] + \\ \left[\frac{(r_3-1)}{6 r_2 r_3} + \frac{1}{6 r_2 r_3}\right] + \left[\frac{1}{6 r_2 r_3} + \frac{(r_2 r_7 - 1)}{2 r_3 r_2} + \frac{\alpha r_6^2 r_3}{2} + \frac{r_1^2 r_5^3}{6 r_3}\right] \end{bmatrix}}$$

Equation 10 for Failure Pattern – 10

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}\left(r_6 + \frac{r_5 r_1}{r_4}\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4 (r_5 + r_7) + I_2 r r_4\right] + \\ \left[\frac{K_x^1}{r}\frac{r_1}{r_1 - 1}\left(\frac{r_1 r_7}{r_4(r_1-1)} + r_6\right) + \frac{I_3}{r}\frac{r_1}{r_1 - 1}\right] + \left[K_y^1 r (r_3^2 r_6) + I_4 r r_3\right] \end{bmatrix}}{R * \begin{bmatrix} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{(r_4 - r_5 r_1 - r_4 r_6) r_1 r_5^2}{2 r_4} + \frac{r_1^2 r_5^3}{6 r_4}\right] + \left[\frac{r_1^2 r_5^3}{6 r_4} + \frac{\alpha r_8^2 r_4}{2} + \frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2}\right] + \\ \left[\frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2} + \frac{r_1 r_7^2}{2(r_1-1)}\left(1 - \frac{r_1 r_7}{r_4(r_1-1)} - r_6\right) + \frac{r_3^2 r_6^3 (r_1-1)}{6 r_1}\right] + \\ \left[\frac{r_3^2 r_6^3 (r_1-1)}{6 r_1} + r_3 r_6\left(\frac{r_1 - r_3 r_6 - (r_1-1) r_3 r_6}{2 r_1}\right) + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{bmatrix}}$$

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Equation 11 for Failure Pattern – 11

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\begin{array}{l}\left[\frac{K_x^1 r_1}{r_1}\left(\frac{r_1r_5+r_3r_8}{r_3}\right)+\frac{I_1r_1}{r}\right]+\left[K_y^1 r r_4\left(\frac{r_4r_8+(r_4r_8)(r_1-1)}{r_1}\right)+I_2 r r_4\right]+\\ \left[\frac{K_x^1 r_1}{r(r_1-1)}\left(\frac{r_1r_7}{r_3(r_1-1)}+r_8\right)+\frac{I_3r_1}{r(r_1-1)}\right]+\left[K_y^1 r r_3(r_5+r_7)+I_4 r r_3\right]\end{array}\right]}{R*\left[\begin{array}{l}\left[\frac{r_1^2r_5^3}{6r_3}+\frac{(r_3-r_1r_5-r_3r_8)r_1r_5^2}{2r_3}+\frac{r_4^2r_8^3}{6r_1}\right]+\left[\frac{r_4^2r_8^3}{6r_1}+\frac{(r_1-r_4r_8-r_4r_8(r_1-1))r_4r_8^2}{2r_1}+\frac{r_4^2r_8^3(r_1-1)}{6r_1}\right]+\\ \left[\frac{r_4^2r_8^3(r_1-1)}{6r_1}+\left(1-\frac{r_1r_7}{r_3(r_1-1)}-r_8\right)\frac{r_1r_7^2}{2r_3(r_1-1)^2}+\frac{r_1^2r_7^3}{6r_3(r_3-1)^2}\right]+\left[\frac{r_1^2r_7^3}{6r_3(r_3-1)^2}+\frac{\alpha r_6^2 r_3}{2}+\frac{r_1^2r_5^3}{6r_3}\right]\end{array}\right]}$$

Equation 12 for Failure Pattern – 12

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\begin{array}{l}\left[\frac{K_x^1 r_1}{r}\left(\frac{r_5r_1}{r_3}+\frac{r_1r_5}{r_4}\right)+\frac{I_1r_1}{r}\right]+\left[K_y^1 r r_4(r_7+r_5)+I_2 r r_4\right]+\\ \left[\frac{K_x^1}{r}\left(\frac{r_1^2r_7}{r_4(r_1-1)^2}+\frac{r_1^2r_7}{r_3(r_1-1)^2}\right)+\frac{I_1r_1}{r(r_1-1)}\right]+\left[K_y^1 r r_3(r_5+r_7)+\frac{I_y r_3}{r}\right]\end{array}\right]}{R*\left[\begin{array}{l}\left[\frac{r_1^2r_5^3}{6r_3}+\frac{r_1r_5^2}{2}\left(1-\frac{r_1r_5}{r_3}-\frac{r_1r_5}{r_4}\right)+\frac{r_1^2r_5^3}{6r_4}\right]+\left[\frac{r_1^2r_5^3}{6r_4}+\frac{\alpha r_4r_8^2}{2}+\frac{r_1^2r_7^3}{6r_4(r_1-1)^2}\right]+\\ \left[\frac{r_1^2r_7^3}{6r_4(r_1-1)^2}+\frac{r_1r_7^2}{2(r_1-1)}\left(1-\frac{r_1r_7}{r_3(r_1-1)}-\frac{r_1r_7}{r_4(r_1-1)}\right)+\frac{r_1^2r_7^3}{6r_3(r_1-1)^2}\right]+\\ \left[\frac{r_1^2r_7^3}{6r_3(r_1-1)^2}+\frac{\alpha r_3r_6^2}{2}+\frac{r_1^2r_5^3}{6r_3}\right]\end{array}\right]}$$

Equation 13 for Failure Pattern – 13

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\begin{array}{l}\left[\frac{k_x^1 r_1}{r}\left(r_8+\frac{r_1r_5}{r_3}\right)+\frac{I_1r_1}{r}\right]+\left[K_y^1 r r_4\left(\frac{r_4r_8}{r_1}+r_7\right)+I_2 r r_4\right]+\\ \left[\frac{k_x^1 r_1^2 r_7}{r(r_1-1)^2}\left(\frac{1}{r_3}+\frac{1}{r_4}\right)+\frac{I_3r_1}{r(r_1-1)}\right]+\left[K_y^1 r r_3(r_5+r_7)++I_4 r r_3\right]\end{array}\right]}{R*\left[\begin{array}{l}\left[\frac{r_1^2r_5^3}{6r_3}+\left(1-r_5-\frac{r_1r_5}{r_3}\right)\frac{r_1r_5^2}{2}+\frac{r_4^2r_8^3}{6r_1}\right]+\left[\frac{r_4^2r_8^3}{6r_1}+\left(1-r_7-\frac{r_4r_8}{r_1}\right)\frac{r_4r_8^2}{2}+\frac{r_1^2r_7^3}{6r_4(r_1-1)^2}\right]+\\ \left[\frac{r_1^2r_7^3}{6r_4(r_1-1)^2}+\left(1-\frac{r_1r_7}{(r_1-1)}\left(\frac{1}{r_3}+\frac{1}{r_4}\right)\right)\frac{r_1r_7^2}{2(r_1-1)}+\frac{r_1^2r_7^3}{6r_3(r_1-1)^2}\right]+\left[\frac{r_1^2r_7^3}{6r_3(r_1-1)^2}+\frac{\alpha r_3r_6^2}{2}+\frac{r_1^2r_5^3}{6r_3}\right]\end{array}\right]}$$

Equation 14 for Failure Pattern – 14

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{k_x^1 r_1}{r}\left(r_6 + \frac{r_1 r_5}{r_4}\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4 (r_5 + r_7) + I_2 r r_4\right] + \\ \left[\frac{k_x^1 r_1^2 r_7}{r(r_1 - 1)^2}\left(\frac{1}{r_3} + \frac{1}{r_4}\right) + \frac{I_3 r_1}{r(r_1 - 1)}\right] + \left[K_y^1 r r_3\left(\frac{r_3 r_6}{r_1} + r_7\right) + + I_4 r r_3\right] \end{array}\right]}{R * \left[\begin{array}{l} \left[\frac{r_3^2 r_6^3}{6 r_1} + \left(1 - r_6 - \frac{r_1 r_5}{r_4}\right)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3}{6 r_4}\right] + \left[\frac{r_1^2 r_5^3}{6 r_4} + \frac{\alpha r_4 r_8^2}{2} + \frac{r_1^2 r_7^3}{6 r_4 (r_1 - 1)^2}\right] + \\ \left[\frac{r_1^2 r_7^3}{6 r_4 (r_1 - 1)^2} + \left(1 - \frac{r_1 r_7}{(r_1 - 1)}\left(\frac{1}{r_3} + \frac{1}{r_4}\right)\right)\frac{r_1 r_7^2}{2(r_1 - 1)} + \frac{r_1^2 r_7^3}{6 r_3 (r_1 - 1)^2}\right] + \\ \left[\frac{r_1^2 r_7^3}{6 r_3 (r_1 - 1)^2} + \left(1 - - r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{array}\right]}$$

Equation 15 for Failure Pattern – 15

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{K_x^1 r_1}{r}(r_8 + r_6) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4\left(r_7 + \frac{r_4 r_8}{r_1}\right) + I_2 r r_4\right] + \\ \left[\frac{K_x^1 r_1^2 r_7}{r(r_1 - 1)^2}\left(\frac{1}{r_4} + \frac{1}{r_3}\right) + \frac{I_3 r_1}{r(r_1 - 1)}\right] + \left[K_y^1 r r_3\left(r_7 + \frac{r_3 r_6}{r_1}\right) + I_4 r r_3\right] \end{array}\right]}{R * \left[\begin{array}{l} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \left(1 - r_7 - \frac{r_4 r_8}{r_1}\right)\frac{r_4 r_8^2}{2} + \frac{r_7^3 r_1^2}{6 r_4 (r_1 - 1)^2}\right] + \\ \left[\frac{r_7^3 r_1^2}{6 r_4 (r_1 - 1)^2} + \frac{r_7^2 r_1}{2(r_1 - 1)}\left(1 - \frac{r_7 r_1}{(r_1 - 1)}\left(\frac{1}{r_4} + \frac{1}{r_3}\right)\right) + \frac{r_7^3 r_1^2}{6 r_3 (r_1 - 1)^2}\right] + \\ \left[\frac{r_7^3 r_1^2}{6 r_3 (r_1 - 1)^2} + \left(1 - - r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{array}\right]}$$

Equation 16 for Failure Pattern– 16

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}(r_6 + r_8) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4^2 r_8 + I_2 r r_4\right] + \left[\frac{K_x^1 r_1}{r(r - 1)}(r_6 + r_8) + \frac{I_4 r_1}{r(r - 1)}\right] + \left[K_y^1 r r_3^2 r_6 + I_4 r r_3\right]\right]}{R * \left[\begin{array}{l} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \frac{r_8^2 r_4}{2}(1 - r_4 r_8) + \frac{r_8^3 r_4^2 (r_1 - 1)}{6 r_1}\right] + \left[\frac{r_8^3 r_4^2 (r_1 - 1)}{6 r_1} + + \frac{\beta r_7^2 r_1}{2(r_1 - 1)} + \frac{r_3^2 r_6^3 (r_1 - 1)}{6 r_1}\right] + \\ \left[\frac{r_3^2 r_6^3 (r_1 - 1)}{6 r_1} + \frac{r_6^2 r_3}{2}(1 - r_3 r_6) + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{array}\right]}$$

Equation 17 for Failure Pattern– 17

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\left[\frac{K_x^1r_1}{r}(r_6+r_8)+\frac{I_1r_1}{r}\right]+\left[K_y^1rr_4+I_2rr_4\right]+\left[\frac{K_x^1r_1}{r(r_1-1)}(r_6+r_8)+\frac{I_3r_1}{r(r_1-1)}\right]+\left[K_y^1r\left(r_3^2r_6\right)+I_4rr_3\right]\right]}{R*\left[\begin{array}{l}\left[\frac{r_3^2r_6^3}{6r_1}+\frac{\beta r_5^2r_1}{2}+\frac{(r_4r_8-1)}{2r_4r_1}+\frac{1}{6r_1r_4}\right]+\left[\frac{1}{6r_1r_4}+\frac{(r_1-1)}{6r_1r_4}\right]+\\\left[\frac{(r_1-1)}{6r_1r_4}+\frac{(r_4r_8-1)(r_1-1)}{2r_4r_1}+\frac{\beta r_1r_7^2}{2(r_1-1)}+\frac{r_3^2r_6^3(r_1-1)}{6r_1}\right]+\left[\frac{r_3^2r_6^2(r_1-1)}{6r_1}+(1-r_3r_6)\frac{r_6^2r_3}{2}+\frac{r_3^2r_6^3}{6r_1}\right]\end{array}\right]}$$

Equation 18 for Failure Pattern– 18

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\left[\frac{K^1r_1}{r}(r_6+r_8)+\frac{I_1r_1}{r}\right]+\left[K_y^1rr_4^2r_8+I_2rr_4\right]+\left[\frac{K_x^1r_1}{r(r_1-1)}(r_8+r_6)+\frac{I_3r_1}{r(r_1-1)}\right]+\left[K_y^1rr_3+I_yrr_3\right]\right]}{R*\left[\begin{array}{l}\left[\frac{1}{6r_1r_3}+\left(\frac{r_6r_3-1}{2r_3r_1}\right)+\frac{\beta r_1r_5^2}{2}+\frac{r_4^2r_8^3}{6r_1}+\frac{r_4^2r_8^3}{6r_1}\right]+\left[\frac{r_4^2r_8^3}{6r_1}+\frac{(r_8r_4-1)r_4r_8^2}{2}+\frac{(r_1-1)r_4^2r_8^3}{6r_1}\right]+\\\left[\frac{(r_1-1)r_4^2r_8^3}{6r_1}+\frac{\beta r_1r_7^2}{2(r_1-1)}+\frac{(r_6r_3-1)(r_1-1)}{2r_3r_1}+\frac{(r_1-1)}{3r_3r_1}\right]+\left[\frac{(r_1-1)}{6r_3r_1}+\frac{1}{6r_3r_1}\right]\end{array}\right]}$$

Equation 19 for Failure Pattern– 19

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\begin{array}{l}\left[\frac{K_x^1r_1}{r}(r_6+r_8)+\frac{I_1r_1}{r}\right]+\left[K_y^1rr_4\left(r_7+\frac{r_4r_8}{r_1}\right)+I_2rr_4\right]+\left[\frac{K_x^1r_1}{r(r_1-1)}\left(r_6+\frac{r_7r_1}{r_4(r_1-1)}\right)+\frac{I_3r_1}{r(r_1-1)}\right]+\\\left[K_y^1rr_3+I_4rr_3\right]\end{array}\right]}{R*\left[\begin{array}{l}\left[\frac{1}{6r_3r_1}+\frac{(r_3r_6-1)}{2r_1r_3}+\frac{\beta r_1r_5^2}{2}+\frac{r_4^2r_8^3}{6r_1}\right]+\left[\frac{r_4^2r_8^3}{6r_1}+\left(1-r_7-\frac{r_4r_8}{r_1}\right)\frac{r_8^2r_4}{2}+\frac{r_1^2r_7^3}{6r_4(r_1-1)^2}\right]+\\\left[\frac{r_1^2r_7^3}{6r_4(r_1-1)^2}+\left(1-r_6-\frac{r_1r_7}{r_4(r_1-1)}\right)\frac{r_1r_7^2}{2(r_1-1)}+\frac{(r_6r_3-1)r_1}{2r_3(r_1-1)}+\frac{(r_1-1)}{6r_3r_1}\right]+\left[\frac{(r_1-1)}{6r_3r_1}+\frac{1}{6r_3r_1}\right]\end{array}\right]}$$

Equation 20 for Failure Pattern – 20

$$\frac{W_{ult}l_y^2}{m_{ult}}=\frac{\left[\begin{array}{l}\left[\frac{K_x^1r_1}{r}(r_6+r_8)+\frac{I_1r_1}{r}\right]+\left[K_y^1rr_4+I_2rr_4\right]+\\\left[\frac{K_x^1r_1}{r(r_1-1)}\left(\frac{r_1r_7}{r_3(r_1-1)}+r_8\right)+\frac{I_3r_1}{r(r_1-1)}\right]+\left[K_y^1rr_3\left(\frac{r_6r_3}{r_1}+r_7\right)+I_4rr_3\right]\end{array}\right]}{R*\left[\begin{array}{l}\left[\frac{r_3^2r_6^3}{6r_1}+\frac{\beta r_5^2r_1}{2}+\frac{(r_8r_4-1)}{2r_4r_1}+\frac{1}{6r_4r_1}\right]+\left[\frac{1}{6r_4r_1}+\frac{(r_1-1)}{6r_4r_1}\right]\\\left[\frac{(r_1-1)}{6r_4r_1}+\frac{(r_8r_4-1)(r_1-1)}{2r_4r_1}+\left(1-r_8-\frac{r_1r_7}{r_3(r_1-1)}\right)\frac{r_1r_7^2}{2(r_1-1)}+\frac{r_1^2r_7^3}{6r_3(r_1-1)^2}\right]+\\\left[\frac{r_1^2r_7^3}{6r_3(r_1-1)^2}+\left(1-r_7-\frac{r_6r_3}{r_1}\right)\frac{r_3r_6^2}{2}+\frac{r_3^2r_6^3}{6r_1}\right]\end{array}\right]}$$

The respective equations for corresponding failure patterns can be obtained for OLC and OLD condition by making respective negative yield moment coefficients zero.

Minimization of theVirtual Work Equations

The value $\frac{W_{ult}L_y^2}{m_{ult}}$ of these equationsconsists of the unknown non dimensional parameters r_1, r_2,r_3 and r_4 which define the positions of the yield lines.A computer program has been developed for various values of the non dimensional parameters r_1, r_2,r_3 and $r_4$within their allowable ranges in order to find the minimum value of $\frac{W_{ult}L_y^2}{m_{ult}}$ for the yield line failure patterns considered. In this computer program, the values of r_1, r_2,r_3 and $r_4$were varied at increments of 0.1. Using the above equations, one can develop useful charts basing on orthogonality which may be used eitherfor design or analysis in general. The governing failure pattern for different edge conditions and for different data is presented in Table-1.

EXAMPLE: One Long sideDiscontinuous Slab (OLD)
(Negative moment to positive moment ratio in both directions is same and unity)

Transform an orthotropic slab to an equivalent isotropic slab in which the ratio of Negative moment to positive moment in both directions is same and unity using the affine theorem.

Since $I_1/K'_x = I_2/K'_y = 1.0$, the transformation of the given orthotropic slab(Fig.2 (a)) in X – direction is transformed to an equivalent isotropic slab (Fig.2 (b)) by dividing with $\sqrt{\mu}$. This principle is illustrated in Fig.2. Using the above methodology, few numerical examples are presented in Table 2.

Table 3shows the strength and the failure pattern forOLD based on the principle $\mu = r^2$for different values of coefficient of orthotropy(μ) and their corresponding orthogonal affine moment coefficients.

<u>Note:</u> In the case of SS, TLC and TSC edge conditions of the slab; the affine theorem cannot be applied because the negative moment is not present along one of the edges of the given slab. Therefore one can design the slab as orthotropic using any available computer program.

Preparation of Charts

Design charts have been prepared for One Long side Discontinuous slab (OLD)slabs.Charts 1-4 are shown in Appendix-B.Charts1-4 are plotted for $\frac{W_{ult}L_y^2}{m_{ult}}$ versus Opening ($\alpha=\beta$) for various ratios of K1, K2 and r for different CS conditions.The values of $r$are taken between 1 and 3. The K1 values are plotted varying between 1 and 5 thus ensuring that greater yield moment is in the direction of short span. This is in accordance with elastic theory distribution of bending moments. The values of K2 are plotted for 1 and 2 for One Long sideDiscontinuous slab (OLD).While preparing the design charts, the least value of $\frac{W_{ult}L_y^2}{m_{ult}}$ given by thirteenfailure patterns is considered. Using these charts one can directly design or do analysis of a slab with ainterior corner opening. In addition, a special charts5-6 is presented for various values of openings which can be used for transforming an orthogonal slab to equivalent isotropic slab for the governing failure patterns only.

A. Analysis Problem:(OLC)

Determine the safe uniformly distributed load on a rectangular two way slab with interior corneropening all sides continuousslab(Fig.3(a)) for the following data:

A slab 6m X 4m with interior corner opening of size 1.2m X 0.4m at a distance of 1.2m from long edge and 0.8m from short edge is reinforced with 10mm diameter bars @ 200mm c/c perpendicular to long span and 8mm diameter bars @150mm c/c perpendicular to short span. Two meshes are used one at top long side continuous and one at bottom, thickness of the slab is 120mm. Characteristic strength of concrete is 20MPa and yield stress of steel is 415MPa.

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Solution:

According to IS 456:2000[9], $m_{ult} = 0.87 f_y A_{st} z$,where $z = d\left(1 - \frac{f_y A_{st}}{f_{ck} bd^2}\right)$------------ (2)

Assuming effective depth of slab in short span direction=100.00 mm
effective depth of slab in long span direction=90.00 mm
Area of the steel perpendicular to long span=374 mm^2

Area of the steel perpendicular to short span=314 mm^2
The ultimate moments in short and long span directions can be found using the expression (2).
Therefore, m_{ult}parallel to long span=13.489 kNm/m
m_{ult} parallel to short span=10.192kNm/m

For aspect ratio of slab, $r = \frac{6.0}{4.0} = 1.5$ and taking m_{ult} = 13.489 kNm/m,

The orthogonal coefficients (Fig.3(b)) will be $K`_x$=0.755, $I_1$=$I_3$=0 and $K`_y$=I_2=1.0, I_4=0.With these orthogonal coefficients and for α=0.2, β=0.1, r =1.5, C_5=1.2m, C_6=0.8m**;**

Twenty predicted failure patterns are evaluated by using computer program to find the governing failure pattern and the final results are as follows.

$\frac{W_{ult} L_y^2}{m_{ult}}$ =19.67773, r_1=3.46191, r_2=3.15185, r_3=3.46191 and **the failure pattern is 7**

W_{ult}= $\frac{19.67773 \times 13.489}{4^2}$ = 16.5895kN/m^2

W_{dl}=(dead load including finishing)= (0.12 x 25) + 0.5=3.5 kN/m^2

W_{ult}=1.5(W_{ll}+W_{dl}) =16.5895kN/m^2

W_{ll}= $\frac{16.5895}{1.5} - 3.5$=7.56kN/m^2

The intensity of live load on the slab is 7.56kN/m^2

B. Design Problem: (OLD)

Designall sides continuous slab of 5.0 m X 2.5 m with interior corner openings of size 1.5 m X 0.75 m at a distance of 0.5m from long edge and 0.5m from short edge to carry a uniformly distributed live load of 4.5kN/m^2 .Use M20 mix and Fe 415 grade steel.

Given data:Aspect ratio of slab(r) = $\frac{\boldsymbol{L_x}}{\boldsymbol{L_y}}$ =5.0/2.5=2.0, αL_x=1.5 m, βL_y=0.75m.

$\therefore$ α=0.3,β=0.3, C_5=0.5m, C_6=0.5m

Twenty predicted failure patterns are evaluated by using the computer program to find the governing failure pattern, **by**assumingK'_x=I_1=I_3=1.0,K'_y=I_2=2.0, I_4=0.

The value of $\frac{W_{ult} L_y^2}{m_{ult}}$ =37.83424andthe failure pattern is 8

Unknown parameters:

$\boldsymbol{r_1}$**=** $\frac{\boldsymbol{L_x}}{\boldsymbol{C_1}}$ **=4.6000,** $\boldsymbol{r_2}$**=** $\frac{\boldsymbol{L_x}}{\boldsymbol{C_2}}$ **=3.55185,** $\boldsymbol{r_3}$**=** $\frac{\boldsymbol{L_y}}{\boldsymbol{C_3}}$ **=2.34783**

$\boldsymbol{C_1}$ **=1.0869 m,** $\boldsymbol{C_2}$ **=1.4077 m,** $\boldsymbol{C_3}$ **=1.0648 m**

Assuming overall thickness of slab =110 mm
Dead load of slab=110 X 25=2.75 kN
Dead loads including finishing's=3.5 kN/m^2
Total load=8.0kN/m^2
Ultimate total load=1.5 X 8.0 =12.0kN/m^2

$m_{ult}=\dfrac{12.0\times 2.5^2}{34.83424}$ =2.15305kNm/m

The orthogonal moments are $K^1_x m_{ult}=I_1 m_{ult}$= 1.0 X 2.15305=2.15305kNm/m

$K^1_y m_{ult}=I_2 m_{ult}$=2.0 X 2.15305= 4.3061kNm/m

Effective depth, $d=\sqrt{\dfrac{2.15305\times 10^6}{(0.138\times 20\times 1000)}}$ =27.9301 mm

Adopt effective depth as 100 mm and overall depth as 110 mm

Area of steel along short span =155.169 mm^2

Minimum area of steel required along short span = 204.82 mm^2

Use 8 mm diameter bars @ 240 mm c/c

Area of steel along long span=84.3189mm^2

Use 6 mm diameter bars @ 300 mm c/c

Details of reinforcement are shown in Fig.4(c).

CONCLUSIONS:

1. The virtual work equations for orthotropic slabs with unequalinterior corner openingwith all sides continuous whose aspect ratio of opening is different from the aspect ratio of slab subjected to udlare presented.
2. Design charts for One LongsideDiscontinuous slabare presented for different aspect ratios of slab.
3. Few numerical examples are presented based on theorem of VI and VII of affine theorem of Johansen. K.W.[1]for orthotropic slabs with interior corner openings.
4. Twouseful charts for affine transformation for different sizes of openings arealso presented.
5. In case of One Long sideDiscontinuous, for K2=1, 2 and K1=1, 2,3 and 5 the strength of the slab is decreasing with increase in aspect ratio when compared to solid slab.
6. The charts can be used either for analysis or design for different size of openings in the interior corner of the slab.

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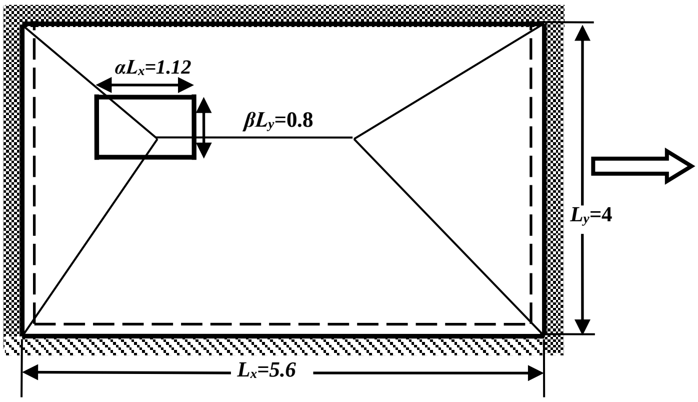

Fig 2 (a): ORTHOTROPIC SLAB

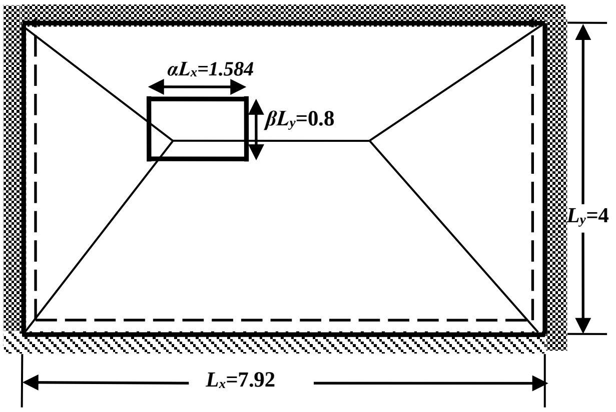

Fig 2 (b): EQUIVALENT ISOTROPIC SLAB

$K'_x=I_1=I_3=0.5$, $K'_y=I_2=1, I_4=0$.
$r5=0.2$, $r6=0.2$, $\alpha=0.2, \beta=0.2$,
$I_3/K'_x=I_4/K'_y=1.0$, $\mu=0.5, \sum K=3$
$r=L_x/L_y=5.6/4=1.4$,
In order to check on affine theorem a computer program is used to evaluate the value of $W_{ult}L_y^2/m_{ult}$ and the value is 22.60740

As per affine theorem the transformed
$K'_x=I_1=I_3=1$, $K'_y=I_2=1, I_4=0$.
$r5=0.2$, $r6=0.2$, $\alpha=0.2, \beta=0.2$,
$L_x'=L_x/\sqrt{\mu}=5.6/\sqrt{0.5}=7.92$m,
$r=7.92/4=1.98$, $\mu=1.0, \sum K=4$,
The value of $W_{ult}L_y^2/m_{ult}$ is obtained from chart – 5 for $r=1.77$. Taking this value one can design the given orthotropic slab without using computer program.

Fig 2 - Orthotropic slab and equivalent isotropic slab as per Theorems VI and VII of Johansen

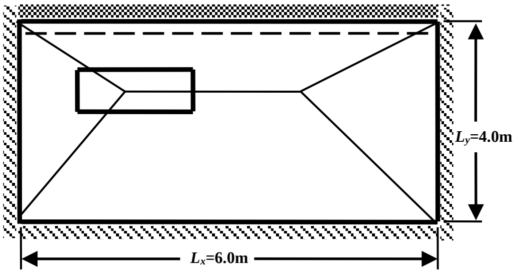

Fig. 3 (a) Analysis problem

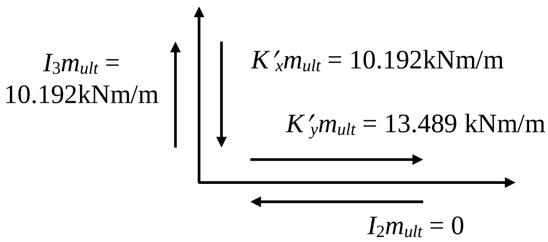

Fig. 3 (b) orthogonal moment coefficients

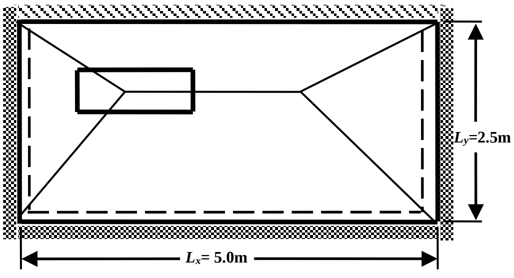

Fig. 4 (a) Design problem

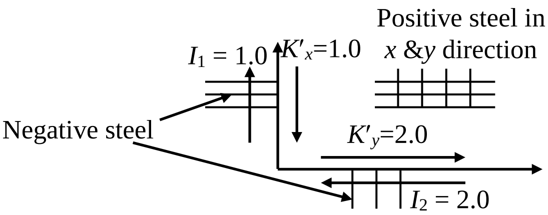

Fig. 4 (b) orthogonal moment coefficients

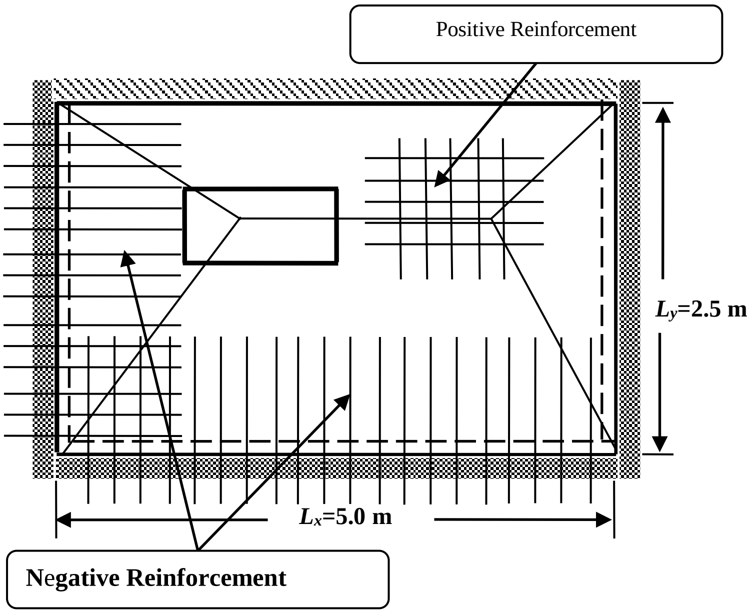

Fig. 4 (c)

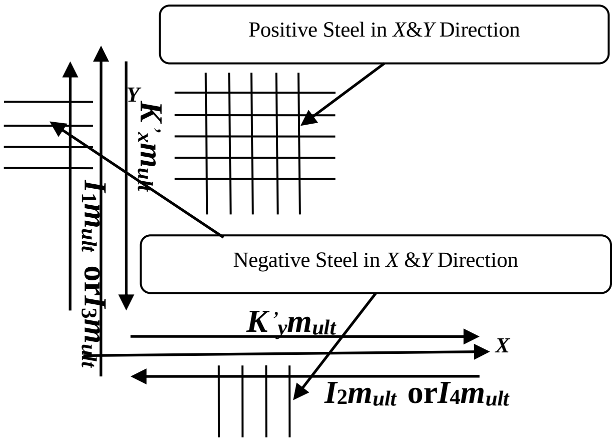

Fig. 5

Table 1

Governing failure patterns for different data for different edge conditions

Failure Patterns	OLC (20) Opening Sizes	OLD (18) Opening Sizes
1	r5 = 0.3, r6 = 0.2, α = 0.2, β = 0.1, r = 1.3 *	r5 = 0.3, r6 = 0.1, α = 0.1, β = 0.3, r = 1
2	r5 = 0.4, r6 = 0.1, α = 0.1, β = 0.2, r = 1.4	r5 = 0.4, r6 = 0.4, α = 0.1, β = 0.1, r = 1 *
3	r5 = 0.2, r6 = 0.1, α = 0.3, β = 0.2, r = 1.3	r5 = 0.2, r6 = 0.1, α = 0.3, β = 0.2, r = 1.5
4	r5 = 0.3, r6 = 0.3, α = 0.2, β = 0.2, r = 1.1	r5 = 0.4, r6 = 0.4, α = 0.1, β = 0.1, r = 1.5
5	r5 = 0.2, r6 = 0.2, α = 0.3, β = 0.1, r = 1	r5 = 0.1, r6 = 0.1, α = 0.4, β = 0.3, r = 1.5
6	r5 = 0.2, r6 = 0.2, α = 0.1, β = 0.3, r = 1.8	r5 = 0.3, r6 = 0.3, α = 0.1, β = 0.2, r = 2
7	r5 = 0.1, r6 = 0.1, α = 0.1, β = 0.2, r = 1	r5 = 0.1, r6 = 0.1, α = 0.1, β = 0.2, r = 1
8	r5 = 0.1, r6 = 0.1, α = 0.1, β = 0.1, r = 1	r5 = 0.1, r6 = 0.1, α = 0.1, β = 0.1, r = 2
9	r5 = 0.2, r6 = 0.1, α = 0.1, β = 0.4, r = 1	r5 = 0.4, r6 = 0.1, α = 0.1, β = 0.1, r = 2
10	r5 = 0.4, r6 = 0.3, α = 0.1, β = 0.2, r = 1.1	---
11	r5 = 0.2, r6 = 0.1, α = 0.2, β = 0.2, r = 1	r5 = 0.2, r6 = 0.1, α = 0.1, β = 0.3, r = 1
12	r5 = 0.4, r6 = 0.1, α = 0.1, β = 0.4, r = 1.2	---
13	r5 = 0.2, r6 = 0.1, α = 0.2, β = 0.4, r = 1	r5 = 0.2, r6 = 0.1, α = 0.3, β = 0.4, r = 1.5
14	r5 = 0.4, r6 = 0.2, α = 0.1, β = 0.3, r = 1.2	r5 = 0.4, r6 = 0.3, α = 0.1, β = 0.2, r = 1.8
15	r5 = 0.1, r6 = 0.1, α = 0.4, β = 0.3, r = 1	r5 = 0.2, r6 = 0.2, α = 0.3, β = 0.3, r = 1.5
16	r5 = 0.3, r6 = 0.2, α = 0.2, β = 0.2, r = 1	r5 = 0.2, r6 = 0.3, α = 0.2, β = 0.2, r = 1
17	r5 = 0.2, r6 = 0.3, α = 0.1, β = 0.1, r = 1 *	r5 = 0.2, r6 = 0.3, α = 0.2, β = 0.1, r = 1
18	r5 = 0.2, r6 = 0.3, α = 0.3, β = 0.1, r = 1	r5 = 0.2, r6 = 0.3, α = 0.1, β = 0.2, r = 1
19	r5 = 0.2, r6 = 0.3, α = 0.3, β = 0.2, r = 1	r5 = 0.2, r6 = 0.4, α = 0.3, β = 0.1, r = 1.3
20	r5 = 0.2, r6 = 0.1, α = 0.3, β = 0.1, r = 1	r5 = 0.2, r6 = 0.2, α = 0.2, β = 0.3, r = 1
COEFFICIENTS	K'x = 1.33, K'y = 0.33, I1 = 0, I2 = 2.33, I3 = 0, I4 = 0, μ = 0.5, ∑K = 4	K'x = 0.9, K'y = 0.5, I1 = 1.5, I2 = 1.1, I3 = 1.5, I4 = 0, μ = 1.5, ∑K = 4
*COEFFICIENTS	K'x = 2, K'y = 0.5, I1 = 0, I2 = 1.5, I3 = 0, I4 = 0, μ = 1, ∑K = 4	K'x = 0.67, K'y = 1.33, I1 = 0.67, I2 = 1.33, I3 = 0.67, I4 = 0, μ = 0.50, ∑K = 4

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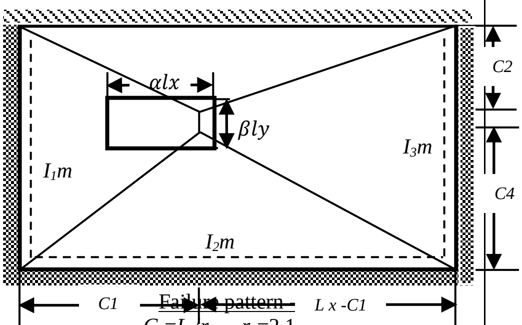
Table- 2					
Numerical examples based on Theorem VI & VII of Affine Theorem of Johansen. K.W.[1]					
Example (OLD)		Orthogonal Moment	Aspect	Strength	Aspect
S.No.	Openingsize	Co-efficients	Ratio (r)	$W_{ult}L_y^2/m_{ult}$	Ratio (r*)
1	α=0.2,β=0.3	K'_x =0.25,K'_y =1.0, I_1=I_3=0.25, I_2=1.0, μ=0.25,,∑K=2.5	1.0	23.04303	2.0
2	α=0.2,β=0.2	K'_x =0.5,K'_y =1.0, I_1= I_3=0.5, I_2=1.0, μ=0.5,∑K=3.0	1.4	22.6074	1.98
3	α=0.2,β=0.1	K'_x =0.667,K'_y =1.0, I_1= I_3=0.667, I_2=1.0, μ=0.667,∑K=3.33	1.6	22.87023	1.96
4	α=0.1,β=0.3	K'_x =1,0,K'_y =1.0, I_1= I_3=1.0, I_2=1.0, μ=1.0,∑K=4.0	1.3	32.89787	1.3
5	α=0.1,β=0.2	K'_x =1.5,K'_y =1.0, I_1= I_3=1.5, I_2=1.0, μ=1.5,∑K=5.0	1.9	27.71505	1.55
6	α=0.3,β=0.2	K'_x =2.0,K'_y =1.0, I_1= I_3=2.0, I_2=1.0, μ=2.0,∑K=6.0	1.7	35.05538	1.20
7	α=0.3,β=0.3	K'_x =4.0,K'_y =1.0, I_1= I_3=4.0, I_2=1.0, μ= I_3=4.0,,∑K=10.0	2.0	44.61118	1.0
NOTE:(1) r*:equivalent isotropic slab aspect ratio, (2)I_1/ K'_x = I_2/ K'_y= 1.0, (3)r5=r6=0.2, (4) I_4=0.					

Table-3				
One Long side continuous slab(OLD), based on the principle $\mu=r^2$				
Coefficient of orthotropy (μ)	Aspect Ratio of Slab $r=\sqrt{\mu}$	Orthogonal moment coefficients (affine)	$\frac{W_{ult}L_y^2}{m_{ult}}$	Failure pattern for all aspect ratios
0.25	0.5	K'_x=0.25,K'_y=1.0 I_1= I_3=0.25, I_2=1.0	44.61118	 Failure pattern C_1=L_x/r_1. r_1=2.14783 C_3= L_y/r_3. r_3=2.54783 C_4= L_y/r_4. r_4=1.84607 Interior corner opening coefficient I_4=0, α = 0.3, β = 0.3,r_5=0.2,r_6=0.2.
0.5	0.707	K'_x =0.5, K'_y =1.0 I_1= I_3=0.5, I_2=1.0	44.61118	
0.667	0.817	K'_x =0.667, K'_y =1.0 I_1=I_3=0.667,I_2=1.0	44.61118	
1.0	1.0	K'_x =1.0, K'_y =1.0 I_1= I_3=1.0, I_2=1.0	44.61118	
1.44	1.2	K'_x =1.44, K'_y =1.0 I_1= I_3=1.44, I_2=1.0	44.61118	
1.5	1.225	K'_x =1.5,K'_y =1.0 I_1= I_3=1.5, I_2=1.0	44.61118	
2.0	1.414	K'_x =2.0, K'_y =1.0 I_1= I_3=2.0, I_2=1.0	44.61118	
2.25	1.5	K'_x =2.25, K'_y =1.0 I_1= I_3=2.25, I_2=1.0	44.61118	
4.0	2.0	K'_x =4.0, K'_y =1.0 I_1= I_3=4.0, I_2=1.0	44.61118	

Notation:

	Continuous edge
	Simply supported edge
	Free edge
	Negative yield line
CS	A slab supported on all sides continuously (restrained)
SS	A slab simply supported on all sides
OLC	A slab restrained on one long sides and simply supported on other three sides
OLD	A slab restrained on three sides and simply supported on one long sides
$K'_x m_{ult}$	Positive ultimate yield moment per unit length provided by bottom tension

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bars placed parallel to X-axis

$K'_y m_{ult}$ — Positive ultimate yield moment per unit length provided by bottom tension bars placed parallel to Y-axis

$I_1 m_{ult}$ and $I_3 m_{ult}$ — Negative ultimate yield moment perunit length provided by top tension reinforcement bars placed parallelto x-axis.

$I_2 m_{ult}$ and $I_4 m_{ult}$ — Negative ultimate yield moment perunit length provided by top tension reinforcement bars placed parallelto y-axis.

K1 — K'_y/K'_x

K2 — I_2/K'_y

L_x, L_y — Slab dimensions in X and Y directionsrespectively

α, β — coefficients of opening in the slab

m_{ult} — Ultimate Yield moment per unit lengthof the slab

r — Aspect ratio of slab defined by L_x/L_y.

r_1, r_2, r_3, r_4 — Non dimensional parameters of yield line propagation

r_5, r_6 — Non dimensional parameters of opening distances

udl — Uniformly Distributed Load

W_{ult} — Ultimate uniformly distributed loadper unit area of slab.

μ — Coefficient of orthotropy $= \dfrac{[K'_x + I_1]}{[K'_y + I_2]}$

Appendix-A

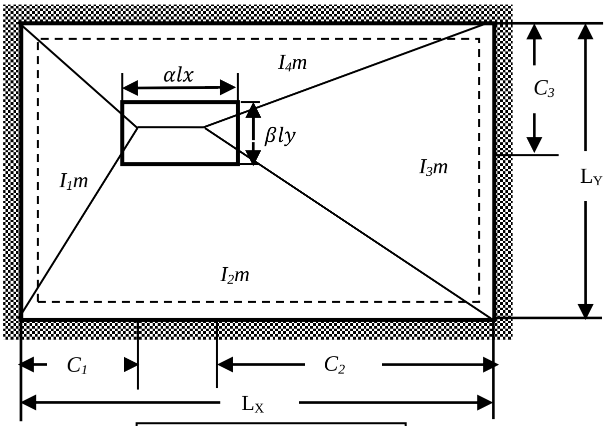

FIG – 2 (PATTERN – 2)

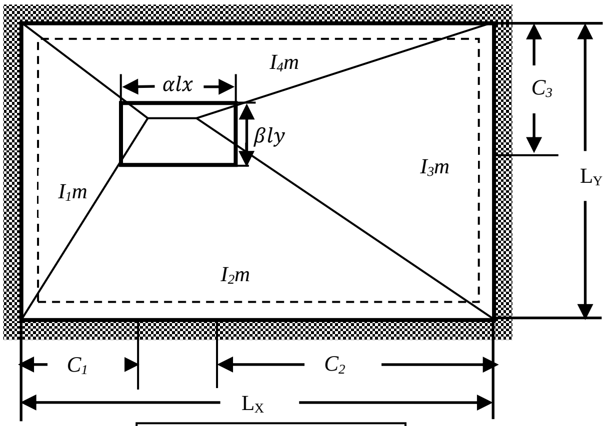

FIG – 4 (PATTERN – 4)

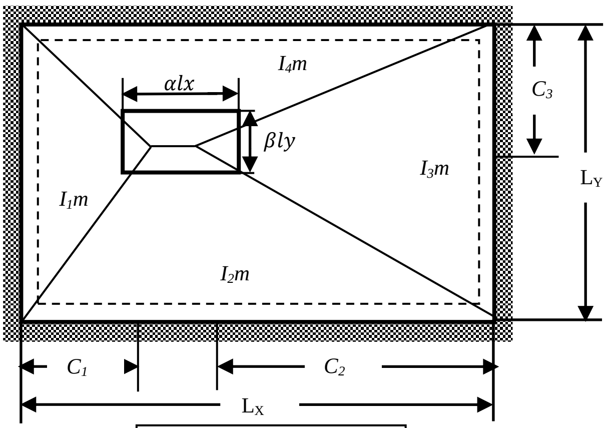

FIG - 3 (PATTERN – 3)

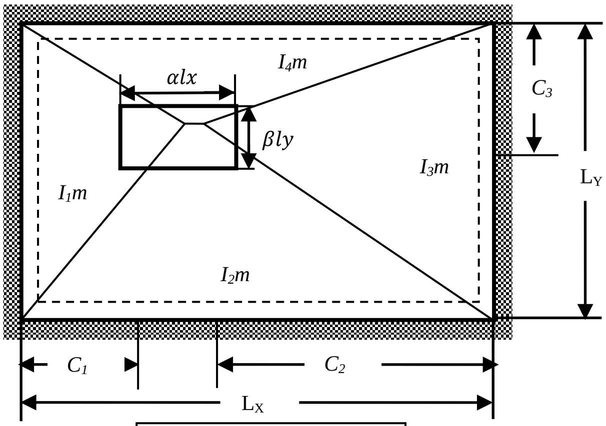

FIG – 5 (PATTERN – 5)

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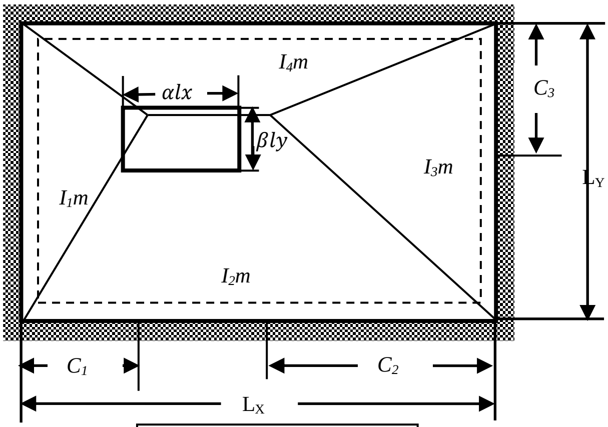

FIG – 6 (PATTERN – 6)

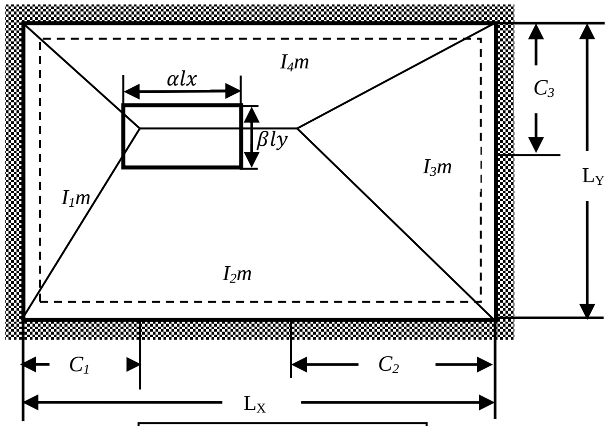

FIG – 9 (PATTERN – 9)

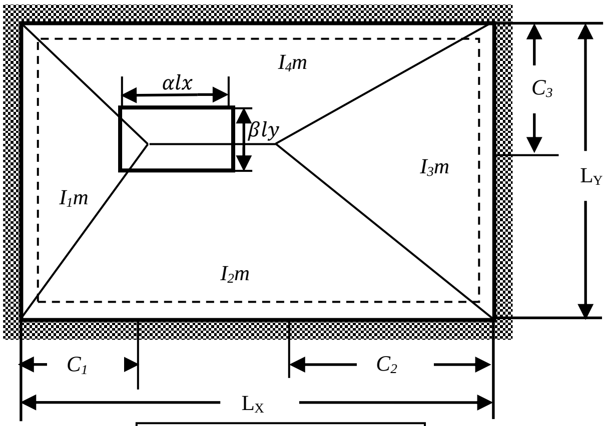

FIG – 7 (PATTERN – 7)

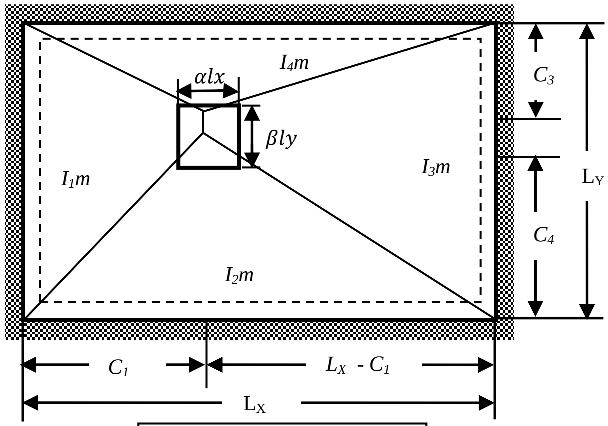

FIG – 10 (PATTERN – 10)

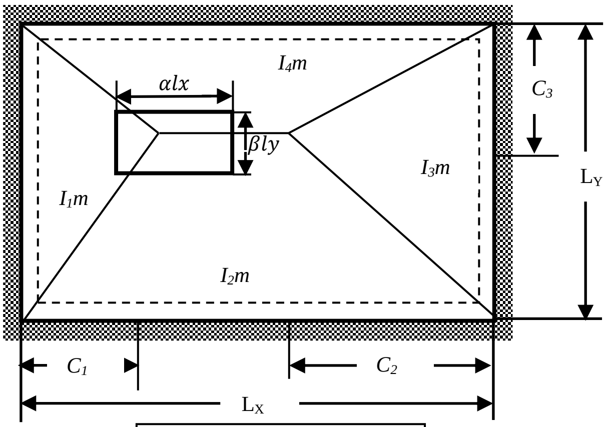

FIG – 8 (PATTERN – 8)

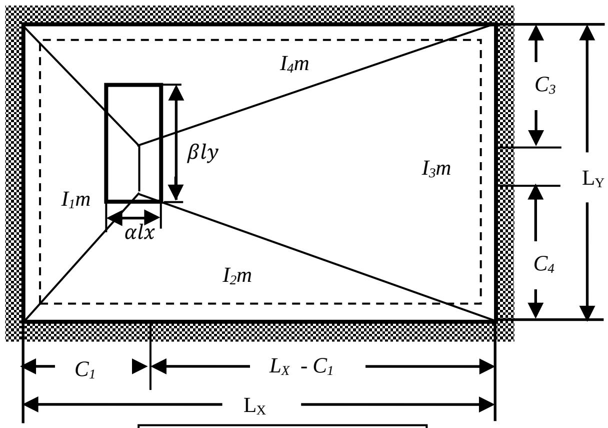

FIG – 11 (PATTERN – 11)

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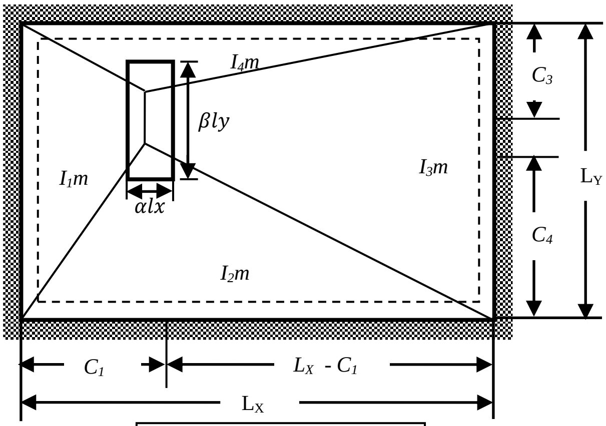

FIG – 12 (PATTERN – 12)

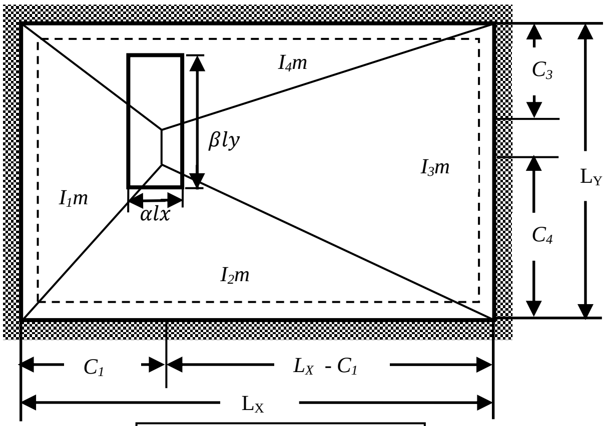

FIG – 13 (PATTERN –13)

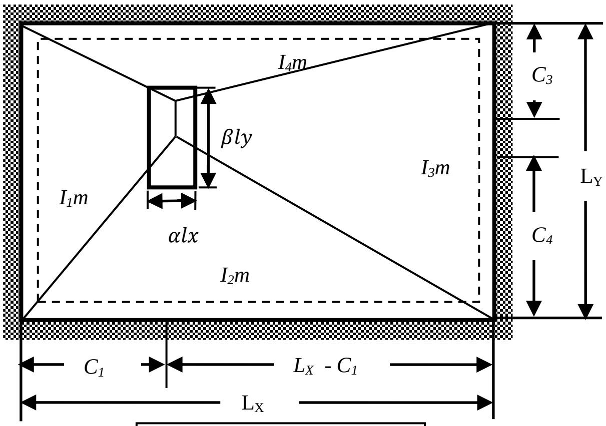

FIG – 14 (PATTERN – 14)

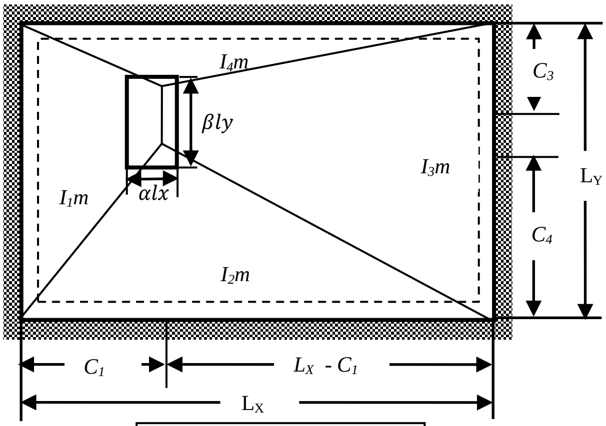

FIG – 15 (PATTERN – 15)

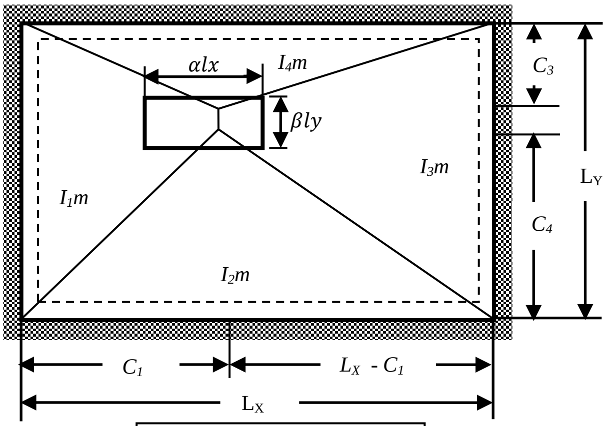

FIG – 16 (PATTERN – 16)

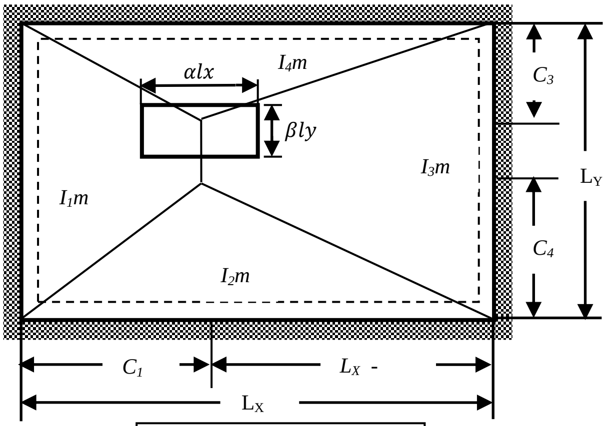

FIG – 17 (PATTERN – 17)

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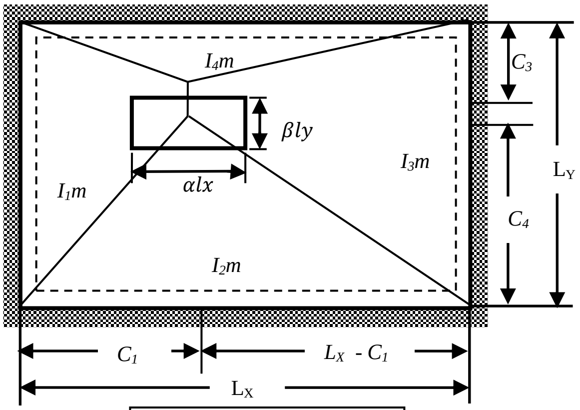

FIG – 18 (PATTERN –

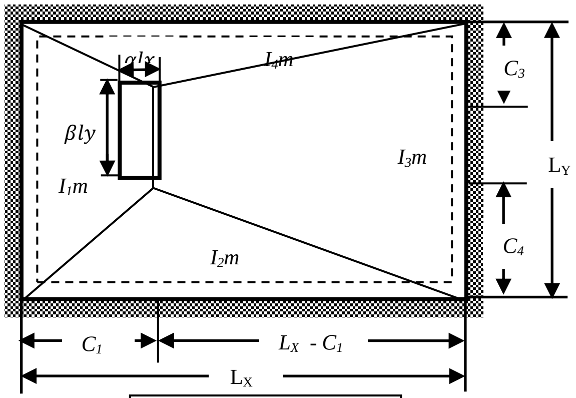

FIG – 20 (PATTERN –

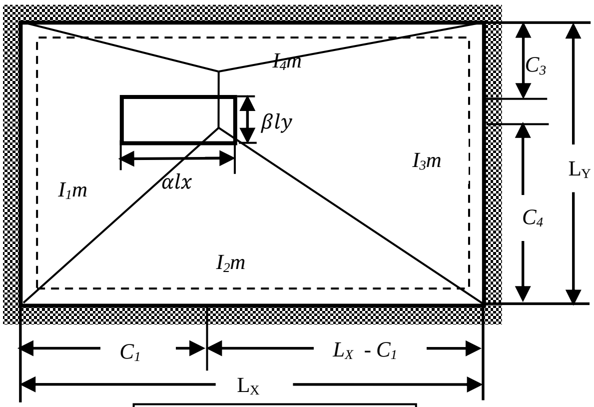

FIG – 19 (PATTERN – 19)

Appendix-B

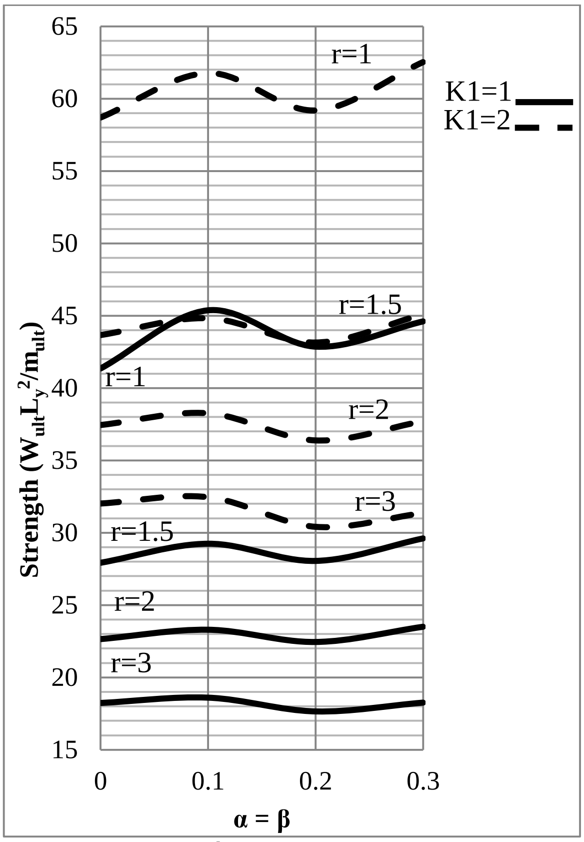

Chart-1: OLD, K2=1

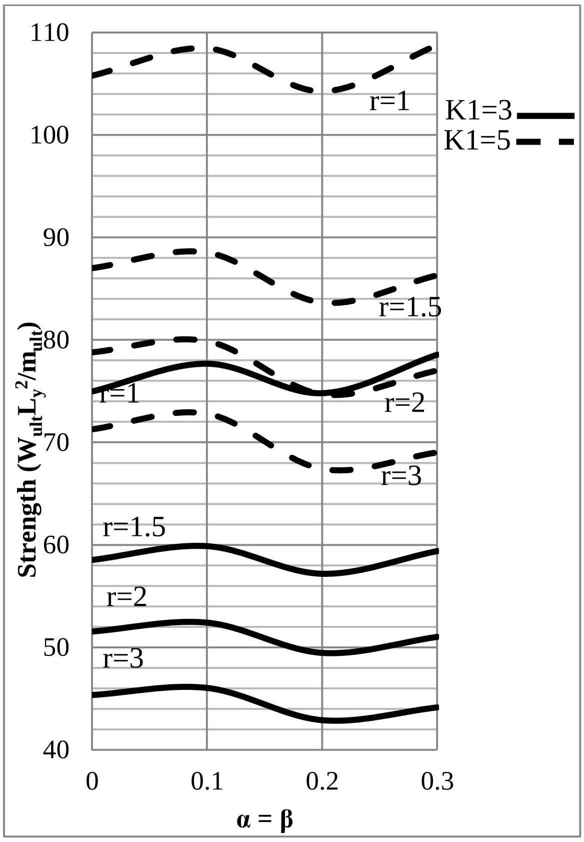

Chart-2: OLD, K2=1

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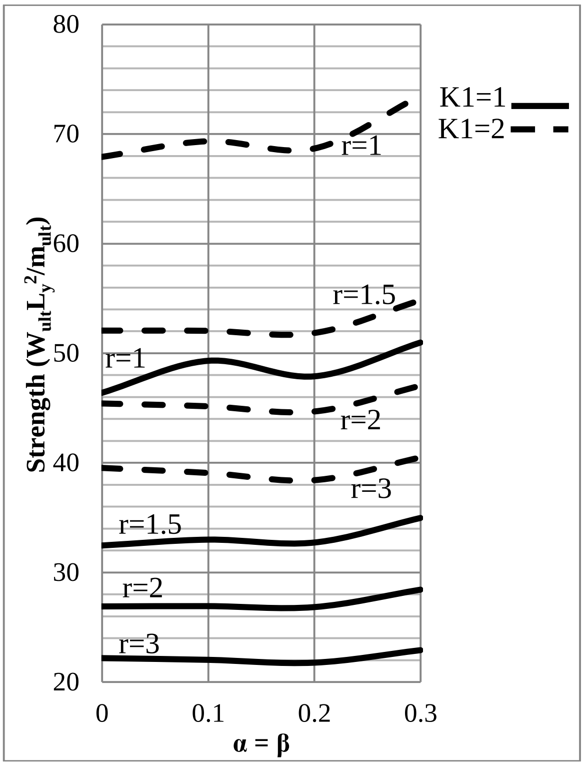

Chart-3: OLD, K2=2

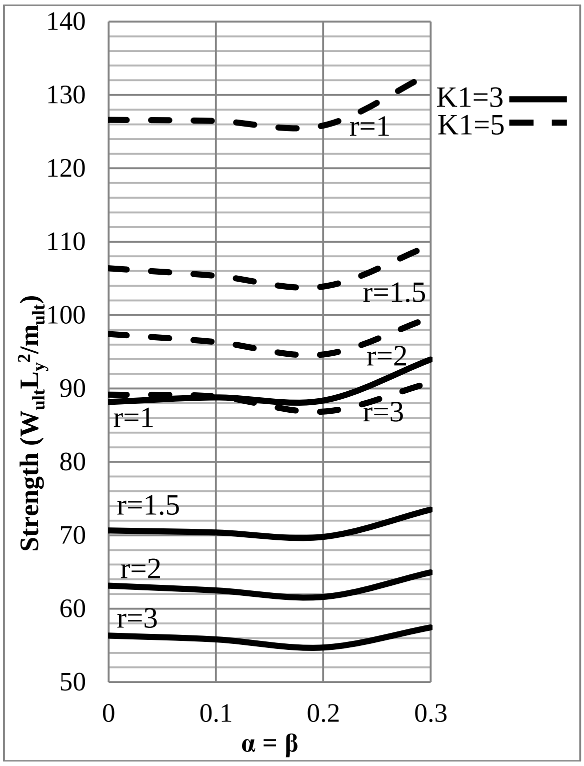

Chart-4: OLD, K2=2

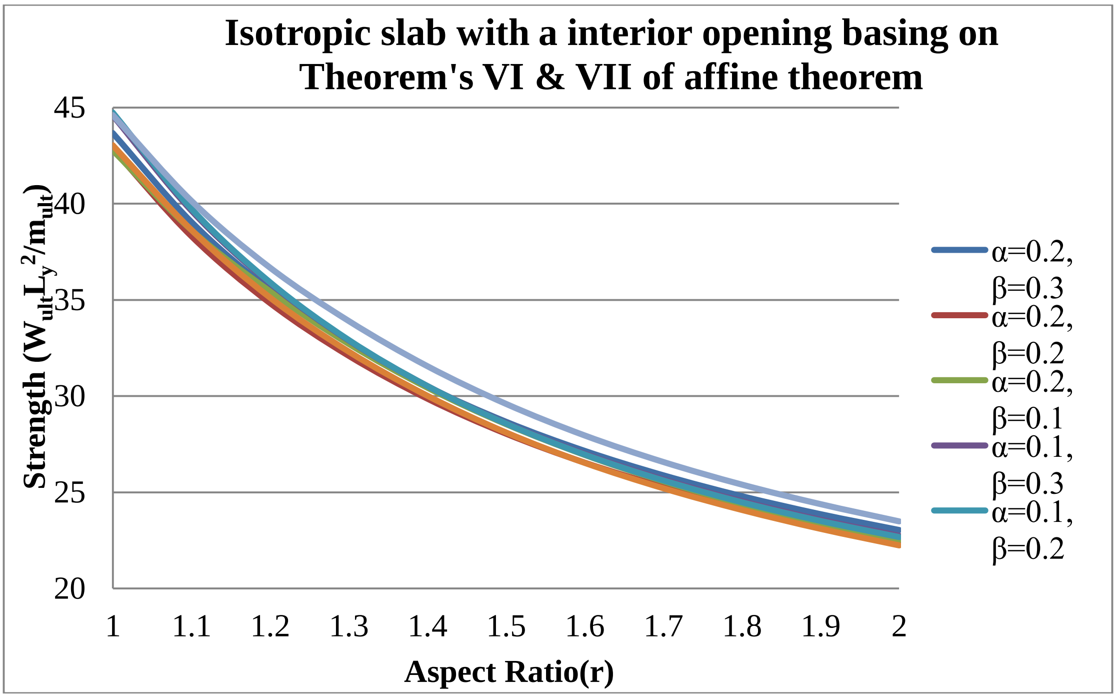

Chart- 5: Strength Vs Aspect Ratio