

# Application of Nonlinear Sliding Mode Disturbance Rejection Technique and Backstepping Controller for PMSM Drives

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**Abstract**—PMSM has been applied to different types of industrial applications as well as robotics because of their high efficiency, low maintenance cost and high power density. However, the PMSM system is laborious to control because of its nonlinear multivariable system. Therefore, it is always necessary to design a high-performance controller which has fast response, high accuracy. The speed controller of PMSM with propeller load is designed using backstepping control and a sliding mode disturbance rejection controller. The Backstepping controller is a systematic and recursive design methodology for nonlinear feedback control. Sliding mode controller is designed for compensation of disturbances. Comparison is done in MATLAB Simulink environment. Finally, it is validated that both the controller shows similar performance in the presence of disturbance.

**Keywords**—Backstepping, permanent magnet synchronous motor, Disturbance rejection

## I. INTRODUCTION

With the accelerated development in permanent magnet material and high efficiency, PMSM has been applied to various propulsion system. There are two main categories of ac motors. These are permanent Magnet synchronous motors and induction motor. PMSM are gradually taking over IMs because of their high efficiency, low maintenance cost and high power density. Yet permanent magnet synchronous motors are difficult to control because of its nonlinear multivariable system. Nonlinear controllers are used for the control of PMSM. These controllers seeks to provide more natural solution to the dynamic performance problems caused to PMSM. The Backstepping controller is a new systematic and recursive design methodology for nonlinear feedback control. Backstepping controller uses a virtual control variable to simplify the original higher order system. Thus final output can be derived systematically based on lyapunov theory. Sliding mode controller is designed for compensation of disturbances. It is a nonlinear controller with remarkable properties such as disturbance rejection, easy implementation and tuning.

## II. MATHEMATICAL MODELING OF PMSM

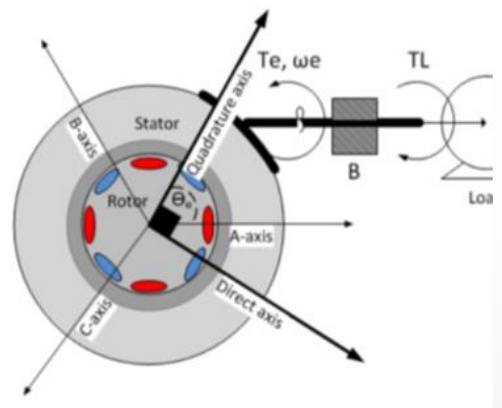


Fig. 1. PMSM drive system

Mathematical model of a surface mounted permanent magnet synchronous motor system in d-q reference frame is shown below:

Voltage equation in d-q reference frame for direct and quadrature axis voltage are:

$$V_q = R_s i_q + \rho \lambda_q + \omega_r \lambda_d \tag{1}$$

$$V_d = R_s i_d + \rho \lambda_d - \omega_r \lambda_q \tag{2}$$

Electromagnetic torque developed:

$$T_e = \frac{3P}{2} (\lambda_d i_q - \lambda_q i_d) \tag{3}$$

The mechanical torque equation is:

$$T_e = B \omega_r + T_L + dt \tag{4}$$

From the above equations dynamic model of pmsm drives system is obtained as:

$$\frac{di_d}{dt} = \frac{V_d}{L_s} - \frac{R_d}{L_s} i_d + \frac{\omega_r \lambda_d}{L_s} i_q \tag{5}$$

$$\frac{di_q}{dt} = \frac{V_q}{L_s} - \frac{R_q}{L_s} i_q + \frac{\omega_r \lambda_q}{L_s} i_q - \frac{\omega_r \phi_m}{L_s} \quad (6)$$

$$\frac{d\omega_r}{dt} = \frac{3P\phi_m}{2J} i_q - \frac{T_L}{J} - \frac{B\omega_r}{J} \quad (7)$$

The system parameters are chosen and it depends on the values of R, Ls, J, B.

$$K_1 = \frac{3P}{2} \phi_m \quad K_2 = \frac{B}{J} \quad K_3 = \frac{1}{J}$$

$$K_4 = \frac{R}{L_s} \quad K_5 = \frac{\phi_m}{L_s} \quad K_6 = \frac{1}{L_s}$$

Then, the PMSM drive system model is rewritten as:

$$\frac{di_d}{dt} = K_6 V_d - K_4 i_d + \omega_r i_q \quad (8)$$

$$\frac{di_q}{dt} = K_6 V_q - K_4 i_q + \omega_r i_d - \omega_r K_5 \quad (9)$$

$$\frac{d\omega_r}{dt} = K_1 i_q - K_2 \omega_r - K_3 T_L \quad (10)$$

### III. BACKSTEPPING CONTROL SCHEME

Nonlinear controllers are capable of providing more accurate solution to the problem possessed by permanent magnet synchronous motor. Backstepping controller is one such controller which is suitable for strict feedback systems. The designer starts the design procedures from a known stable subsystem and back out all other subsystem .when the final external controller is reached the process get terminated. Here an error variable is stabilized by choosing suitable lyapunov function.

Speed tracking error dynamics are defined as:

$$e_\omega = x_1 - x_1^* \quad (11)$$

$$\dot{e}_\omega = \dot{x}_1 - \dot{x}_1^* \quad (12)$$

Lyapunov functions are defines as:

$$V_1 = \frac{1}{2} e_\omega^2 \quad (13)$$

Derivatives of lyapunov function is defined as:

$$\dot{V}_1 = e_\omega \dot{e}_\omega \quad (14)$$

$$\dot{V}_1 = - \dot{e}_\omega (K_1 x_2 - K_2 x_1 - K_3 T_L) \quad (15)$$

The stabilizing functions are chosen in order for tracking error to converge to origin and it is give as:

$$x_2^* = x_2 \text{ des} = \frac{(K_\omega e_\omega - K_2 x_1 - K_3 T_L)}{K_1}$$

$$\dot{x}_3 = x_3 \text{ des} = 0 \quad (16)$$

On equating we get,

$$\dot{V}_1 = -K_\omega e_\omega^2 \quad (17)$$

Speed error converges to origin.

Current Tracking error dynamics is defines as:

$$e_q = x_2 - x_2^* \quad (18)$$

Lyapunov functions are defines as:

$$V_2 = V_1 + \frac{1}{2} e_q^2 \quad (19)$$

Control law is defined:

$$u_1 = \frac{1}{K_6} \left( \frac{(K_\omega e_\omega - K_2 x_1 - K_3 T_L)}{K_1} - K_4 x_2 + x_1 x_3 - K_5 x_1 + K_q e_q \right)$$

Equating control law we get,

$$\dot{V}_2 = -K_\omega e_\omega^2 - K_2 e_q^2 \quad (20)$$

$$\dot{V}_2 < 0$$

Current tracking error dynamics are defined as:

$$e_d = x_3 - x_3^* \quad (21)$$

Lyapunov functions are defined as:

$$V_3 = V_2 + \frac{1}{2} e_d^2 \quad (22)$$

Derivatives of lyapunov functions are:

$$\dot{V}_3 = \dot{V}_2 + e_d \dot{e}_d \quad (23)$$

$$\dot{V}_3 = -K_\omega e_\omega^2 - K_q e_q^2 + e_d(-K_6 u_2 - K_4 i_d + \omega_r i_q + K_d e_d)$$

Providing  $K_a > 0$ , guarantees that  $\dot{V}_1 < 0$  or negative definite

Stabilizing control law is defined as:

$$u_2 = \frac{1}{K_6} (K_4 i_d + \omega_r i_q + K_d e_d) \quad (24)$$

Error signal  $e_q$  will be represented by the sliding surface  $s_2$

On equating:

$$\begin{aligned} \dot{V}_3 &= -K_\omega e_\omega^2 - K_q e_q^2 - K_d e_d^2 \\ \dot{V}_3 &< 0 \end{aligned} \quad (25)$$

Lyapunov functions are defines as:

$$V_2 = \frac{1}{2} s_2^2 \quad (31)$$

Derivatives of lyapunov functions are:

$$\dot{V}_2 = s_2 \dot{s}_2 < 0 \quad \forall s \quad (32)$$

Sliding surface ' $s_2$ ' is of the Form

$$s_2 = \dot{x}_2 - x_2 \quad (33)$$

#### IV. SLIDING MODE CONTROL SCHEME

Nonlinear Sliding mode controller is one with remarkable features such as accuracy, disturbance rejection, easy tuning and implementation etc. control objective is to track reference speed.

Error  $e_\omega$  will be represented by the sliding surface  $s_1$ .

$$\begin{aligned} V_1 &= \frac{1}{2} s_1^2 \\ \dot{V}_1 &= s_1 \dot{s}_1 < 0 \quad \forall s, \end{aligned} \quad (26)$$

Current tracking error dynamics is derived as:

$$\begin{aligned} \dot{s}_2 &= \dot{x}_2 - \dot{x}_2 \\ \dot{s}_2 &= \dot{x}_2 - (K_6 u_1 - K_4 x_2 + x_1 x_3 - K_5 x_1) \\ \dot{x}_2 - (K_6 u_1 - K_4 x_2 + x_1 x_3 - K_5 x_1) &= 0 \end{aligned} \quad (34)$$

The Required sliding surface is of the form:

$$s_1 = \dot{x}_1 - x_1 \quad (27)$$

Here  $u_1 = \frac{1}{K_6} (x_2 - K_4 x_2 + x_1 x_3 - K_5 x_1)$

$$\text{Then, } u_1^* = \frac{1}{K_6} (x_2 - K_4 x_2 + x_1 x_3 - K_5 x_1) + C_2 \text{sgn}(s_2) \quad (35)$$

Speed tracking error dynamics are defined as:

$$\text{Where } \text{sgn}(s_2) = \begin{cases} 1 & s_2 > 0 \\ 0 & s_2 = 0 \\ -1 & s_2 < 0 \end{cases}$$

$$\begin{aligned} \dot{s}_1 &= \dot{x}_1 - \dot{x}_1 \\ \dot{s}_1 &= \dot{x}_1 - (K_1 x_2 - K_2 x_1 - K_3 T_L) \\ \dot{x}_1 - (K_1 x_2 - K_2 x_1 - K_3 T_L) &= 0 \end{aligned} \quad (28)$$

On equating we get,

$$\begin{aligned} \dot{s}_2 &= -K_b \text{sgn}(s_2) \\ \text{Thus,} \\ \dot{V}_2 &= s_2 (-K_b \text{sgn}(s_2)) \end{aligned} \quad (36)$$

$$\text{Then, } I_q^* = \frac{(x_1 - K_2 x_1 - K_3 T_L)}{K_1} + C_1 \text{sgn}(s_1) \quad (29)$$

Providing  $K_b > 0$  in equation (65), guarantees that  $\dot{V}_2 < 0$  or negative definite.

$$\text{Where } \text{sgn}(s_1) = \begin{cases} 1 & s_1 > 0 \\ 0 & s_1 = 0 \\ -1 & s_1 < 0 \end{cases}$$

Error signal  $e_d$  will be represented by the sliding surface  $s_3$ . Taking lyapunov function as:

$$V_3 = \frac{1}{2} s_3^2 \quad (37)$$

On equating we get,

Derivative can be expressed as:

$$\begin{aligned} \dot{s}_1 &= -K_a \text{sgn}(s_1) \\ \dot{V}_1 &= s_1 (-K_a \text{sgn}(s_1)) \end{aligned} \quad (30)$$

$$\dot{V}_3 = s_3 \dot{s}_3 < 0, \quad \forall s, \quad (38)$$

Sliding surface ' $s_3$ ' is of the form

$$\dot{s}_3 = \dot{x}_3 - \dot{x}_3 \tag{39}$$

Current tracking error dynamics is derived as:

$$\begin{aligned} \dot{s}_3 &= \dot{x}_3 - \dot{x}_3 \\ \dot{s}_3 &= \dot{x}_3 - (K_6 u_2 - K_4 i_d + \omega_r i_q) \\ \dot{x}_3 - (K_6 u_2 - K_4 i_d + \omega_r i_q) &= 0 \end{aligned} \tag{40}$$

Stabilizing control law is defined as:

$$u_2^* = \frac{1}{K_6} ( \dot{x}_3 - \frac{1}{4} i_d + \omega_r i_q + K_d e_d ) + C_3 \text{sgn}(s_3)$$

Where  $\text{sign}(s_3) = \begin{cases} 1 & s_3 > 0 \\ 0 & s_3 = 0 \\ -1 & s_3 < 0 \end{cases}$

$$\dot{s}_3 = -K_c \text{sgn}(s_3)$$

$$\dot{V}_3 = s_3 (-K_b \text{sgn}(s_3)) \tag{41}$$

Providing  $K_b > 0$ , guarantees that  $\dot{V}_3 < 0$  or negative definite.

### VI PROPELLER LOAD

Propeller is a type of fan that produce a relative motion by forcing the surrounding fluid backwards by using blades. Pressure difference is formed between the near and farther end of blade. Produces a thrust force which is transmitted to engine of the vehicle.

According to the working principle load torque produced can be:

$$T_L = K_t \rho \omega^2 D_p^3 \tag{42}$$

Where  $\rho$ ,  $D_p$ ,  $\omega$ , are sea water density, speed of propeller and diameter of propeller.

### VI SIMULATION RESULTS AND DISCUSSIONS

To confirm the validity of theoretical analysis and investigate the effectiveness of system with proposed scheme, simulation is carried out in MATLAB Simulink Fig. 3 and Fig. 4 shows the simulation result for system and system with propeller load .The reference speed was set at 1000 rpm and it is evident from the graph that, it shows considerable static error. Fig. 5 and Fig. 6 shows the tracking and regulation of system with backstepping controller .System shows fast response and

accurate tracking. Fig. 7 and Fig. 8 shows the tracking and regulation of system with SMC System shows fast response and accurate tracking. Fig. 2 shows the block diagram of proposed scheme. The desired speed was set at 1000 rpm .In the absence of controller system shows considerable static error. In the presence of controller speed is approximately equal to 1000rpm.

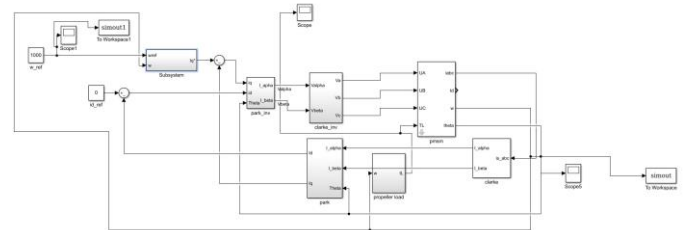


Fig. 2. Block diagram of system with controller

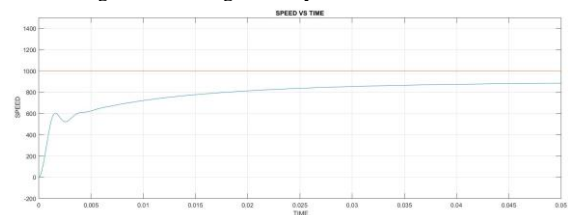


Fig. 3. Simulation result for PMSM system.

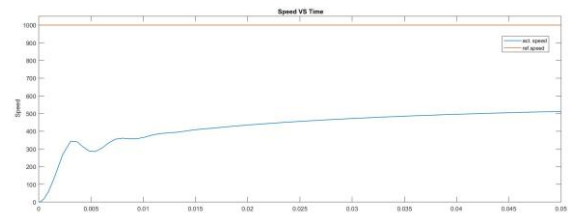


Fig. 4. System with propeller load only.

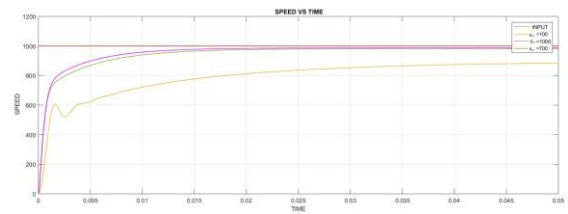


Fig. 5. Tracking curve of system with backstepping.

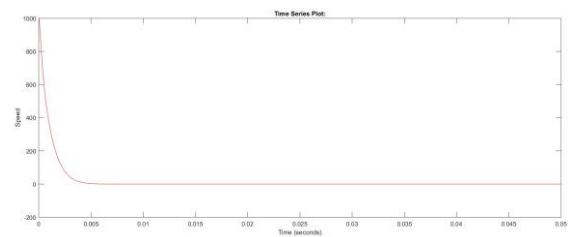


Fig. 6. Speed regulation at 1000 rpm for system with backstepping controller.

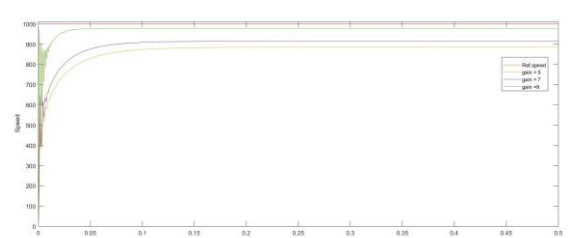


Fig. 7. Tracking curve of system with SMC

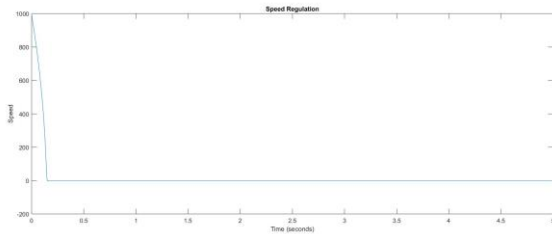


Fig. 8. Speed regulation at 1000 rpm for system with SMC

## VII CONCLUSIONS

Speed control technique for PMSM drives with propeller load is discussed. A nonlinear backstepping controller and disturbance rejection sliding mode controller was proposed. Simulation results confirms the effectiveness of the system with both the controllers. It is validated that both controllers shows similar performance.

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