Application of Nonlinear Sliding Mode Disturbance Rejection Technique and Backstepping Controller for PMSM Drives

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Abstract—PMSM has been applied to different types of industrial applications as well as robotics because of their high efficiency, low maintenance cost and high power density. However, the PMSM system is laborious to control because of its nonlinear multivariable system. Therefore, it is always necessary to design a high-performance controller which has fast response, high accuracy. The speed controller of PMSM with propeller load is designed using backstepping control and a sliding mode disturbance rejection controller. The Backstepping controller is a systematic and recursive design methodology for nonlinear feedback control. Sliding mode controller is designed for compensation of disturbances. Comparison is done in MATLAB Simulink environment. Finally, it is validated that both the controller shows similar performance in the presence of disturbance.

Keywords—Backstepping, permanent magnet synchronous motor, Disturbance rejection

I. INTRODUCTION

With the accelerated development in permanent magnet material and high efficiency, PMSM has been applied to various propulsion system. There are two main categories of ac motors. These are permanent Magnet synchronous motors and induction motor. PMSM are gradually taking over IMs because of their high efficiency, low maintenance cost and high power density. Yet permanent magnet synchronous motors are difficult to control because of its nonlinear multivariable system. Nonlinear controllers are used for the control of PMSM. These controllers seeks to provide more natural solution to the dynamic performance problems caused to PMSM. The Backstepping controller is a new systematic and recursive design methodology for nonlinear feedback control. Backstepping controller uses a virtual control variable to simplify the original higher order system. Thus final output can be derived systematically based on lyapunov theory. Sliding mode controller is designed for compensation of disturbances. It is a nonlinear controller with remarkable properties such as disturbance rejection, easy implementation and tuning.

II. MATHEMATICAL MODELING OF PMSM

Mathematical model of a surface mounted permanent magnet synchronous motor system in d-q reference frame is shown below:

Voltage equation in d-q reference frame for direct and quadrature axis voltage are:

\[ V_d = R_s i_d + \rho \lambda_q + \omega_s \lambda_d \] (1)

\[ V_q = R_s i_q + \rho \lambda_d - \omega_s \lambda_q \] (2)

Electromagnetic torque developed:

\[ T_e = \frac{3B}{2} (\lambda_d i_q - \lambda_q i_d) \] (3)

The mechanical torque equation is:

\[ T_e = B \omega_s + T_L + \frac{J d\omega_s}{dt} \] (4)

From the above equations dynamic model of pmsm drives system is obtained as:

\[ \frac{di_d}{dt} = \frac{V_d}{L_d} - \frac{R_d}{L_d} i_d + \frac{\omega_s L_q}{L_d} i_q \] (5)
\[
\frac{d}{dt} i_3 = \frac{v_3}{L_3} - R_3 i_3 + \frac{\omega_3}{L_3} i_4 - \frac{v_3}{L_3}
\]
\[
\frac{d}{dt} \omega_3 = \frac{3PQ_3}{2J} i_1 - \frac{v_3}{f} \frac{E_{0k}}{f}
\]

The system parameters are chosen and it depends on the values of \( R, L_s, J, B \).

\[
K_s = R \omega_3
\]
\[
K_f = L_s \omega_3
\]
\[
K_b = \frac{1}{L_s}
\]

Then, the PMSM drive system model is rewritten as:

\[
\frac{d}{dt} \omega_3 = K_s V_d - K_4 i_4 + \omega_3 i_4
\]
\[
\frac{d}{dt} i_4 = K_s V_q - K_4 i_4 + \omega_3 i_4 - \omega_4 K_5
\]
\[
\frac{d}{dt} i_5 = K_4 i_4 - K_5 \omega_3 - K_5 T_L
\]

### III. BACKSTEPPING CONTROL SCHEME

Nonlinear controllers are capable of providing more accurate solution to the problem possessed by permanent magnet synchronous motor. Backstepping controller is one such controller which is suitable for strict feedback systems. The designer starts the design procedures from a known stable subsystem and back out all other subsystems when the final external controller is reached the process get terminated. Here an error variable is stabilized by choosing suitable lyapunov function.

Speed tracking error dynamics are defined as:

\[
e_d = x_2 - \dot{x}_2
\]
\[
\dot{e}_d = \dot{x}_2 - \dot{\dot{x}}_2
\]
\[
\ddot{e}_d = K_s e_d
\]

Lyapunov functions are defined as:

\[
V_1 = \frac{1}{2} e_d^2
\]

Derivatives of lyapunov function is defined as:

\[
\dot{V}_1 = e_d \dot{e}_d = \frac{1}{2} \ddot{e}_d^2
\]

Equating control law we get,

\[
V_2 > 0
\]

Current tracking error dynamics is defined as:

\[
\dot{e}_2 = x_2 - \dot{x}_2
\]
\[
\dot{e}_2 = \dot{x}_2 - \dot{\dot{x}}_2
\]
\[
\ddot{e}_2 = (K_6 e_2 - K_5 e_2)
\]

Lyapunov functions are defined as:

\[
V_2 = \frac{1}{2} e_2^2
\]

Control law is defined:

\[
u_1 = \frac{1}{K_6} \left( \frac{K_6 e_2 - K_5 e_2}{K_5} - K_4 x_2 + x_1 x_3 + K_5 x_3 + K_5 e_2 \right)
\]

Equating control law we get,

\[
V_2 > 0
\]

Current tracking error dynamics are defined as:

\[
\dot{e}_d = x_2 - x_2
\]
\[
\dot{e}_2 = x_2 - \dot{x}_2
\]
\[
\ddot{e}_2 = (K_6 e_2 - K_5 e_2)
\]

Lyapunov functions are defined as:

\[
V_3 = \frac{1}{2} \dot{e}_d^2
\]

Derivatives of lyapunov functions are:

\[
\dot{V}_3 = \frac{1}{2} \dot{e}_d^2
\]
\[
\dot{V}_3 = -K_\alpha e_\theta^2 - K_\beta e_{\theta q}^2 + e_d (- (K_\theta u_2 - K_\eta l_2 + \omega_\eta l q + K_\delta e_d))
\]

Stabilizing control law is defined as:
\[
u_2 = \frac{1}{K_\eta} (K_\theta l_2 + \omega_\eta l q + K_\delta e_d)
\]
(24)

On equating:
\[
\dot{V}_3 = -K_\alpha e_\theta^2 - K_\beta e_{\theta q}^2 + K_\delta e_d^2
\]
\[
\dot{V}_3 < 0
\]
(25)

IV. SLIDING MODE CONTROL SCHEME

Nonlinear Sliding mode controller is one with remarkable features such as accuracy, disturbance rejection, easy tuning and implementation etc. control objective is to track reference speed.

Error \( e_{\theta 0} \) will be represented by the sliding surface \( s_1 \).

\[
V_{12} = \frac{1}{2} s_1^2
\]
(26)

The Required sliding surface is of the form:
\[
s_1 = x_1 - x_l
\]
(27)

Speed tracking error dynamics are defined as:
\[
\dot{s}_1^2 = x_1^2 - \dot{x}_l^2
\]
\[
\dot{s}_1^2 = x_1^2 - (K_\alpha x_2 - K_\beta x_1 - K_\eta T_1 l)
\]
\[
\dot{x}_1 = (K_\alpha x_2 - K_\beta x_1 - K_\eta T_1 l) = 0
\]
(28)

Then,
\[
I_q = \frac{(x_1 - K_\delta x_2 - K_\eta T_1 l)}{K_\delta} + C_1 sgn (s_1)
\]
Where \( sgn (s_1) = \begin{cases} 1 & s_1 > 0 \\ 0 & s_1 = 0 \\ -1 & s_1 < 0 \end{cases} \)
(29)

On equating we get,
\[
\dot{s}_1 = -K_\alpha sgn (s_1)
\]
\[
\dot{V}_1 = s (-K_\delta sgn (s_1))
\]
(30)

Providing \( K_\alpha > 0 \), guarantees that \( \dot{V}_1 < 0 \) or negative definite

Error signal \( e_d \) will be represented by the sliding surface \( s_2 \).

Lyapunov functions are defines as:
\[
V_{22} = \frac{1}{2} s_2^2
\]
(31)

Derivatives of lyapunov functions are:
\[
\dot{V}_2 = s_2 \dot{s}_2 < 0 \forall s
\]
(32)

Sliding surface \( s_2 \) is of the Form
\[
s_2 = x_2 - x_l
\]
(33)

Current tracking error dynamics is derived as:
\[
\dot{x}_2 = x_2 - x_2
\]
\[
\dot{s}_2 = x_2 - (K_\delta x_2 + x_1 x_3 + K_\eta x_3)
\]
(34)

Here \( u_1 = \frac{1}{K_\delta} (x_2 - K_\delta x_2 + x_1 x_3 - K_\eta x_3) \)

Then, \( \dot{u}_1 = \frac{1}{K_\delta} (x_2 - K_\delta x_2 + x_1 x_3 - K_\eta x_3) + C_2 sgn (s_2) \)
(35)

Where \( sgn (s_2) = \begin{cases} 1 & s_2 > 0 \\ 0 & s_2 = 0 \\ -1 & s_2 < 0 \end{cases} \)

On equating we get,
\[
\dot{u}_1 = -K_\delta sgn (s_2)
\]

Thus,
\[
V_{22} = \frac{1}{2} (K_\delta sgn (s_2))^2
\]
(36)

Providing \( K_\delta > 0 \) in equation (65), guarantees that \( \dot{V}_2 < 0 \) or negative definite.

Error signal \( e_d \) will be represented by the sliding surface \( s_3 \).

Taking lyapunov function as:
\[
V_{33} = \frac{1}{2} s_3^2
\]
(37)

Derivative can be expressed as:
\[
\dot{V}_3 = s_3 \dot{s}_3 < 0, \forall s
\]
(38)

Sliding surface \( s_3 \) is of the form
Current tracking error dynamics is derived as:

\[ s_3 = x_3 - \dot{x}_3 \]

(39)

\[ \dot{s}_3 = x_3 - \dot{x}_3 \]

(39)

\[ \dot{s}_3 = x_3 - (K_q u_2 - K_q i_d + \omega_r i_q) \]

\[ x_3 - (K_q u_2 - K_q i_d + \omega_r i_q) = 0 \]

(40)

Stabilizing control law is defined as:

\[ u_d = \frac{1}{K_q} (x_3 - \dot{x}_3 i_d + \omega_r i_q + K_d e_d) + C_2 \text{sgn} (s_3) \]

Where \( \text{sgn}(s_3) = \begin{cases} 
1 & s_3 > 0 \\
0 & s_3 = 0 \\
-1 & s_3 < 0 
\end{cases} \)

\[ \dot{s}_3 = K_c \text{sgn} (s_3) \]

\[ \dot{V}_3 = s_3 (K_c \text{sgn} (s_3)) \]

(41)

Providing \( K_c > 0 \), guarantees that \( \dot{V}_3 < 0 \) or negative definite.

V PROPELLER LOAD

Propeller is a type of fan that produce a relative motion by forcing the surrounding fluid backwards by using blades. Pressure difference is formed between the near and farther end of blade. Produces a thrust force which is transmitted to engine of the vehicle.

According to the working principle load torque produced can be:

\[ T_L = K_c \rho \omega^2 D_p^3 \]

(42)

Where \( \rho \), \( D_p \), \( \omega \), are sea water density, speed of propeller and diameter of propeller.

VI SIMULATION RESULTS AND DISCUSSIONS

To confirm the validity of theoretical analysis and investigate the effectiveness of system with proposed scheme, simulation is carried out in MATLAB Simulink Fig. 3 and Fig. 4 shows the simulation result for system and system with propeller load. The reference speed was set at 1000 rpm and it is evident from the graph that, it shows considerable static error. Fig. 5 and Fig. 6 shows the tracking and regulation of system with backstepping controller. System shows fast response and accurate tracking. Fig. 7 and Fig. 8 shows the tracking and regulation of system with SMC. System shows fast response and accurate tracking. The desired speed was set at 1000 rpm. In the absence of controller system shows considerable static error. In the presence of controller system shows considerable static error.
II CONCLUSIONS

Speed control technique for PMSM drives with propeller load is discussed. A nonlinear backstepping controller and disturbance rejection sliding mode controller was proposed. Simulation results confirms the effectiveness of the system with both the controllers. It is validated that both controllers shows similar performance.

REFERENCES


