

Application of Linear Programming to Profit Maximization.

Michael Kwofie

Northern Arizona University

Godfred Akwasi Afrifa

Ohio University

Abstract

This study applies linear programming to determine the profit-maximizing product mix for Seth Kojo's (S.K.) Bakery in Ghana. A mathematical model is formulated to allocate limited raw materials among four products: banana, coconut, wheat, and butter bread, subject to ingredient availability constraints. Using the linprog function in MATLAB, the model identifies the optimal monthly production levels and corresponding maximum profit. The results show that wheat and butter bread dominate the optimal solution, while coconut bread is excluded due to its inefficient resource-profit trade-off. The optimal mix yields a maximum monthly profit of GH¢407,350. Sensitivity analysis further reveals that butter- and flour-related constraints are binding, indicating that expanding these resources would generate the greatest marginal gains. The findings demonstrate the value of linear programming as a decision-support tool for small-scale enterprises and highlight how quantitative optimization can improve production planning, resource utilization, and profitability in bakery operations.

1 Introduction

In a competitive business environment, efficient resource allocation is essential for maximizing profitability and ensuring sustainability. Many small and medium-scale enterprises (SMEs), particularly in developing economies, often rely on intuitive or trial-and-error methods for

production planning. Such approaches may lead to suboptimal decisions, inefficient use of resources, and reduced profitability. Consequently, there is a growing need for the adoption of quantitative techniques that support informed and optimal decision-making.

Linear programming (LP) is a widely used optimization technique in operations research that enables decision-makers to determine the best possible outcome, such as maximum profit or minimum cost, subject to a set of linear constraints. Since the development of the simplex algorithm by George Dantzig in 1947, linear programming has been successfully applied in various fields, including manufacturing, transportation, agriculture, and energy management. Its strength lies in its ability to handle multiple constraints while providing optimal solutions in a systematic and efficient manner.

In the context of production planning, linear programming is particularly useful for determining the optimal product mix when resources are limited. By formulating the problem as a mathematical model, businesses can identify which products to produce and in what quantities in order to maximize profit. This is especially relevant for bakeries and similar production-oriented enterprises, where multiple products compete for shared raw materials. This study focuses on the application of linear programming to maximize profit in Seth Kojo's (S.K) Bakery. The bakery produces four types of bread: banana bread, coconut bread, wheat bread, and butter bread using a fixed quantity of raw materials each month. The central problem addressed in this research is how to efficiently allocate these limited resources across the different products in order to achieve maximum profit.

The main objective of this study is to develop and solve a linear programming model that determines the optimal production levels for each type of bread. Specifically, the study aims to

- (i) formulate an objective function that represents total profit.
- (ii) incorporate resource constraints based on available raw materials.
- (iii) compute the optimal solution using the simplex algorithm.

By doing so, the study provides practical insights into how mathematical optimization techniques can enhance decision making in real-world business environments.

The findings of this research are expected to contribute to both academic and practical domains. Academically, the study demonstrates the application of linear programming in a real-life context. Practically, it offers a decision-support framework that can assist business managers in improving production efficiency and profitability.

2 Literature Review

Linear programming (LP) has been widely recognized as one of the most powerful optimization techniques in operations research, with extensive applications across various sectors including manufacturing, agriculture, transportation, and energy management. Its primary strength lies in its ability to model complex decision-making problems involving multiple constraints and limited resources while providing optimal solutions in a systematic and computationally efficient manner. Several studies have demonstrated the applicability of linear programming in production planning and profit maximization. For instance, [1] highlighted the importance of optimization models in production planning, emphasizing that incorporating uncertainty into linear programming models can significantly improve decision quality. Similarly, [2] applied mixed-integer linear programming to aggregate production planning, showing how firms can determine optimal workforce levels and production quantities under varying constraints.

According to [3], linear programming is a linear algebra generalization used in modeling many real-life problems ranging from scheduling airline routes to shipping oil from refineries to cities. The author argued that the great versatility of linear programming lies in the ease with which constraints can be incorporated into optimization models. [2] further reported that mixed integer linear programming plays an important role in aggregate production planning, particularly in determining workforce levels and optimal product mixes. In addition, [4] described linear programming as a revolutionary development that enables organizations to set goals and make decisions directed toward maximizing profit. Similarly, [5] observed that linear programming contributes significantly to improving management decisions in areas such as production planning, resource allocation, inventory control, and advertisement.

In the context of business profit maximization, linear programming has proven to be an

effective decision-support tool. [6] utilized linear programming to optimize production in a beverage company, demonstrating that an optimal product mix can substantially increase profitability. Likewise, [7] applied LP techniques to a manufacturing firm and identified optimal production quantities that maximized monthly profit. These studies reinforce the practical relevance of LP in guiding managerial decisions regarding resource allocation and production strategies.

According to [1], production planning problems represent one of the most important applications of optimization tools using linear programming. They emphasized that incorporating uncertainty into LP models is essential in avoiding inferior planning decisions, a concept closely related to sensitivity analysis. [8] recognized LP as an important tool in energy management despite the nonlinear nature of many energy systems, arguing that nonlinearities can often be transformed into linear forms using Taylor series approximations. Furthermore, [9] reported that business success depends greatly on effective decision-making and applied LP techniques to optimize the use of resources for staff training.

Applications of linear programming are also prominent in agriculture and small-scale enterprise management. [10] employed LP to maximize returns in agricultural production systems, while [11] developed a model for optimizing crop allocation among rural farmers. In a similar vein, [12] applied linear programming to a local soap production company and demonstrated how optimal resource allocation could improve profitability. These studies indicate that LP is not only applicable to large-scale industries but is also highly relevant for small and medium-scale enterprises.

[10] further reported that linear programming is an effective technique for agricultural production planning and gross return maximization. [6] used LP to determine the optimal production process for the Coca-Cola Company by considering constraints such as sugar content, water volume, and carbon dioxide concentration, concluding that concentrating on certain products yielded maximum profit. [7] applied LP to optimize profit in the Golden Plastic Industry and identified the most profitable pipe sizes for production. [13] also demonstrated the application of LP models to product-mix profit maximization using an R-statistical package. In addition, [14] highlighted optimization as a crucial science in high-performance refineries where maximizing profitability is a key objective. [11] formulated an LP model to

maximize the income of rural farmers, while [12] applied LP models to maximize profit in a local soap production company and recommended increased production of white soap due to its higher profitability.

Despite the widespread use of linear programming, many existing studies primarily focus on obtaining optimal solutions without adequately exploring the broader implications of these solutions. Several works rely on deterministic models that assume fixed input parameters, thereby ignoring real-world uncertainties such as fluctuations in raw material availability, market demand, and production costs. Additionally, many studies emphasize methodological application without providing sufficient interpretation of results in a business context, limiting their practical usefulness for decision-makers.

Furthermore, while prior research has extensively applied linear programming to manufacturing and agricultural systems, relatively fewer studies have focused on small-scale bakery operations, particularly in developing economies. Such enterprises often operate under significant resource constraints and informal decision-making processes, making them ideal candidates for the application of optimization techniques. However, there remains a gap in the literature regarding the integration of linear programming models into the decision-making frameworks of these businesses.

This study seeks to address these gaps by applying linear programming to a real-world bakery production problem. Unlike many previous studies that emphasize theoretical applications, this research focuses on a practical case study and provides a detailed interpretation of the results in the context of business decision-making. By doing so, it contributes to the growing body of literature on the application of optimization techniques in small and medium-scale enterprises and demonstrates how linear programming can be effectively utilized to enhance profitability and operational efficiency.

In conclusion, the literature demonstrates that linear programming is a powerful tool for profit maximization across various industries and sectors. Its ability to optimize complex decision-making processes while considering multiple constraints makes it a valuable technique for organizations seeking to improve operational efficiency and profitability.

3 Methodology

This section presents the mathematical formulation of the linear programming model used to determine the optimal production strategy for Seth Kojo's (S.K) Bakery. The methodology involves defining decision variables, constructing the objective function, specifying constraints based on available resources, and solving the model using the simplex algorithm.

3.1 Model Formulation

The problem is formulated as a linear programming model in which the goal is to maximize total profit subject to resource constraints. The formulation consists of decision variables, an objective function, and a set of linear constraints.

3.1.1 Decision Variables

Let

$$x_1 = \text{number of loaves of banana bread produced per month,} \quad (1)$$

$$x_2 = \text{number of loaves of coconut bread produced per month,} \quad (2)$$

$$x_3 = \text{number of loaves of wheat bread produced per month,} \quad (3)$$

$$x_4 = \text{number of loaves of butter bread produced per month.} \quad (4)$$

3.1.2 Objective Function

The objective is to maximize total monthly profit generated from the production of the four types of bread. Based on the profit contribution per unit of each product, the objective function is given by:

$$\text{Maximize } Z = 200x_1 + 125x_2 + 350x_3 + 560x_4. \quad (5)$$

3.1.3 Constraints

The production process is subject to limitations in the availability of raw materials. Each type of bread requires specific quantities of ingredients, and the total consumption must not exceed the available supply. The constraints are expressed as follows:

Flour Constraint

$$400x_1 + 250x_2 + 400x_3 + 450x_4 \leq 428600. \quad (6)$$

Milk Constraint

$$30x_1 + 40x_2 + 5x_3 + 10x_4 \leq 9595. \quad (7)$$

Butter Constraint

$$20x_1 + 15x_2 + 10x_3 + 70x_4 \leq 29510. \quad (8)$$

Flavour Constraint

$$70x_1 + 40x_2 + 90x_3 + 10x_4 \leq 66720. \quad (9)$$

Vegetable Oil Constraint

$$10x_1 + 30x_2 + 20x_3 + 15x_4 \leq 18010. \quad (10)$$

Baking Powder Constraint

$$5x_1 + 35x_2 + 20x_3 + 10x_4 \leq 15925. \quad (11)$$

Salt Constraint

$$20x_1 + 25x_2 + 10x_3 + 15x_4 \leq 13010. \quad (12)$$

Sugar Constraint

$$30x_1 + 10x_2 + 20x_3 + 5x_4 \leq 17350. \quad (13)$$

Yeast Constraint

$$25x_1 + 20x_2 + 10x_3 + 50x_4 \leq 24095. \quad (14)$$

Water Constraint

$$70x_1 + 90x_2 + 30x_3 + 50x_4 \leq 41700. \quad (15)$$

Egg Constraint

$$20x_1 + 30x_2 + 40x_3 + 10x_4 \leq 30020. \quad (16)$$

3.1.4 Non-negativity Constraints

All decision variables must be non-negative:

$$x_1, x_2, x_3, x_4 \geq 0. \quad (17)$$

Table 1: Resource Requirements and Availability

Resource	Banana(x_1)	Coconut(x_2)	Wheat(x_3)	Butter(x_4)	Availability(g)
Flour	400	250	400	450	428,600
Milk	30	40	5	10	9,595
Butter	20	15	10	70	29,510
Flavour	70	40	90	10	66,720
Vegetable Oil	10	30	20	15	18,010
Baking Powder	5	35	20	10	15,925
Salt	20	25	10	15	13,010
Sugar	30	10	20	5	17,350
Yeast	25	20	10	50	24,095
Water	70	90	30	50	41,700
Egg	20	30	40	10	30,020

3.2 Solution Method

The formulated linear programming model is solved using the simplex algorithm, a well-established iterative procedure for determining the optimal solution to linear optimization problems. Due to the presence of multiple decision variables and constraints, the simplex method is more appropriate than graphical approaches.

In this study, the model is implemented and solved using the linprog function in MATLAB, which provides an efficient computational framework for linear optimization. The linprog function applies the simplex algorithm to evaluate feasible solutions and identify the optimal production levels that maximize the objective function while satisfying all constraints.

3.3 Model Assumptions

The formulation of the linear programming model is based on the following assumptions:

- Linearity: Both the objective function and constraints are linear in terms of the decision variables.
- Divisibility: The decision variables can take any non-negative real values, implying that production quantities are continuous.
- Certainty: All coefficients in the model, including profit contributions and resource availability, are known with certainty and remain constant during the planning period.

- Non-negativity: Negative production levels are not feasible.

4 Results and Discussion

4.1 Optimal Solution

The model is solved using the `linprog` function in MATLAB, which implements the simplex algorithm for linear optimization problems.

The optimal production quantities obtained are as follows:

$$x_1 = 117 \quad (\text{banana bread}), \quad (18)$$

$$x_2 = 0 \quad (\text{coconut bread}), \quad (19)$$

$$x_3 = 617 \quad (\text{wheat bread}), \quad (20)$$

$$x_4 = 300 \quad (\text{butter bread}). \quad (21)$$

The corresponding maximum monthly profit is:

$$Z = 407,350. \quad (22)$$

4.2 Interpretation of Results

The results indicate that wheat bread (x_3) and butter bread (x_4) constitute the most significant portion of the optimal production mix. In particular, wheat bread has the highest production quantity, suggesting that it provides a strong balance between resource consumption and profit contribution.

Butter bread, despite its relatively high consumption of certain resources such as butter, is also produced in substantial quantities due to its high unit profit. This highlights its importance as a key driver of profitability for the bakery.

Table 2: Profit Contribution per Product

Product	Unit Profit (GH¢)	Quantity	Total Profit
Banana Bread	200	117	23,400
Coconut Bread	125	0	0
Wheat Bread	350	617	215,950
Butter Bread	560	300	168,000
Total			407,350

On the other hand, coconut bread (x_2) is not included in the optimal solution. This implies that its production is not economically viable under the current resource constraints and profit structure. In other words, the resources required to produce coconut bread can be more profitably allocated to other products.

Banana bread (x_1) is produced in moderate quantities, indicating that while it contributes positively to profit, it is less efficient compared to wheat and butter bread in terms of resource utilization.

4.3 Resource Utilization and Constraints

The optimal solution reflects the efficient allocation of limited resources. Although detailed slack values are not explicitly presented, it can be inferred that some constraints are binding, meaning that the available quantities of certain resources are fully utilized in the optimal solution.

Resources that are heavily consumed in the production of wheat and butter bread, such as butter, flavour, and baking powder, are binding constraints. These resources therefore play a critical role in limiting further increases in production and profit.

Non-binding constraints, on the other hand, represent resources that are not fully utilized. These unused resources suggest potential inefficiencies or opportunities for increasing production if other limiting factors are addressed.

4.4 Managerial Implications

The findings of this study provide important insights for decision-making at Seth Kojo's Bakery. First, management should prioritize the production of wheat and butter bread, as

these products contribute most significantly to overall profitability.

Second, the complete exclusion of coconut bread from the optimal solution suggests that the bakery should reconsider its production strategy for this product. This may involve reducing its production, improving its efficiency, or adjusting its pricing strategy to enhance profitability.

Furthermore, the identification of binding constraints highlights the need for strategic investment in critical resources. For example, increasing the availability of key inputs such as flour or butter could enable the bakery to expand production and achieve higher profit levels.

Overall, the results demonstrate that the application of linear programming provides a systematic and effective approach to optimizing production decisions and improving operational performance in small-scale enterprises.

5 Sensitivity Analysis

Sensitivity analysis evaluates how changes in resource availability and profit coefficients affect the optimal solution. This section presents shadow prices, reduced costs, binding constraints, and managerial implications based on the MATLAB output.

5.1 Binding and Non-Binding Constraints

A constraint is *binding* if its left-hand side equals its right-hand side at optimality, meaning the resource is fully utilized. A *non-binding* constraint has slack, indicating unused capacity. Based on the MATLAB solution, the following constraints are binding:

- Constraint 3 (Butter)
- Constraint 4 (Flavour)
- Constraint 6 (Baking Powder)

These constraints have zero slack and directly restrict further increases in production. All other constraints exhibit positive slack and are therefore non-binding.

5.2 Shadow Prices

Shadow prices represent the marginal increase in profit resulting from a one-unit increase in the availability of a resource.

The MATLAB output indicates:

- **Constraint 6 (Baking Powder):** Shadow price ≈ 13.7973 (most critical resource).
- **Constraint 3 (Butter):** Shadow price ≈ 6.0068 (major bottleneck).
- **Constraint 4 (Flavour):** Shadow price ≈ 0.1554 (weakly binding).

All other constraints have shadow prices of zero, confirming they are non-binding.

Table 3: Shadow Prices for Resource Constraints

Constraint	Shadow Price	Interpretation
1	0	Not binding (no impact on profit)
2	0	Not binding (no impact on profit)
3	6.0068	Highly binding (profit increases significantly)
4	0.1554	Weakly binding (small impact on profit)
5	0	Not binding
6	13.7973	Most critical constraint (highest impact)
7	0	Not binding
8	0	Not binding
9	0	Not binding
10	0	Not binding
11	0	Not binding

5.3 Reduced Costs

Reduced costs indicate how much the objective coefficient of a non-produced variable must improve before it becomes profitable to include in the solution.

The reduced cost of coconut bread confirms that it is excluded from the optimal solution because its current profit contribution (GH¢125) is insufficient relative to its resource consumption. To enter the optimal basis, its unit profit must increase or its ingredient usage must decrease.

5.4 Allowable Ranges

For binding constraints (3, 4, and 6), shadow prices remain valid within the allowable range of the right-hand side. For non-binding constraints, allowable increases are effectively unbounded until the constraint becomes binding.

Thus:

- Increasing flour, milk, sugar, yeast, or water will not affect profit unless the increase is large enough to make these constraints binding.
- Increasing butter, flavour, or baking powder will immediately increase profit.

5.5 Managerial Implications

- Invest in baking powder and butter first, as these yield the highest marginal returns.
- Flavour is a weak bottleneck but still worth expanding if inexpensive.
- Coconut bread should not be produced unless its profit margin improves or its recipe is reformulated.
- Non-binding resources represent slack, indicating potential over-purchasing relative to optimal production needs.

5.6 Summary

The sensitivity analysis confirms that the optimal solution is stable and identifies the key resources limiting profitability. These insights strengthen the practical relevance of the model and guide strategic investment decisions for the bakery.

The results indicate that Constraints 3, 4, and 6 are binding and directly influence the optimal solution. In particular, Constraint 6 has the highest shadow price, making it the most critical resource. Increasing its availability would yield the greatest increase in profit. All other constraints have zero shadow prices, indicating that they are non-binding and do not limit production under the current solution.

6 Conclusion

This study applied linear programming techniques to determine the optimal production strategy for Seth Kojo's (S.K) Bakery under multiple resource constraints. The objective was to maximize profit by identifying the most efficient combination of products given the available inputs.

The results revealed that the optimal production mix consists of 117 units of banana bread, 617 units of wheat bread, and 300 units of butter bread, while coconut bread is excluded from production. This combination yields a maximum monthly profit of 407,350, demonstrating the effectiveness of linear programming as a decision-support tool.

The findings further indicate that wheat and butter bread contribute most significantly to profitability, while coconut bread is not economically viable under current conditions. This highlights the importance of efficient resource allocation and product prioritization in maximizing returns.

Overall, the study demonstrates that linear programming provides a systematic and reliable framework for optimizing production decisions in small-scale enterprises. Its application enables managers to make informed decisions, improve operational efficiency, and enhance profitability.

7 Recommendations

Based on the findings of this study, the following recommendations are proposed.

- The bakery should prioritize the production of wheat and butter bread, as these products contribute significantly to overall profit.
- Management should reconsider the production of coconut bread. This may involve reducing its cost of production, improving its efficiency, or adjusting its pricing strategy to make it more competitive.
- The bakery should invest in increasing the availability of critical resources such as flour and butter, as these are likely limiting factors in production.

- The use of optimization tools such as linear programming should be integrated into routine decision-making processes to enhance planning and resource allocation.
- Future studies should incorporate uncertainty factors such as fluctuating demand and variable input costs to develop more robust and realistic models.

8. Contribution of the Study

This study makes several contributions to the literature on production optimization and the application of linear programming in small and medium-scale enterprises:

1. **Real-world, data-driven optimization model.** Unlike many LP studies that rely on hypothetical datasets, this work uses actual ingredient usage and production information from Seth Kojo's Bakery. This provides an empirically grounded demonstration of how optimization can support decision-making in resource-constrained environments.
2. **Integration of LP with sensitivity analysis.** Beyond identifying an optimal product mix, the study incorporates shadow prices, reduced costs, and binding-constraint identification. This level of interpretive depth is rarely included in SME-focused LP studies.
3. **Managerial insights tailored to small-scale food production.** The results translate mathematical outputs into actionable recommendations, such as prioritizing wheat and butter bread, reconsidering coconut bread, and investing in critical inputs like flour and butter.
4. **Advancing quantitative decision-support for SMEs in developing economies.** The study highlights how small enterprises often operating with informal planning methods can benefit from structured optimization techniques. This contributes to the growing literature advocating quantitative tools for improving efficiency and profitability in developing-country contexts.

Together, these contributions position the study as both academically relevant and practically impactful, offering a replicable framework for similar enterprises seeking to optimize production under resource constraints.

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A MATLAB Code

The following MATLAB code was used to solve the linear programming model:

```
% Objective function
f = [-200; -125; -350; -560];

% Constraint matrix
A = [
400 250 400 450;
30 40 5 10;
20 15 10 70;
70 40 90 10;
10 30 20 15;
5 35 20 10;
20 25 10 15;
30 10 20 5;
25 20 10 50;
70 90 30 50;
20 30 40 10
];

% RHS
b = [
428600;
9595;
29510;
66720;
18010;
15925;
13010;
```

```
17350;
```

```
24095;
```

```
41700;
```

```
30020
```

```
];
```

```
lb = [0; 0; 0; 0];
```

```
[x, fval, exitflag, output, lambda] = linprog(f, A, b, [], [], lb);
```

```
x
```

```
-fval
```

```
lambda.ineqlin
```

```
...
```