

Application of Linear Programming Model for Production Planning in an Engineering Industry-A Case Study

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Abstract: In the era of globalization, integrated planning for production, workforce and capacity are the key factors for attaining success in any industry. This paper develops a mathematical model for determining the best possible required capacity, workforce and lot-size. Here, we go through the existing practices of production planning in a single piece flow based cellular manufacturing unit producing auto electrical parts. It is a linear programming model with three objectives namely, Production cost minimization, production quantity maximization and maximization of capacity utilization. It is to be solved by considering each objective sequentially as a Lexicographic approach. The results obtained from the model are compared with actual observed values for validation.

Keywords: Cellular Manufacturing, Linear Programming, Lexicographic approach, Production planning, Mathematical Model.

1. INTRODUCTION

Optimization and Operational research techniques are extensively used to derive better solutions and decisions to industrial problems. In any type of manufacturing industry, production planning is the most effective tool to meet customer satisfaction. Controlling daily production and workforce deployments are very difficult tasks for a production manager in a volatile market and other uncertain situations. This paper mainly focuses on the uncertain factors occurring in production planning in a real-case situation. While formulating production planning, several key questions arise that may include: How to define the best set of design using available and actual capacity levels? How to define the best set of workforce from permanent and temporary workforce including their overtime? What are the major factors to be considered when assigning the best set of workforce and capacity planning?

In this paper, we are presenting an optimization model developed for production planning in a cellular manufacturing with a single piece flow production type of auto electrical manufacturing industry. For operations associated with alternator manufacturing process, a suitable mathematical programming model has been developed by considering different production channels, which assign workforce in a priority manner. A suitable multi-objective

optimization mathematical model has been developed for this type of problem.

This paper is organized as follows. Product and process details about the case study organization are provided in Section 2. Operational conditions, variables details, and proposed mathematical model is discussed in Section 3. Analysis of results is presented in Section 4. Section 5 concludes the paper.

2. CASE STUDY

The case study industry selected for the production planning problem is an auto electrical part manufacturing industry in south-India. This is the first company in India to demonstrate cellular manufacturing with an adaptation to the Toyota production system and single piece flow manufacturing system based on the lean principles. Major products are alternator, car starter, commercial starter, and wiper motor for two/three wheelers, cars, commercial vehicles, tractors, and various engines. Major customers are Maruti Suzuki, Tata Motors, Ashok Leyland, Hyundai, etc. The yearly turnover is more than USD 200 million with seven production plants located in seven places in India. The total workforce of the industry is around 8,000, which include 2,000 permanent employees. "Alternator" product was selected for the detailed study, which has combine fourteen major platforms with more than 200 product varieties. The alternator production unit faces problem in meeting the daily requirement, as a result causing the backlog in monthly and yearly production.

2.1. Production line flow

The alternator has a large production volume which faces a lot of planning issues, demand more than one lakh alternators per month. So we selected the alternator production flow lines for the case study analysis. Each production line is a multi-model, multi-product, and multi-stage cellular manufacturing system (CMS) with just in time (JIT) environment. In the cellular manufacturing, there are clusters of dissimilar but sequentially related machines (i.e., module) for meeting the processing needs of a family of products. In the JIT, each group or cell is further amended by moving employees, workstations, or both in a U-shaped type layout that increases the possible interaction

among employees with the single piece flow (i.e., move one—check one—finish one). In this case study industry, employees also move along with the job. Workforce allocation is a most important decision in Production planning in this case study unit.

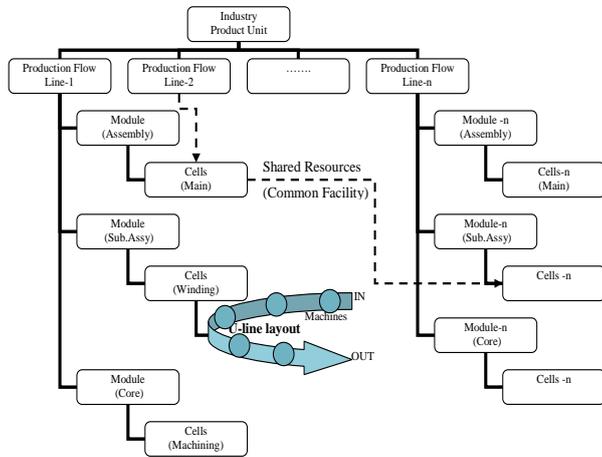


Fig-1: Product flow line layout structure

2.2. Existing workforce planning :

The marketing department of the case study organization sends the forecast plan on a monthly basis, then material planning department decides a production plan based on resource availability mainly focusing on the suppliers-end and available working days. After making the final production plan, the material planning department gets approval from the top management and number of working days required. The final production plan is shared with the manufacturing systems engineering, production, and purchase departments. The manufacturing systems engineering department updates the cycle time in a standard excel worksheet for calculating workforce and gets the existing manpower strength from the human resource (HR) department to consolidate available effective strength considering attrition of temporary trainees, transfers, voluntarily retirements (VRS), retirees, etc., for a detailed workforce requirement statement. The above procedures are regularly followed throughout the year. The manufacturing systems engineering department calculates a weekly workforce plan involving the Production and HR departments.

2.3 Production planning problem

Despite shared and individual facilities in all stages, following an integrated planning approach in all stages is difficult in this industry. If a delay occurs in subassembly stages, organization immediately concentrates on the concerned subassembly/delay stage and puts in a great deal of overtime to achieve the targeted forecasted production. Thus, the problem of allocation of demand from different stages is an uneven production situation that increases extra cost due to additional workforce deployment in main and subassembly levels, the machine idle time, and may lead to unmet demand, low level machine utilization and subsequently customer dissatisfaction.

Yeu et al. (2015) provided the concept of multi-production channels to meet an irregular and unpredictable order. If demand is less than the capacity, a normal production channel produces the required demand; if demand is greater than the capacity, the contingency channel produces along with the normal production channel after determining the optimal lot size and production rate. Here in this case study organization, the same strategy is followed. Workforce planning is considered instead of capacity planning to control the production planning. Therefore, the proposed model has three different production channels—normal, contingency, and overtime—to balance the demand based on workforce level of permanent employees, trainees/temporary employees, and employees doing overtime, which is the practice in the case study organization.

The main motivation behind this paper is to find a better methodology to solve the real-life production planning problem in a single piece flow based CMS. In the existing literature, the Lexicographic approach is majorly used for solving scheduling and layout design. Here, we have attempted to solve the production planning problem using the lexicographic approach in a single piece flow cellular manufacturing type production system. The following section provides the framework of the proposed mathematical model.

3. MATHEMATICAL MODEL FORMULATION

The proposed mathematical model integrates the regular, contingency, and overtime production in a real-world manufacturing situation.

3.1. Notations used in the mathematical model

Parameters

- PC_{np} Normal production cost
- PC_{cp} Contingency production cost
- PC_{op} Overtime production cost
- OP Design production rate
- Oee Overall equipment efficiency
- η_{np} Normal production channel efficiency
- η_{cp} Contingency production channel efficiency
- η_{op} Overtime production channel efficiency
- F_d Forecasted demand
- C_d Bottleneck capacity of the production line

System Variables

- W_d Number of working days
- W Number of permanent workforce

W_{np} Workforce for normal production

Decision Variables

OP_{np} Normal production rate per shift

OP_{cp} Contingency production rate per shift

OP_{op} Overtime production rate per shift

W_r Required workforce

W_{cp} Workforce for contingency production

W_{op} Workforce for overtime production

PU_{np} Normal production quantity (i.e., achieve through permanent employees)

PU_{cp} Contingency production quantity (i.e., achieve through trainees/temporary)

PU_{op} Overtime production quantity (i.e., achieve through permanent employees and trainees/temporary)

3.2. Assumptions considered in the model

- i. Working hours per shift per person is 8, maximum allowable shifts per day are 3, and the number of working days per month is 22. Design production rate per shift per person (OP) is 34 units of final assembly of alternator production.
- ii. Considered overall equipment efficiency (Oee) as 83%.
- iii. In real-world cellular manufacturing with JIT environment, bottlenecks can be a combination of more than one element. For example, the bottleneck may not essentially be the slowest stage or least capacity operation but the high cycle time. The bottleneck capacity of the production line (C_d) per month is 27,786 units. After considering the Oee , the value of available capacity is 23,062 units.
- iv. Available capacity is assumed always higher than the demand (F_d) per month, deterministic and known.
- v. Processing time per product is deterministic and known.
- vi. The number of available permanent workforce is considered the normal production channel with production rate efficiency (η_{np}) of 90%. Similarly, temporary/trainee workforce is considered as contingency production channel with production rate efficiency (η_{cp}) of 80%. Both types of workforce working overtime are considered as overtime production channel with production rate efficiency (η_{op}) of 100%.
- vii. Normal production cost (PC_{np}) is ₹147 per unit, contingency production cost (PC_{cp}) is ₹ 74 per unit, and overtime production cost (PC_{op}) is ₹ 191 per unit. These costs are computed based on

case study industry practices (All cost units are in rupees ₹. 1 € = 84.42 ₹).

- viii. Case study industry production line consists of multi-model, multi-product, multi-period, and multi-stage CMS. For model simplification, only main assembly with a single product and single period is considered. The number of permanent workforce (W) is 8 in the main assembly.

3.3. Model components

The production cost is the most commonly used criterion for measuring production planning performance. In the proposed model, we suggested three major criteria of minimizing the production cost, maximizing the production unit, and maximizing the utilization level to optimize the performance. So the model integrates these three major objective functions: (a) minimize the production cost, (b) maximize the production quantity, and (c) maximize the utilization in a sequential priority order. These three objective functions are handled sequentially as a lexicographic approach.

3.3.1. Objective function-1: Minimize the production cost

$$\min Z_1 = [(PU_{np} \times PC_{np})^a + (PU_{cp} \times PC_{cp})^b + (PU_{op} \times PC_{op})^c] \quad (OF-1)$$

The following three constraints (1)–(3) establish the production rate of individual channel. The channel production rate is controlled by the individual stage of production rate, overall equipment efficiency, and channel efficiency. Constraints (1)–(3) are rewritten as Equations (1a), (2a), and (3a) after adding values for parameters OP [30 units/shift], η_{np} [0.9], η_{cp} [0.8], η_{op} [1], and Oee [0.83]

$$OP_{np} - [OP \times \eta_{np} \times Oee] \geq 0 \quad (1)$$

$$OP_{cp} - [OP \times \eta_{cp} \times Oee] \geq 0 \quad (2)$$

$$OP_{op} - [OP \times \eta_{op} \times Oee] \geq 0 \quad (3)$$

$$OP_{np} \geq 22 \quad (1a)$$

$$OP_{cp} \geq 20 \quad (2a)$$

$$OP_{op} \geq 25 \quad (3a)$$

Constraint (4) defines the required workforce for the forecasted demand. Equation (5) imposes the demand level within the bottleneck capacity of the production line.

$$W_r \leq F_d / OP \quad (4)$$

$$(C_d \times W_d \times Oee) \geq F_d \quad (5)$$

Similarly, constraints (4) and (5) are rewritten as Equations (4a) and (5a) after adding the values for parameters F_d [18,051 units/month] and OP [30 units/shift].

$$W_r \leq 602 \quad (4a)$$

$$(C_d \times W_d \times Oee) \geq 18,051 \quad (5a)$$

Detailed workforce distribution plan is derived from Equations (7) and (8). Here, the number of the permanent workforce, W , is considered as a system variable. The system variables of the case study industry are identified from the historical data. The normal production workforce (W_{np}) equals the product of available permanent workforce, W [8 permanent employees/day] and working days per month, W_d [22 days/month]. So, W_{np} becomes 176 permanent employees per month. Constraint (6) restricts daily overtime per worker is not more than 2 hours per day.

$$[W_{np} + W_{cp}] \times 2 - [W_{op}] \times 8 \leq 0;$$

then the balanced overtime constraint is

$$[W_{np} + W_{cp}] - 4[W_{op}] \leq 0 \quad (6)$$

After adding the values of W_{np} , constraint (6) becomes

$$[176 + W_{cp}] - 4[W_{op}] \leq 0;$$

then,

$$[-176 - W_{cp}] + 4[W_{op}] \geq 0;$$

Finally, the overtime constraint is rewritten as

$$4[W_{op}] - [W_{cp}] \geq 176 \quad (6a)$$

The required workforce must be greater than the normal and contingency including overtime channel.

$$[W_{np} + W_{cp} + W_{op}] \geq W_r \quad (7)$$

$$[W_{np} + W_{cp}] \leq W_r \quad (8)$$

The main aim is to control the workforce requirement for the maximums achievable production quantity at a minimum cost. As a result, the required workforce must be less than the permanent employees and trainees/temporary without considering the overtime.

After adding the values of, W_{np} and W_r , constraint (7) becomes

$$[176 + W_{cp} + W_{op}] \geq 602, \text{ which is equivalent to}$$

$$[W_{cp} + W_{op}] \geq 426 \quad (7a)$$

After adding the values of W_{np} and W_r , constraint (8) becomes

$$[176 + W_{cp}] \leq 602, \text{ which is equivalent to}$$

$$[W_{cp}] \leq 426 \quad (8a)$$

Decision variables (9)–(11) confirm that the production quantity is not more than planned deployed workforce production rate. The purpose is to limit the workforce as per the demand. After adding the values of W_{np} , constraint (9) is converted to (9a).

$$PU_{np} - [OP_{np} \times W_{np}] \geq 0 \quad (9)$$

$$PU_{np} - [OP_{np} \times 176] \geq 0; \quad (9a)$$

$$PU_{cp} - [OP_{cp} \times W_{cp}] \geq 0 \quad (10)$$

$$PU_{op} - [OP_{op} \times W_{op}] \geq 0 \quad (11)$$

Constraints (10) and (11) are nonlinear, which are converted to linear form by taking log on both sides

$$\log(LP_{cp}) = \log(OP_{cp} \times W_{cp}) \quad (10a)$$

$$\log(OP_{cp} \times W_{cp}) = \log(OP_{cp}) + \log(W_{cp}) \quad (10b)$$

$$PU_{cp} - [\log(LP_{cp})] \geq 0 \quad (10c)$$

Similarly, for overtime channel,

$$PU_{op} - [\log(LP_{op})] \geq 0 \quad (11a)$$

The above described mathematical model (Model-1) with objective function (OF-1) and constraints (1)–(11a) was solved using Lindo solver and the results are tabulated in Table 1.

Objective function: Z_1 (Minimize production cost), (First priority)				Units
Objective value Z_1	16×10^5			₹
Decision variables	Normal production quantity	PU_n p	3,94 4	Quantity units
	Contingency production quantity	PU_c p	6,08 3	
	Overtime production quantity	PU_o p	2,99 6	
	Normal production rate/shift	OP_n p	22	Quantity units
	Contingency production rate/shift	OP_c p	20	
	Overtime production rate/shift	OP_o p	25	
System variables — Workforce	Number of permanent employees engaged	W_{np}	176	Number of employees
	Number of trainees/temporary workforce engaged	W_{cp}	305	
	Number of overtime workforce engaged	W_{op}	121	

Table 1 Optimal values of Model-1

3.3.2. Objective function-2: Maximize the production quantity

Second priority objective function (OF-2) is to maximize the production quantity of three different production channels—normal, contingency, and overtime.

$$\text{Max } Z_2 = (PU_{np} + PU_{cp} + PU_{op}) \text{ (OF-2)}$$

This is subjected to constraints (1)–(11a) along with additional constraint derived from the objective function value of Z_1 , shown as constraint (12).

$$[PU_{np} \times PC_{np} + PU_{cp} \times PC_{cp} + PU_{op} \times PC_{op}] \leq 16 \times 10^5 \quad (12)$$

After adding the values of PC_{np} [₹ 147], PC_{cp} [₹ 74], and PC_{op} [₹ 191]; constraint (12) is rewritten as (12a).

$$[PU_{np} \times 147 + PU_{cp} \times 74 + PU_{op} \times 191] \leq 16 \times 10^5 \quad (12a)$$

Calling this as Model-2, with objective function OF-2 and constraints (1)–(12a), solved using Lindo solver. The results are tabulated in Table 2.

Objective function: Z_2 : (Maximize production quantity) , (Second priority)				Units
Objective function value Z_2	13,024			Quantity units
Decision variables	Normal production quantity	PU_{np}	3,944	Quantity units
	Contingency production quantity	PU_{cp}	6,084	
	Overtime production quantity	PU_{op}	2,996	
	Total produced quantity	13,024		

Table 2 Optimal values of Model-2

Since we have used constraint value of ₹ 16×10^5 in Equation (12a) which is the exact objective function value of Z_1 after consulting the company's management. So we got the same production quantities in Model-2 also as in Model-1. But we have the option of changing the constraint value as per the need of the industry and Model-2 will give appropriate optimal production quantities.

3.3.1. Objective function-3: Maximize the capacity utilization

Third priority objective function (OF-3) is to maximize the capacity utilization at three different production channels—normal, contingency, and overtime.

$$\text{max } Z_3 = [PU_{np} + PU_{cp} + PU_{op}] / C_d \text{ (OF-3)}$$

This is subjected to constraints (1)–(12a) and (13).

The additional constraint (13) derived from Z_2 value is

$$[PU_{np} + PU_{cp} + PU_{op}] \geq 13,024 \quad (13)$$

Solving this third stage optimization problem as Model-3 with objective function (OF-3) and constraints (1)–(13), results are tabulated in Table 3.

Objective function: Z_3 : (Maximize capacity utilization) , (Third priority)		Units
Objective function values Z_3	72	%

Table 3 Optimal values of Model-3

Results derived from the Lexicographic approach of solving the mathematical models (Model-1, -2, -3) are summarized in Table 4.

Objective function values		
Production cost	Z_1	₹ 16×10^5 per 13,024 units
Production quantity	Z_2	13,024 units per month
Capacity utilization	Z_3	72%
Optimal values of decision variables*		
Normal production rate per shift	OP_{np}	22 units
Contingency production rate per shift	OP_{cp}	20 units
Overtime production rate per shift	OP_{op}	25 units
Required workforce	W_r	602 workers
Workforce for contingency production	W_{cp}	305 workers
Workforce for overtime production	W_{op}	121 workers
Normal production quantity	PU_{np}	3,944 units
Contingency production quantity	PU_{cp}	6,084 units
Overtime production quantity	PU_{op}	2,996 units

Table 4 Summary of results

4. CONCLUSIONS

In this paper, we presented a production planning problem from a case study organization where single piece flow based cellular manufacturing is being operated. Selected product is “Alternator” used in automobiles, which is one of many products being manufactured in that industry. The three objective functions considered in the mathematical model formulation of production planning problem are: (a) minimize the production cost (Z_1), (b) maximize the production quantity (Z_2), and (c) maximize the capacity utilization (Z_3). The set of 11 constraints were formulated as a function of decision variables. This Linear programming model has been solved using LINDO solver. The model-1, with objective function-1 and 11 constraints was solved, optimal solution is recorded. For model-2,

objective function-2 with 11 constraints along with 12th constraint framed from the objective function value of model-1 has been solved to get optimal solution at stage-2. For model-3, objective function-3 with above 12 constraints along with 13th constraint derived from the objective function value of model-2 has been solved to get optimal solution at stage-3. This final solution was validated by comparing with the observed values of the industry for 12 months. This validated model is useful for the industry to derive suitable production plans.

This approach of handling multiple objectives sequentially is referred to as Lexicographic approach. This is one way of solving multi objective linear programming problem. We can also handle the same problem with Goal programming approach. In this paper we limited our production planning problem for a single product, multi period, and single stage. This is being extended to multi product, multi period, and multi stage production planning problem. Work in this direction is in progress.

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