International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 06, June-2020

# Application of Haar Wavelet Discretization Method for Free Vibration Analysis of Rectangular Plate

Kwanghun Kim[1], Hyeyong Pak[2]
[1,2]department of Engineering Machine,
Pyongyang University of Mechanical Engineering,
Pyongyang, Democratic People's Republic of Korea

Chol Nam[4]
[4]Department of Resource Development Machine,
Pyongyang University of Mechanical Engineering,
Pyongyang, Democratic People's Republic of Korea

Suchol O[3]
[3]Institute of Science,
Chongjin Mine & Metal University,
Chongjin, Democratic People's Republic of Korea

Jonggil K O[5]
[5]Department Of Machine Manufacturing,
Pyongyang University of Mechanical Engineering,
Pyongyang, Democratic People's Republic of Korea

***Abstract*—This paper focuses on the Haar wavelet discretization method (HWDM) for solving the problem of free vibration behaviour of rectangular plate. The displacements are expressed as Haar wavelet series and their integral. Since the constants generated during the integration process are decided as the setting of boundary conditions, therefore, the equations of motion of the entire system including boundary conditions are expressed as a series of algebraic equations. By solving the eigenvalue problem of these algebraic equation, the natural frequencies of the rectangular plate can be obtained. The accuracy and convergence of HWDM is verified against the results of the previous data, and the comparison results agree well. The effects of several geometric dimensions on the natural frequencies of rectangular plate under various boundary conditions are investigated. The numerical results for rectangular plate obtained by Haar wavelet discretization method may be served as benchmark solutions for future research.**

***Keywords—Rectangular plate; Haar wavelet discretization method; Free vibration; Numerical analysis; Eigenvalue problem***

## I. INTRODUCTION

Plates are widely used in various engineering fields, and specially, in the case of ship and ocean structures they can be considered as one of the fundamental structural elements. It is very important to investigate the vibration behavior of plates which are widely used in various engineering fields. A lot of scholars have published a number of papers on the free vibration of rectangular plates [1-13]. Different methods such as Rayleigh-Ritz method, Galerkin method, finite element method and the method of separation of variables have been used to analyze the free vibration of rectangular plate. The results of the review show that although many studies have been conducted to analyze the free vibrations of rectangular plates, finding a reasonable approach to obtain the natural frequencies of rectangular plates is still an important problem. The Haar wavelet discretization method is a very simple and powerful method to solve the eigenvalue problem and the application in engineering of this method will be reviewed below.

In 1909, A. Haar proposed the Haar function, which made a great contribution to the emergence of wavelet theory. In 1981, J. Morlet [14] proposed the wavelet concept, which laid a good foundation for the formation of wavelet theory. Since then, the research of wavelet theory has entered a stage of rapid development. In 1981, Stromberg improved on the basis of the Haar function and found the first orthogonal wavelet. In 1982, expert Marr formed the "Mexican Hat" wavelet. In 1985, Meyer obtained a smooth wavelet orthonormal basis with a certain attenuation, namely Meyer base[15], which laid a good foundation for the further development of wavelet theory. In 1988, wavelet analyst Daubechies[16, 17] constructed an iterative method to construct a wavelet base that is non-zero only in a finite region (ie Daubechies base) and gave 10 presentations at a wavelet conference held in the United States. It caused a sensation, and then the book "Small Waves Ten" became a great book with great influence in contemporary mathematics, which pushed the development and practical application of wavelet theory to a climax. In 1989, Mallat[18, 19] proposed the famous "Mallat algorithm", which opened the development space for the engineering application of wavelet theory. After the 1990s, wavelet theory gradually matured. At present, research on wavelet theory has penetrated a wide variety of fields. Majak et al. [20, 21] developed Haar wavelet-based discretization method for solving differential equations, discussed both, strong and weak formulations based apporaches, and introduced this method to solve solid mechanics problems. Recently, Hein and Feklistova[22, 23] applied HWDM for solving the vibration problems of functionally graded beams under some boundary conditions.

As can be seen from the literature review, the Haar method was used for vibration analysis of various types of structures, but two-dimensional development is difficult, so there are very few examples applied to the vibration of plate structures. Therefore, the main purpose of this paper is to establish a solution method and system to apply conveniently and efficiently the Haar wavelet discretization method to the free vibration of a rectangular plate.

## II. APPLICATION OF HWDM

Haar wavelet which is a group of square waves that has size of +1 and -1 in some intervals and has zeros in elsewhere is one of the simplest compactly supported orthogonal wavelet among the wavelet families. The details of Haar wavelet series and their integrals can be found in the following references [25-28].

HWDM are used to discretize the derivatives in entire governing equations including boundary conditions. Since the Haar wavelet series is defined in the interval [0, 1], firstly, in order to apply the HWDM, a linear transformation statute may be introduced for coordinate conversion from length interval $[0, L]$of the rectangular plate to the interval[0,1]of the Haar wavelet series, that is,

$$\xi=\frac{x-x_1}{x_2-x_1}=\frac{x-x_1}{L_x},\qquad \eta=\frac{y-y_1}{y_2-y_1}=\frac{y-y_1}{L_y} \tag{1}$$

In the HWDM, highest order derivatives of the displacements are expressed by Haar wavelet series, the lower order derivatives can be obtained by integrating Haar wavelet series.

The transverse vibration differential equation of the plate is expressed as

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}-k^4 w=0 \tag{2}$$

where $k^4=\frac{\rho h}{D}\omega^2$, $D$ is the flexural stiffness of the plate, is defined as $D=E_1 h^3/12\left(1-\mu^2\right)$.

Substituting equation (2) into equation (1), the vibration equation of the plate at local coordinates is defined as:

$$\frac{1}{L_x^4}\frac{\partial^2 w}{\partial \xi^2}+2\frac{1}{L_x^4}\frac{1}{L_y^4}\frac{\partial^4 w}{\partial \xi^2\partial \eta^2}+\frac{1}{L_y^4}\frac{\partial^2 w}{\partial \eta^2}-k^4 w=0 \tag{3}$$

By taking $n=2m$, $f(\xi)=w(\xi,\eta)$ (when $\eta$ is fixed, $w$ is a function containing only the variable $\xi$), and the highest order derivative can be approximated by Haar wavelet along parallel to the $\xi$ axis:

$$\frac{d^4 f(\xi)}{d\xi^4}=\sum_{i=1}^{2m}a_i h_i(\xi) \tag{4}$$

By integrating the Eq. (4), the following derivative terms can be obtained.

$$\frac{d^3 f(\xi)}{d\xi^3}=\sum_{i=1}^{2m}a_i p_{1,i}(\xi)+\frac{d^3 f(0)}{d\xi^3} \tag{5,a}$$

$$\frac{d^2 f(\xi)}{d\xi^2}=\sum_{i=1}^{2m}a_i p_{2,i}(\xi)+\xi\frac{d^3 f(0)}{d\xi^3}+\frac{d^2 f(0)}{d\xi^2} \tag{5,b}$$

$$\frac{df(\xi)}{d\xi}=\sum_{i=1}^{2m}a_i p_{3,i}(\xi)+\frac{\xi^2}{2}\frac{d^3 f(0)}{d\xi^3}+\xi\frac{d^2 f(0)}{d\xi^2}+\frac{df(0)}{d\xi} \tag{5,c}$$

$$f(\xi)=\sum_{i=1}^{2m}a_i p_{4,i}(\xi)+\frac{\xi^3}{6}\frac{d^3 f(0)}{d\xi^3}+\frac{\xi^2}{2}\frac{d^2 f(0)}{d\xi^2}+\xi\frac{df(0)}{d\xi}+f(0) \tag{5,d}$$

The displacement function $v(\xi)$ can be written in the form of a matrix as following:

$$\boldsymbol{f}(\xi)=\boldsymbol{P}_1\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix} \tag{6}$$

where

$$\boldsymbol{f}(\xi)=\left[f(\xi_1)\quad f(\xi_2)\quad \cdots\quad f(\xi_n)\right]^T$$

$$\boldsymbol{a}=\left[a_1\quad a_2\quad \cdots\quad a_n\right]^T,$$

$$\boldsymbol{b}=\left[\frac{d^3 f(0)}{d\xi^3}\quad \frac{d^2 f(0)}{d\xi}\quad \frac{df(0)}{d\xi}\quad f(0)\right]^T$$

$$\boldsymbol{P}_1=\begin{bmatrix} p_{4,1}(\xi_1) & p_{4,2}(\xi_1) & \cdots & p_{4,n}(\xi_1) & \frac{\xi_1^3}{6} & \frac{\xi_1^3}{2} & \xi_1 & 1\\ p_{4,1}(\xi_2) & p_{4,2}(\xi_2) & \cdots & p_{4,n}(\xi_2) & \frac{\xi_2^3}{6} & \frac{\xi_2^3}{2} & \xi_2 & 1\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ p_{4,1}(\xi_n) & p_{4,2}(\xi_n) & \cdots & p_{4,n}(\xi_n) & \frac{\xi_n^3}{6} & \frac{\xi_n^2}{2} & \xi_n & 1 \end{bmatrix}$$

In practical process applications, the clamped boundary condition, simply-supported boundary condition and the free boundary condition are widely used, these boundary conditions can be written as follows in equation form.

*Clamped boundary condition*

$$\begin{aligned} &w(\xi)=0,\quad \frac{dw(\xi)}{d\xi}=0,\quad (\xi=0,\ \xi=1),\\ &w(\eta)=0,\quad \frac{dw(\eta)}{d\eta}=0,\quad (\eta=0,\ \eta=1) \end{aligned} \tag{7}$$

*Simply-supported boundary condition*

$$\begin{aligned} &w(\xi)=0,\quad \frac{d^2 w(\xi)}{d\xi^2}=0,\quad (\xi=0,\ \xi=1),\\ &w(\eta)=0,\quad \frac{d^2 w(\eta)}{d\eta^2}=0,\quad (\eta=0,\ \eta=1) \end{aligned} \tag{8}$$

*Free boundary condition*

$$\begin{aligned} &\frac{d^2 w(\xi)}{d\xi^2}=0,\quad \frac{d^3 w(\xi)}{d\xi^3}=0,\quad (\xi=0,\ \xi=1)\\ &\frac{d^2 w(\eta)}{d\eta^2}=0,\quad \frac{d^3 w(\eta)}{d\eta^3}=0,\quad (\eta=0,\ \eta=1) \end{aligned} \tag{9}$$

Four boundary condition equations can be obtained by introducing the boundary condition, and can be written as follows in matrix form:

$$\boldsymbol{f}_{\mathbf{b}}=\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix} \tag{10}$$

where, for the clamped boundary condition

$$\boldsymbol{f}_{\mathbf{b}}=\left[f(0)\quad f(1)\quad \frac{df(0)}{d\xi}\quad \frac{df(1)}{d\xi}\right]^T,$$

$$\boldsymbol{P}_2=\begin{bmatrix} p_{4,1}(0) & p_{4,2}(0) & \cdots & p_{4,n}(0) & 0 & 0 & 0 & 1\\ p_{4,1}(1) & p_{4,2}(1) & \cdots & p_{4,n}(1) & \frac{1}{6} & \frac{1}{2} & 1 & 1\\ p_{1,1}(0) & p_{1,2}(0) & \vdots & p_{1,n}(0) & 0 & 0 & 1 & 0\\ p_{1,1}(1) & p_{1,2}(1) & \cdots & p_{1,n}(1) & \frac{1}{2} & 1 & 1 & 0 \end{bmatrix}$$

for the simply supported boundary condition,

$$\boldsymbol{f}_{\mathbf{b}}=\left[f(0)\quad f(1)\quad \frac{d^2 f(0)}{d\xi^2}\quad \frac{d^2 f(1)}{d\xi^2}\right]^T,$$

$$\boldsymbol{P}_2=\begin{bmatrix} p_{4,1}(0) & p_{4,2}(0) & \cdots & p_{4,n}(0) & 0 & 0 & 0 & 1 \\ p_{4,1}(1) & p_{4,2}(1) & \cdots & p_{4,n}(1) & \frac{1}{6} & \frac{1}{2} & 1 & 1 \\ p_{2,1}(0) & p_{2,2}(0) & \vdots & p_{2,n}(0) & 0 & 1 & 0 & 0 \\ p_{2,1}(1) & p_{2,2}(1) & \cdots & p_{2,n}(1) & 1 & 1 & 0 & 0 \end{bmatrix}$$

and, for the free boundary condition

$$\boldsymbol{f}_{\mathbf{b}}=\left[\frac{d^2f(0)}{d\xi^2} \quad \frac{d^2f(1)}{d\xi^2} \quad \frac{d^3f(0)}{d\xi^3} \quad \frac{d^3f(1)}{d\xi^3}\right]^T,$$

$$\boldsymbol{P}_2=\begin{bmatrix} p_{2,1}(0) & p_{2,2}(0) & \cdots & p_{2,n}(0) & 0 & 1 & 0 & 0 \\ p_{2,1}(1) & p_{2,2}(1) & \cdots & p_{2,n}(1) & 1 & 1 & 0 & 0 \\ p_{3,1}(0) & p_{3,2}(0) & \vdots & p_{3,n}(0) & 1 & 0 & 0 & 0 \\ p_{3,1}(1) & p_{3,2}(1) & \cdots & p_{3,n}(1) & 1 & 0 & 0 & 0 \end{bmatrix}$$

By combining Eq. (6) and Eq. (10) the following equation can be obtained.

$$\begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{f}_{\mathbf{b}} \end{bmatrix}=\begin{bmatrix} \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \end{bmatrix}\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}=\boldsymbol{Q}_1\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix} \tag{11}$$

From Eq. (11), an unknown coefficient matrix is defined as:

$$\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}=\boldsymbol{Q}_1^{-1}\begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{f}_{\mathbf{b}} \end{bmatrix} \tag{12}$$

Therefore, the fourth order derivative of displacement $f$ can be expanded into the following as:

$$\frac{\boldsymbol{d}^4\boldsymbol{f}}{\boldsymbol{d}\xi^4}=\boldsymbol{H}_1\boldsymbol{Q}_1^{-1}\begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{f}_{\mathbf{b}} \end{bmatrix}=\boldsymbol{L}_1\boldsymbol{f}+\boldsymbol{L}_2\boldsymbol{f}_{\mathbf{b}} \tag{13}$$

Where $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ are the first $n$ columns and the last four columns of the matrix $\boldsymbol{H}_1\boldsymbol{Q}_1^{-1}$.

$$\frac{\boldsymbol{d}^4\boldsymbol{f}}{\boldsymbol{d}\xi^4}=\left[\frac{d^4f(\xi_1)}{d\xi^4},\frac{d^4f(\xi_2)}{d\xi^4},\cdots,\frac{d^4f(\xi_n)}{d\xi^4}\right]^T,$$

$$\boldsymbol{P}_1=\begin{bmatrix} h_1(\xi_1) & h_2(\xi_1) & \cdots & h_n(\xi_1) & 0 & 0 & 0 & 0 \\ h_1(\xi_1) & h_2(\xi_2) & \cdots & h_n(\xi_2) & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_1(\xi_n) & h_2(\xi_n) & \cdots & h_n(\xi_n) & 0 & 0 & 0 & 0 \end{bmatrix}$$

By using the tensor multiplying, $\boldsymbol{d}^4\boldsymbol{f}/\boldsymbol{d}\xi^4$can be extend to two dimensions

$$\frac{\partial^4\boldsymbol{w}}{\partial\xi^4}=\left(\boldsymbol{L}_1\otimes\boldsymbol{I}_y\right)\boldsymbol{w}+\left(\boldsymbol{L}_2\otimes\boldsymbol{I}_y\right)\boldsymbol{f}=\boldsymbol{K}_x\boldsymbol{w}+\boldsymbol{M}_x \tag{14}$$

Considering the boundary conditions, it is easy to know that $f$ is a zero vector with a length equal to $2n$, and it will be omitted. $\boldsymbol{Iy}$ is the unit matrix.

Similarly,

$$\frac{\partial^4\boldsymbol{w}}{\partial\eta^4}=\left(\boldsymbol{I}_x\otimes\boldsymbol{L}_1\right)\boldsymbol{w}=\boldsymbol{K}_y\boldsymbol{w} \tag{15}$$

The following expression will be used to calculate the Haar wavelet expression of the fourth-order mixed partial derivative of $w$.

$$2\frac{\partial^4 w}{\partial\xi^2\partial\eta^2}=\frac{\partial^2}{\partial\xi^2}\left(\frac{\partial^2 w}{\partial\eta^2}\right)+\frac{\partial^2}{\partial\eta^2}\left(\frac{\partial^2 w}{\partial\xi^2}\right) \tag{16}$$

If $g(\xi)=\partial^2w/\partial\eta^2$, the second derivative of $g(\xi)$ along the parallel axis $\xi$ can be approximated by Haar wavelet.

$$\frac{d^2g(\xi)}{d\xi^2}=\sum_{i=1}^{n}c_ih_i(\xi) \tag{17}$$

By integrating second derivative of k in turn, the following expressions can be obtained.

$$\frac{dg(\xi)}{d\xi}=\sum_{i=1}^{n}c_ip_{1,i}(\xi)+\frac{dg(0)}{d\xi},$$

$$g(\xi)=\sum_{i=1}^{n}c_ip_{2,i}(\xi)+\xi\frac{dg(0)}{d\xi}+dg(0)$$

$g(\xi)$ can be written in matrix form as

$$\boldsymbol{g}(\xi)=\begin{bmatrix} p_{2,1}(\xi_1) & p_{2,2}(\xi_1) & \cdots & p_{2,n}(\xi_1) & \xi_1 & 1 \\ p_{2,1}(\xi_2) & p_{2,2}(\xi_2) & \cdots & p_{2,n}(\xi_2) & \xi_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ p_{2,1}(\xi_n) & p_{2,2}(\xi_n) & \cdots & p_{2,n}(\xi_n) & \xi_n & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix}=\boldsymbol{R}_1\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix} \tag{18}$$

where, $\boldsymbol{g}(\xi)=\left[g(\xi_1) \quad g(\xi_2) \quad \cdots \quad g(\xi_n)\right]^T$

Two boundary condition equations can be obtained by introducing the boundary condition, and can be written as follows in matrix form (in this example the boundary condition is set simply supported boundary condition, in simply supported boundary condition $g(\xi)=\partial^2w/\partial\eta^2=0$, ($\eta=0$, $\eta=1$)):

$$\boldsymbol{g}_{\mathbf{b}}=\begin{bmatrix} p_{2,1}(0) & p_{2,2}(0) & \cdots & p_{2,n}(0) & 0 & 1 \\ p_{2,1}(1) & p_{2,2}(1) & \cdots & p_{2,n}(1) & 1 & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix}=\boldsymbol{R}_2\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix} \tag{19}$$

where $\boldsymbol{c}=\left[c_1 \quad c_2 \quad \cdots \quad c_n\right]^T$, $\boldsymbol{d}=\left[\frac{dg(0)}{d\xi} \quad g(0)\right]^T$

By combining Eq. (18) and Eq. (19) the following equation can be obtained.

$$\begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{g}_{\mathbf{b}} \end{bmatrix}=\begin{bmatrix} \boldsymbol{R}_1 \\ \boldsymbol{R}_2 \end{bmatrix}\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix}=\boldsymbol{Q}_2\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix} \tag{20}$$

From Eq. (20), an unknown coefficient matrix is defined as:

$$\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix}=\boldsymbol{Q}_2^{-1}\begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{g}_{\mathbf{b}} \end{bmatrix} \tag{21}$$

Therefore, the second order derivative of displacement $g$ can be expanded into the following as:

$$\frac{\boldsymbol{d}^2\boldsymbol{g}}{\boldsymbol{d}\xi^2}=\begin{bmatrix} h_1(\xi_1) & h_2(\xi_1) & \cdots & h_n(\xi_1) & 0 & 0 \\ h_1(\xi_2) & h_2(\xi_2) & \cdots & h_n(\xi_2) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_1(\xi_n) & h_2(\xi_n) & \cdots & h_n(\xi_n) & 0 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix} \tag{22}$$

$$=\boldsymbol{H}_2\boldsymbol{Q}_2^{-1}\begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{g}_{\mathbf{b}} \end{bmatrix}=\boldsymbol{N}_1\boldsymbol{g}+\boldsymbol{N}_2\boldsymbol{g}_{\mathbf{b}}$$

Where $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ are the first $n$ columns and the last two columns of the matrix $\boldsymbol{H}_2\boldsymbol{Q}_2^{-1}$.

Similarly, for $k(\eta)=w(\xi,\eta)$

$$\boldsymbol{l}(\eta)=\begin{bmatrix} p_{2,1}(\eta_1) & p_{2,2}(\eta_1) & \cdots & p_{2,n}(\eta_1) & \eta_1 & 1 \\ p_{2,1}(\eta_2) & p_{2,2}(\eta_2) & \cdots & p_{2,n}(\eta_2) & \eta_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ p_{2,1}(\eta_n) & p_{2,2}(\eta_n) & \cdots & p_{2,n}(\eta_n) & \eta_n & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{n} \end{bmatrix} \tag{23}$$

$$=\boldsymbol{S}_1\begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{n} \end{bmatrix}$$

where, $\boldsymbol{l}(\eta)=\left[l(\eta_1) \quad l(\eta_2) \quad \cdots \quad l(\eta_n)\right]^T$

$\boldsymbol{m}=\left[m_1 \quad m_2 \quad \cdots \quad m_n\right]^T$, $\boldsymbol{n}=\left[\frac{dl(0)}{d\eta} \quad l(0)\right]^T$

In a similar way to equation (19)

$$\boldsymbol{l}_{\mathbf{b}}=\begin{bmatrix} p_{2,1}(0) & p_{2,2}(0) & \cdots & p_{2,n}(0) & 0 & 1 \\ p_{2,1}(1) & p_{2,2}(1) & \cdots & p_{2,n}(1) & 1 & 1 \end{bmatrix}\begin{bmatrix}\boldsymbol{m}\\ \boldsymbol{n}\end{bmatrix}=\boldsymbol{S}_2\begin{bmatrix}\boldsymbol{m}\\ \boldsymbol{n}\end{bmatrix} \quad (24)$$

By combining Eq. (23) and Eq. (24) the following equation can be obtained.

$$\begin{bmatrix}\boldsymbol{l}\\ \boldsymbol{l}_{\mathbf{b}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{S}_1\\ \boldsymbol{S}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{m}\\ \boldsymbol{n}\end{bmatrix}=\boldsymbol{Q}_3\begin{bmatrix}\boldsymbol{m}\\ \boldsymbol{n}\end{bmatrix} \quad (25)$$

Therefore, the second order derivative of displacement $l$ can be expanded into the following as:

$$\frac{d^2\boldsymbol{l}}{d\eta^2}=\begin{bmatrix} h_1(\eta_1) & h_2(\eta_1) & \cdots & h_n(\eta_1) & 0 & 0 \\ h_1(\eta_2) & h_2(\eta_2) & \cdots & h_n(\eta_2) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_1(\eta_n) & h_2(\eta_n) & \cdots & h_n(\eta_n) & 0 & 0 \end{bmatrix}\begin{bmatrix}\boldsymbol{m}\\ \boldsymbol{n}\end{bmatrix} \quad (26)$$

$$=\boldsymbol{H}_3\boldsymbol{Q}_3^{-1}\begin{bmatrix}\boldsymbol{l}\\ \boldsymbol{l}_{\mathbf{b}}\end{bmatrix}=\boldsymbol{T}_1\boldsymbol{l}+\boldsymbol{T}_2\boldsymbol{l}_{\mathbf{b}}$$

Where $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ are the first $n$ columns and the last two columns of the matrix $\boldsymbol{H}_3\boldsymbol{Q}_3^{-1}$.

From above equations, we can obtain as following expression.

$$\frac{\partial^2}{\partial\xi^2}\left(\frac{\partial^2\boldsymbol{w}}{\partial\eta^2}\right)=\left(\boldsymbol{N}_1\otimes\boldsymbol{I}_y\right)\frac{\partial^2\boldsymbol{w}}{\partial\eta^2}=\left(\boldsymbol{N}_1\otimes\boldsymbol{I}_y\right)\left(\boldsymbol{I}_x\otimes\boldsymbol{T}_1\right)=\boldsymbol{K}_{xy}\boldsymbol{w} \quad (27)$$

In this way, we can get an expression similar to Eq. (27) for $\frac{\partial^2}{\partial\eta^2}\left(\frac{\partial^2\boldsymbol{w}}{\partial\xi^2}\right)$.

Therefore, a governing equation for an orthogonal isotropic plate expressed by Haar wavelet and its integral can be obtained as:

$$\left(\boldsymbol{K}_x+2\boldsymbol{K}_{xy}+\boldsymbol{K}_y\right)\boldsymbol{z}=\boldsymbol{k}^4\left(\boldsymbol{I}_x\otimes\boldsymbol{I}_{y1}\right)\boldsymbol{z} \quad (28)$$

Therefore, the natural frequency of the plate can be easily obtained according to equation (28).

## III. NUMERICAL RESULTS

In this section, new numerical data that future researchers can use as benchmarks are presented along with parameter studies.

TEBLE I. FREQUENCY PARAMETERS Ω OF PLATE WITH VARIOUS BOUNDARY CONDITIONS $\Omega=\omega L^2/h(\rho/(E_1)^{1/2}$

| Boundary Condition | Mode | J=2 | J=3 | J=4 | J=5 | Ref. [24] |
|---|---|---|---|---|---|---|
| | | Present | Present | Present | Present | |
| CCCC | $\Omega_1$ | 3.65939 | 3.65014 | 3.64713 | 3.6463 | 3.6467 |
| | $\Omega_2$ | 7.52806 | 7.46223 | 7.44301 | 7.438 | 7.4416 |
| | $\Omega_3$ | 7.52806 | 7.46223 | 7.44301 | 7.438 | 7.4416 |
| | $\Omega_4$ | 11.1062 | 11.0136 | 10.9777 | 10.968 | 10.974 |
| SSSS | $\Omega_1$ | 2.00959 | 2.00241 | 2.0006 | 2.0002 | 2.00 |
| | $\Omega_2$ | 5.06696 | 5.01684 | 5.00422 | 5.0011 | 5.00 |
| | $\Omega_3$ | 5.06696 | 5.01684 | 5.00422 | 5.0011 | 5.00 |
| | $\Omega_4$ | 8.15074 | 8.03836 | 8.00963 | 8.0024 | 8.00 |
| CSCS | $\Omega_1$ | 2.94584 | 2.93673 | 2.9342 | 2.9336 | 2.9336 |
| | $\Omega_2$ | 5.60883 | 5.56345 | 5.55091 | 5.5477 | 5.5484 |
| | $\Omega_3$ | 7.12473 | 7.05063 | 7.03096 | 7.026 | 7.0285 |
| | $\Omega_4$ | 9.73665 | 9.62799 | 9.59499 | 9.5864 | 9.5888 |

Table 1 lists the first four order natural frequency parameters of square thin plates with constant thickness under different boundary conditions using Haar wavelet, where $Lx=Ly=1$m, $h=0.1$m and material is steel ($E=210$Gpa, $\mu=0.3$, $\rho=7800$kg/m$^3$). It can be seen that a small number of matching points can achieve good numerical accuracy, and with the increase of the scaling factor $J$, the numerical results are getting closer and closer to the literature[24], thus proving that the structure is solved discretely using Haar wavelets in both directions. The natural frequency is feasible, and the boundary conditions can be accurately applied, which proves the feasibility of the method and provides a basis for the use of this method in more complex structures. $\Omega$

Table 2 to 5 shows the natural frequencies of the plates for different thicknesses under several boundary conditions using presented method.

TABLE2. NATURAL FREQUENCIES OF PLATE WITH CCCC BOUNDARY CONDITION

| Mode | h | | | | | |
|---|---|---|---|---|---|---|
| | 0.02 | 0.04 | 0.06 | 0.08 | 0.1 | 0.2 |
| 1 | 179.9086 | 359.8172 | 539.7258 | 1468.621 | 899.543 | 1799.086 |
| 2 | 367.1552 | 734.3104 | 1101.466 | 1468.621 | 1835.776 | 3671.552 |
| 3 | 367.1552 | 734.3104 | 1101.466 | 2166.072 | 1835.776 | 3671.552 |
| 4 | 541.518 | 1083.036 | 1624.554 | 2635.765 | 2707.59 | 5415.18 |
| 5 | 658.9412 | 1317.882 | 1976.824 | 2648.261 | 3294.706 | 6589.412 |
| 6 | 662.0652 | 1324.13 | 1986.196 | 719.6344 | 3310.326 | 6620.652 |

TABLE NO 3. NATURAL FREQUENCIES OF PLATE WITH CSCS BOUNDARY CONDITION

| Mode | h | | | | | |
|---|---|---|---|---|---|---|
| | 0.02 | 0.04 | 0.06 | 0.08 | 0.1 | 0.2 |
| 1 | 144.7407 | 289.4814 | 434.2221 | 578.9627 | 723.7034 | 1447.407 |
| 2 | 273.82 | 547.64 | 821.46 | 1095.28 | 1369.1 | 2738.2 |
| 3 | 346.829 | 693.6581 | 1040.487 | 1387.316 | 1734.145 | 3468.29 |
| 4 | 473.3095 | 946.619 | 1419.929 | 1893.238 | 2366.548 | 4733.095 |
| 5 | 511.7808 | 1023.562 | 1535.342 | 2047.123 | 2558.904 | 5117.808 |
| 6 | 646.5281 | 1293.056 | 1939.584 | 2586.113 | 3232.641 | 6465.281 |

TABLE NO 4. NATURAL FREQUENCIES OF PLATE WITH SSSS BOUNDARY CONDITION

| Mode | h | | | | | |
|---|---|---|---|---|---|---|
| | 0.02 | 0.04 | 0.06 | 0.08 | 0.1 | 0.2 |
| 1 | 98.68739 | 197.3748 | 296.0622 | 394.7495 | 493.4369 | 986.8739 |
| 2 | 246.8521 | 493.7043 | 740.5564 | 987.4085 | 1234.261 | 2468.521 |
| 3 | 246.8521 | 493.7043 | 740.5564 | 987.4085 | 1234.261 | 2468.521 |
| 4 | 395.1056 | 790.2112 | 1185.317 | 1580.422 | 1975.528 | 3951.056 |
| 5 | 494.19 | 988.38 | 1482.57 | 1976.76 | 2470.95 | 4941.9 |
| 6 | 494.19 | 988.38 | 1482.57 | 1976.76 | 2470.95 | 4941.9 |

TABLE NO 5. NATURAL FREQUENCIES OF PLATE WITH CFFF BOUNDARY CONDITION

| Mode | h | | | | | |
|---|---|---|---|---|---|---|
| | 0.02 | 0.04 | 0.06 | 0.08 | 0.1 | 0.2 |
| 1 | 144.5568 | 289.1136 | 433.6705 | 578.2273 | 722.7841 | 1445.568 |
| 2 | 267.354 | 534.708 | 802.0619 | 1069.416 | 1336.77 | 2673.54 |
| 3 | 346.8095 | 693.619 | 1040.429 | 1387.238 | 1734.048 | 3468.095 |
| 4 | 469.3112 | 938.6224 | 1407.934 | 1877.245 | 2346.556 | 4693.112 |
| 5 | 472.3318 | 944.6637 | 1416.996 | 1889.327 | 2361.659 | 4723.318 |
| 6 | 646.5246 | 1293.049 | 1939.574 | 2586.098 | 3232.623 | 6465.246 |

Fig. 1 shows the natural frequency change of the plate with increasing A under several boundary conditions. As shown in the Fig.1, the natural frequencies are decreased as $Ly/Lx$ increases. In particular, it can be seen that the natural frequencies are decreased rapidly until $Ly/Lx=1$, then are gradually decreased.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 06, June-2020

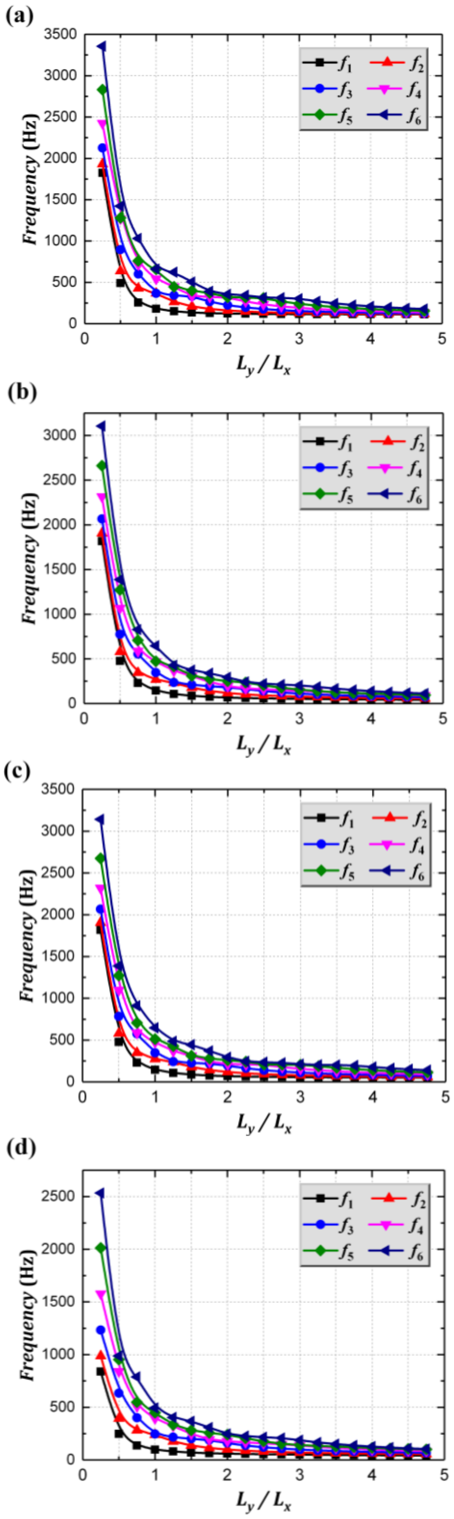


Fig.1 The change of natural frequencies as increasing of *Ly*/*Lx*
(a)-CCCC, (b)-CFCF, (c)-CSCS, (d)-SFSF

Lastly, Fig. 2 shows the change in natural frequency of a plate with a completely clamped boundary with increasing thickness *h*. It can be clearly seen from the figure that as the thickness *h* of the plate increases, the natural frequency also increases.

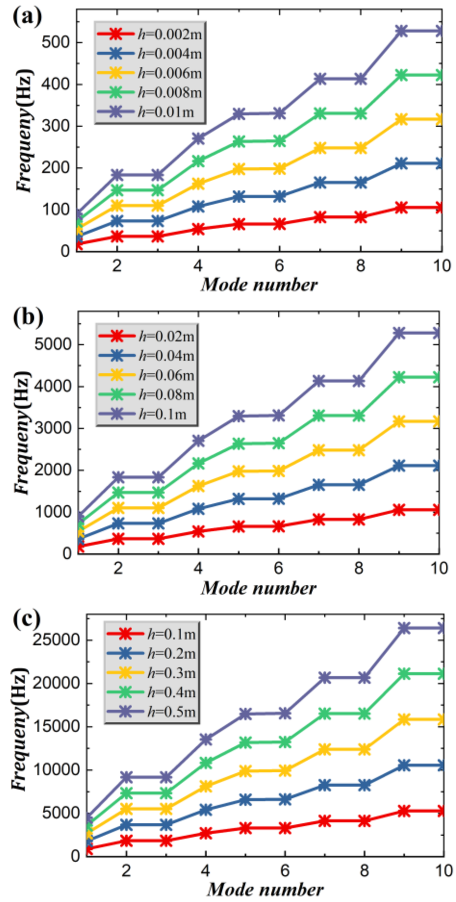


Fig.2 The change of natural frequencies as increasing of thickness *h*

## IV. NUMERICAL RESULTS

In this paper, a reasonable analysis method based on the Haar wavelet has been presented to obtain the natural frequencies of rectangular plate with several boundary conditions. The basic principles and formulas of the Haar wavelet collocation method are described, and its approximation to displacement functions using HWDM are discussed an in-depth. The solution process of Haar wavelet used in free vibration analysis of rectangular plate is given in detail. The efficiency and accuracy of presented method are proved for natural frequencies of rectangular plate with several boundary conditions. Some conclusions obtained through numerical examples and free vibration analysis results for the rectangular plate are presented, these data may be may be served as benchmark solutions for future research.

## ACKNOWLEDGMENT

The authors also gratefully acknowledge the supports from Pyongyang University of Mechanical Engineering of DPRK.

## REFERENCES

[1] F.K. Bogner, R.L. Fox, L.A. Schmit, The generation of inter-element-compatible stiffness and mass matrices by the use of interpolation formulas, Proc. Conf. Matrix Meth. Struct. Mech., Wright-Patterson Air Force Base, Ohio, AFFDL-TR-66-80, 1966, 397-443.
[2] P.A.A. Laura, R.H. Gutierrez, Fundamental frequency of vibration of clamped plates of arbitrary shape subjected to a hydrostatic state of plane stress, J. Sound Vib. 48 (1976) 327-32.
[3] R. Schinzin, P.A.A. Laura, Conformal Mapping: Methods and Applications, Elsevier, Amsterdam, 1991.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 9 Issue 06, June-2020

[4] J.L. Chen, K.P. Chong, Vibration of irregular plates by finite strip method with splined functions in: A.P. Boresi, K.P. Chong (Eds.), Engineering Mechanics in Civil Engineering, Proc. 5th Engineering Mechanics Div. ASCE 1(1984), 256—260.
[5] Y.K. Cheung, L.G. Tham, W.Y. Li, Free vibration and static analysis of general plate by spline finite strip, Comput. Mech. 3 (1989) 187-197.
[6] G.N. Geannakakes, Natural frequencies of arbitrarily shaped plates using the Rayleigh-Ritz method together with natural co-ordinate regions and normalized characteristic orthogonal polynomials, J. Sound Vib. 182 (1995) 441-478.
[7] R.G. Anderson, B.M. Irons, O.C. Zienkiewicz, Vibration and stability of plates using finite elements, Int. J. Solids Struct. 4 (1968) 1031-1055.
[8] J.H. Argyris, Continua and discontinua, Proc. Conf. Matrix Meth. Struct. Mech., Wright-Patterson Air Force Base, Ohio, AFFDL-TR-66-80, 1966, 11-189.
[9] L.R. Herrman, A bending analysis for plates, Proc. Conf. Matrix Meth. Struct. Mech., Wright-Patterson Air Force Base, Ohio, AFFDL-TR-66-80, 1966, 577-602.
[10] R.J. Melosh, A flat triangular shell element stiffness matrix, Proc. Conf. Matrix Meth. Struct. Mech., Wright-Patterson Air Force Base, Ohio, AFFDL-TR-66-80, 1966, 503-514.
[11] M. Petyt, Finite element vibration analysis of cracked plates in tension, Technical Report AFML-TR-67-396, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio, 1968.
[12] M.D. Olson, G.M. Lindberg, Annual and circular sector finite elements for plate bending, Int. J. Meth. Sci. 11 (1969) 17-23.
[13] M.W. Chernuka, G.R. Cowper, G.M. Lindberg, M.D. Olson, Finite element analysis of plates with curved edges, Int. J. Numer. Methods Eng. 4 (1972) 49-65.
[14] Grossman A, Morlet J. Decomposition of hardly functions into square integrable wavelets of constant shape [J]. SIAM J. Math. 15(1984), 723-736.
[15] Meyer Y.Principe d' incertitude bases hilbertiennes et algèbras d' operateurs, Borubaki Séminaire. 662(1985), 1271-1283.
[16] Daubechies I. Orthonormal bases of compactly supported wavelets, Communications on Pure and Applied Mathematics.1988, XII: 909-996.
[17] Daubechies I. Ten Lectures on Wavelets. CBMS-NSF Regional Conference Series in Applied Mathmatics, Philadephia, SIAM, 1992.
[18] Mallat S G. Multiresolution representation and wavelet. Philadelphia, PA: University of Pennsylvania, 1988.
[19] Mallat S G. A theory for multiresolution signal decomposition: the wavelet representation. IEEE Transactions on Pattern Analysis and Machine Intelligence.11(1989), 674~693.
[20] Majak J, Pohlak M, Eerme M. Application of the Haar wavelet-based discretization technique to problems of orthotropic plates and shells. Mech Compos Mater. 45(2009), 631–42.
[21] Majak J, Pohlak M, Eerme M, Lepikult T. Weak formulation based Haar wavelet method for solving differential equations. Appl Math Comput. 211(2009), 488–94.
[22] Hein H, Feklistova L. Free vibrations of non-uniform and axially functionally graded beams using Haar wavelet. Eng Struct. 33(2011), 3696–701.
[23] Hein H, Feklistova L. Computationally efficient delamination detection in composite beams using Haar wavelets. Mech Syst Signal Process. 25(2011), 2257–70.
[24] Timoshenko S P, Goodier J N. Theory of Elasticity.3rd edition. New York: McGraw-Hill Press, 1970.
[25] Jin GY, Xie X, Liu ZG. The Haar wavelet method for free vibration analysis of functionally graded cylindrical shells based on the shear deformation theory. Compos Struct. 108(2014), 435–48.
[26] Xie X, Jin GY, Liu ZG. Free vibration analysis of cylindrical shells using the Haar wavelet method. Int J Mech Sci. 77(2013), 47–56.
[27] Xie X, Jin GY, Yan YQ, Shi SX, Liu ZG. Free vibration analysis of composite laminated cylindrical shells using the Haar wavelet method. Compos Struct. 109(2014), 169–77.
[28] Xie Xiang, Jin Guoyong, Li Wanyou, Liu Zhigang. A numerical solution for vibration analysis of composite laminated conical, cylindrical shell and annular plate structures. Compos Struct. 111 (2014), 20–30.