Application of Differential Equations

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Abstract - **In this paper we show that a method of embedding for a class of non-linear Volterra equations can be used in a novel fashion to obtain variation of parameters formulas for Volterra integral equations subjected to a general type of variation of the equation. The approach is of intrinsic interest. Our variation of parameters formulas generalize classical formulas for ordinary differential equations and for linear Volterra integral equations (based on resolvents). Illustrative examples are related to known results.**

Keywords: Volterra equation, integral equation, variation of parameters, ordinary differential equations, linear equations, non linear equations.

INTRODUTION

Sir Isaac Newton (1642 - 1727) English scientist and mathematician famous for his discovary of the law of gravity and three laws of motion. Today these laws are known as Newton's laws of motion and descripe the motion of all objects on the scale we experience in our everyday lives. Solved his first differential equation, by the use of infinite series, eleven years after his discovery of calculus in 1665. Gottfried Wilhelm Leibniz (also known as von Leibniz) was a prominent German mathematician, philosopher, physicist and statesman. Noted for his independent invention of the differential and integral calculus, Gottfried Leibniz remains one of the greatest and most influential metaphysicians, thinkers and logicians in history. He also invented the Leibniz wheel and suggested important theories about force, energy and time.

DEFINITIONS

Differential equation:

Any relation involving the dependent variable, independent variable (or variables) and the differential coefficient (or coefficients) of the dependent variable with respect to the independent variable(or variables) is known as a differential equation. (Or)

A differential equation is a relationship between a function of time and its derivative.

For example:

1. $\frac{dy}{dx} = \cot x$

5. $\frac{d^2y}{dx^2} + 4y = \tan 2x$

2. $\frac{d^2y}{dx^2} + y = 0$

6. $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

3. $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$

7. $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial x}$

4. $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$

8. $\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$

Are all differential equations.

VARIATION OF PARAMETER

Variation of parameter is another method for finding a particular solution of the nth-order linear differential equation

$$L(y) = \varphi(x) \qquad --------\rightarrow (1)$$

Once the solution of the associated homogeneous equation

$L(y) = 0$ is known. If $y_1(x), y_2(x), \cdots, y_n(x)$ are n linearly independent solutions of $L(y) = 0$,then the general solution of $L(y) = 0$ is

$$y_h = c_1y_1(x) + c_2y_2(x) + \cdots c_ny_n(x) ---\rightarrow (2)$$

The method:

A particular solution of $L(y) = \varphi(x)$ has the form

$$y_p = v_1 y_1 + v_2 y_2 + \cdots v_n y_n$$

Where $y_i = y_i(x)(i = 1,2,3,\cdots,n)$is given in (2) and $v_i(i = 1,2,\cdots,n)$ is an unknown function of x which still must be determined.

To find v_i, first solve the following linear equations simultaneously forv'_i.

$$v'_1 y_1 + v'_2 y_2 + \cdots + v'_n y_n = 0$$
$$v'_1 y'_1 + v'_2 y'_2 + \cdots + v'_n y'_n = 0$$
$$\vdots$$
$$v'_1 y_1^{(n-2)} + v'_2 y_2^{(n-2)} + \cdots + v'_n y_n^{(n-2)} = 0$$
$$v'_1 y_1^{(n-1)} + v'_2 y_2^{(n-1)} + \cdots + v'_n y_n^{(n-1)} = \varphi(x)$$

Then integrate each $v'_i$to obtain v_i, disregarding all constraints of integration. This is permissible because we are seeking only one particular solution.

Example :

For the special case n=3, reduce to

$$v'_1 y_1 + v'_2 y_2 + v'_3 y_3 = 0$$
$$v'_1 y'_1 + v'_2 y'_2 + v'_3 y'_3 = 0$$
$$v'_1 y''_1 + v'_2 y''_2 + v'_3 y''_3 = \varphi(x)$$

Problems

1. $solve\ y'' + y = \sec x.$

Solution:

Given $y'' + y = \sec x.$

This is a third-order equation with

$$y_h = c_1 \cos x + c_2 \sin x \quad ------\rightarrow (1)$$

It follows from equation,

$$y_p = v_1 \cos x + v_2 \sin x \quad ------\rightarrow (2)$$

Using linear equation,

$$v'_1 y_1 + v'_2 y_2 = 0$$
$$v'_1 y'_1 + v'_2 y'_2 = \varphi(x)$$

Here $y_1 = \cos x, y_2 = \sin x$ & $\varphi(x) = \sec x$

$$\therefore v'_1 \cos x + v'_2 \sin x = 0$$

$\Rightarrow$ $$v'_1 \cos x = -v'_2 \sin x$$
$$v'_1 = -v'_2 \tan x ---\rightarrow (3)$$
$$v'_1(-\sin x) + v'_2 \cos x = \sec x$$

$\Rightarrow$ $$v'_2(\tan x \sin x) + v'_2 \cos x = \sec x$$

$\Rightarrow$ $$v'_2 \left[\frac{\sin^2 x + \cos^2 x}{\cos x}\right] = \sec x$$
$$v'_2 = \frac{1}{\cos x} \times \cos x = 1$$
$$\therefore \quad v'_2 = 1$$
$$(3) \Rightarrow \quad v'_1 = -\tan x$$

Thus $$v_1 = \int v'_1 dx = \int(-\tan x)dx = \log(\cos x)$$
$$v_2 = \int v'_2 dx = \int dx = x$$
$$\therefore \quad v_1 = \log(\cos x) \quad \& \quad v_2 = x$$

Substituting these values into (2) we obtain

$$y_p = \log(\cos x)\cos x + x \sin x$$

The general solution is

$$y = y_h + y_p$$
$$= c_1 \cos x + c_2 \sin x + \cos x(\log(\cos x)) + x \sin x$$

2. $solve\ y'' - y' - 2y = e^{3x}$

Solution:

This is second-order equation

Here n=2 & $y_h = c_1e^{-x} + c_2e^{2x}$

Hence $y_p = v_1e^{-x} + v_2e^{2x}$ $-----------\rightarrow (1)$

Using linear equation is

$$v'_1y_1 + v'_2y_2 \quad = 0$$

$$v'_1y'_1 + v'_2y'_2 = \varphi(x)$$

$here\ y_1 = e^{-x}, y_2 = e^{2x}$ & $\varphi(x) = e^{3x}$,

$$\therefore v'_1e^{-x} + v'_2e^{2x} \quad = 0 \quad ------------\rightarrow (2)$$

$$\Rightarrow \quad v'_1e^{-x} = -v'_2e^{2x}$$

$$v'_1 = -v'_2e^{3x}$$

$$\& \quad -v'_1(-e^{-x}) + 2v'_2e^{2x} \quad = 0$$

$$\Rightarrow \quad v'_2e^{2x} + 2v'_2e^{2x} = e^{3x}$$

$$3v'_2e^{2x} = e^{3x}$$

$$v'_2 \quad = \frac{e^x}{3}$$

$$(2) \Rightarrow \quad v'_1e^{-x} = -\frac{e^{3x}}{3}$$

$$v'_1 = -\frac{e^{4x}}{3}$$

$$\therefore \quad v'_1 = -\frac{e^{4x}}{3} \quad \& \quad v'_2 = \frac{e^x}{3}$$

Thus $v_1 = \int v'_1dx = \int -\frac{e^{4x}}{3}dx$

$$= -\frac{1}{3}\frac{e^{4x}}{4} = -\frac{e^{4x}}{12}$$

$$v_2 = \int v'_2dx = \int \frac{e^x}{3}dx$$

$$v_2 = \frac{e^x}{3}$$

$$\therefore \quad v_1 = -\frac{e^{4x}}{12} \quad \& \quad v_2 = \frac{e^x}{3}$$

Substituting these values into (1) we obtain

$$y_p = -\frac{e^{4x}}{12}e^{-x} + \frac{e^x}{3}e^{2x}$$

$$= -\frac{e^{3x}}{12} + \frac{e^{3x}}{3} = \frac{-e^{3x}+4e^{3x}}{12}$$

$$y_p = \frac{e^{3x}}{4}$$

The general solution is,

$$y = y_h + y_p = c_1e^{-x} + c_2e^{2x} + \frac{e^{3x}}{4}$$

CONCLUSION

I am very glad that I had an opportunity to do an independent paper in differential equation & their application. In the short span of time I have done best of my level. I scincerely hope that this small paper work will inspire the reader to do further reading in this field. This paper is mainly concerned with many interesting chapters such as basic concepts of differential equation , variation of parameter & etc.

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