International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 14 Issue 11 , November - 2025

Application of a Semi-Analytical Method for the Free Vibration and Damping Analysis of Sandwich Shell Structures

Kwang-Hun Kim, Ju-Song Kim
Faculty of Mechanical Engineering No.1, Pyongyang University of Mechanical Engineering, Democratic People's Republic of Korea

Hyon-Sik Hong
Institute of Mechanics, State Academy of Science, Democratic People's Republic of Korea

Song Ri
Faculty of Electronics and Automation, **Kim Il Sung** University, Pyongyang, Democratic People's Republic of Korea

Jong-Hun Kye
Department of Physics, University of Sciences, Pyongyang, Democratic People's Republic of Korea

Abstarct - **In this paper, the free vibration and damping characteristics of sandwich cylindrical and conical shells with viscoelastic core are investigated by using a semi-analytical method. Donnel's shell theory and the energy principle are applied to establish the theoretical formulations of the sandwich shell with viscoelastic core. The displacement components of the viscoelastic core are expressed as those of base and constraining layers by using continuity condition. The displacement field of the sandwich shell expanded by the Legendre polynomials in axial direction and Fourier series in circumferential direction. The presented method are verified to have enough reliability and accuracy for predicting the natural frequencies and modal loss factors of the sandwich cylindrical and conical shells with viscoelastic layer by comparing to the vibrational analysis results of published article.**

Keywords: Sandwich shell, a semi-analytical method, Free vibration, Damping characteristics, Numerical analysis

I. INTRODUCTION

Sandwich structures are widely used in various engineering applications duo to the high vibration and noise reduction ability caused by the large shear deformation of the viscoelastic material layer [1]. Therefore, the study on the dynamic characteristics of the sandwich structures have attracted a lot of interest of many researchers [2, 3]. Yang et al. [4] studied the vibration and damping characteristics of the sandwich conical shells and annular plates with arbitrary boundary conditions including classical and elastic ones by using a simple and efficient modified Fourier solution. Wang et al. [5] presented a semi-analytical method for the free vibration analysis of the functionally graded (FG) sandwich doubly-curved panels and shells of revolution with arbitrary boundary conditions. Bardell et al. [6] presented the vibration study of a general three-layer conical sandwich panel based on the h-p version of the finite element method. In their study, the h-p finite element formulation of sandwich panel was derived based on a set of trigonometric assumed displacement functions. Singha et al. [7] investigated the free vibration behavior of rotating pretwisted sandwich conical shell panels with functionally graded graphene-reinforced composite (FG-GRC) face sheets and homogenous core using finite element method in conjunction with HSDT. Sofiyev and Osmancelebioglu [8] demonstrated the effectiveness of functionally graded coatings in the vibration of sandwich truncated conical shells. In their study, the governing equations were established by using FSDT and Donnell kinematics assumptions. Jin et al. [9] developed an accurate solution for the vibration and damping characteristics of a three-layered passive constrained layer damping (PCLD) cylindrical shell with general elastically restrained boundaries by means of the modified Fourier–Ritz method in conjunction with Donnell shell assumptions and linear viscoelastic theory. Sahu et al. [10] conducted the free vibration study of doubly curved sandwich shell panels having a core of viscoelastic material, constrained by a functionally graded material (FGM) layer by using finite element method (FEM) in framework of FSDT.

Meanwhile, researchers proposed different numerical methods such as FEM [10, 11], differential quadrature method [12], pb-2 Ritz method [13], Non-Uniform Rational B-Splines (NURBS) method [14], spectral-Tchebychev solution technique [15], dynamic stiffness method [16-18], meshfree method [19, 20] for the dynamic analysis of composite shells and plates. The meshfree method is attracting attention from researchers due to their excellent behaviors [21-23].

In this study, a semi-analytical method for the free vibration and damping analysis of sandwich cylindrical and conical shells with viscoelastic core with Legendre polynomials as displacement functions is presented. The theoretical formulations of the sandwich shell are established by using the energy principle, FSDT and Donnel's shell thoery. Using the continuity condition in interface between the layers, the displacement components of the viscoelastic core are replaced by those of base and constraining layers. The displacement component of the sandwich shell are approximated by using Fourier series in conjunction with Legendre polynomials. The accuracy and reliability of the presented method are verified through the comparison with the results of published literature.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 14 Issue 11 , November - 2025

II. THEORETICAL FORMULATIONS

Fig. 1 shows the geometry and coordinate system of a sandwich conical shell which is composed of the base layer, the viscoelastic core and the constraining layer. Orthogonal curvilinear coordinate systems (x, θ, z) are located on the middle surfaces of each layer of the conical shell. The symbols L and α are the length and semi-vertex angle of the sandwich conical shell. R_{i0} and h_i (i=s,v,c) denote small edge radius and thickness of each layer, and the subscripts s, v, and c are indicated for base layer, viscoelastic core and constraining layer, respectively. The cylindrical shell is considered as a conical shell with semi-vertex angle α=0.

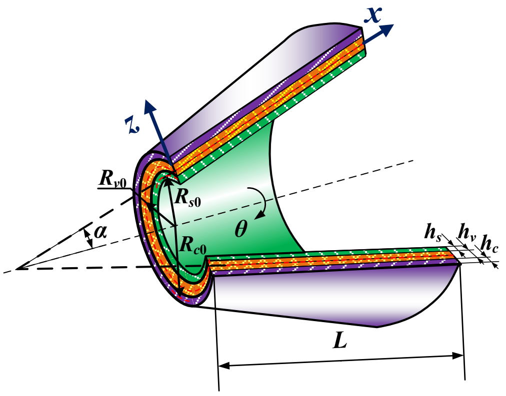


Fig. 1. Geometry and coordinate system of sandwich conical shell with viscoelastic core.

Based on the FSDT and Donnel's shell theory, the displacement components at any point of individual layer are expressed as [9]

$$\begin{cases} \bar{u}_i = u_i + z\psi_{xi} \\ \bar{v}_i = v_i + z\psi_{\theta i} \quad i = s,v,c \\ \bar{w}_i = w \end{cases} \tag{1}$$

where u_i, v_i and w represent the middle plane displacements of each layer in the x, θ and z directions, respectively. In the base and constraining layers, the shear rotations ψ_{xi} and $\psi_{\theta i}$ can be written as follows.

$$\begin{cases} \psi_{xi} = -\dfrac{\partial w}{\partial x}, \\ \psi_{\theta i} = -\dfrac{\partial w}{R_i \partial \theta} \end{cases} \quad i = s,c \tag{2}$$

where R_i=R_{i0}+$x\sin\alpha$.

By using the displacement continuity between layers, the displacements of the viscoelastic layer can be expressed as:

$$\begin{aligned} u_v &= \frac{1}{2}\left(u_s + u_c + \frac{h_s}{2}\psi_{xs} - \frac{h_c}{2}\psi_{xc}\right), \\ \psi_{xv} &= \frac{1}{h_v}\left(u_c - u_s - \frac{h_s}{2}\psi_{xs} - \frac{h_c}{2}\psi_{xc}\right), \\ v_v &= \frac{1}{2}\left(v_s + v_c + \frac{h_s}{2}\psi_{\theta s} - \frac{h_c}{2}\psi_{\theta c}\right), \\ \psi_{\theta v} &= \frac{1}{h_v}\left(v_c - v_s - \frac{h_s}{2}\psi_{\theta s} - \frac{h_c}{2}\psi_{\theta c}\right) \end{aligned} \tag{3}$$

Generally, the Young's modulus of the viscoelastic layer is much smaller than that of the base and constraining layers. Therefore, it can be assumed that the viscoelastic material layer undergoes only shear strains while the other two layers are only allowed flexural and axial deformations. Considering Eq. (3), the strains of the core layer are expressed as:

$$\begin{aligned} \gamma_{xz}^v &= \frac{1}{h_v}(u_c - u_s) + c_x\frac{\partial w}{\partial x} \\ \gamma_{\theta z}^v &= \left(\frac{1}{h_v} - \frac{\cos\alpha}{2R_v}\right)v_c - \left(\frac{1}{h_v} + \frac{\cos\alpha}{2R_v}\right)v_s + c_\theta\frac{\partial w}{\partial \theta} \end{aligned} \tag{4}$$

where

$$\begin{aligned} c_x &= \frac{h_s + h_c + 2h_v}{2h_v} \\ c_\theta &= \begin{bmatrix} \dfrac{1}{2h_v}\left(\dfrac{h_c}{R_c} + \dfrac{h_s}{R_s}\right) \\ + \dfrac{1}{4R_v}\left(\dfrac{h_s\cos\alpha}{R_s} - \dfrac{h_c\cos\alpha}{R_c} + 4\right) \end{bmatrix} \end{aligned} \tag{5}$$

The shear stresses of the core layer are expressed as:

$$\tau_{xz}^v = G_v\gamma_{xz}^v \quad \tau_{\theta z}^v = G_v\gamma_{\theta z}^v \tag{6}$$

where the shear modulus G_v of the viscoelastic material is composed of real and imaginary parts.

$$G_v = G_r + iG_i \tag{7}$$

where G_r and G_i denote the real part and imaginary part of the complex shear modulus of the viscoelastic material layer, respectively.

Meanwhile, the stress-strain relationships of the base and constraining layers can be written as follows.

$$\begin{bmatrix} \sigma_x^i \\ \sigma_\theta^i \\ \tau_{x\theta}^i \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{11} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^i \\ \varepsilon_\theta^i \\ \gamma_{x\theta}^i \end{bmatrix} \quad i = s,c \tag{8}$$

where the stiffness coefficients Q_{mn} of the isotropic material are as

$$Q_{11} = \frac{E_i}{1-\mu_i^2}, \quad Q_{12} = \frac{\mu_i E_i}{1-\mu_i^2}, \quad Q_{66} = \frac{E_i}{2(1+\mu_i)} \tag{9}$$

where E_i and μ_i are the Young's modulus and Poisson's ratios of the base and constraining layers. The strain-displacement relationships of the base and constraining layers can be written as:

$$\begin{aligned} \varepsilon_x^i &= \frac{\partial u_i}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_\theta^i &= \frac{\sin\alpha}{R_i}u_i + \frac{1}{R_i}\frac{\partial v_i}{\partial \theta} + \frac{\cos\alpha}{R_i}w \\ &\quad - z\frac{\sin\alpha}{R_i}\frac{\partial w}{\partial x} - \frac{z}{R_i^2}\frac{\partial^2 w}{\partial \theta^2} \\ \gamma_{x\theta}^i &= \frac{1}{R_i}\frac{\partial u_i}{\partial \theta} + \frac{\partial v_i}{\partial x} - \frac{\sin\alpha}{R_i}v_i \\ &\quad + 2z\frac{\sin\alpha}{R_i^2}\frac{\partial w}{\partial \theta} - \frac{2z}{R_i}\frac{\partial^2 w}{\partial x\partial \theta} \end{aligned} \tag{10}$$

The strain energies of the sandwich shell can be described as follows.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 14 Issue 11 , November - 2025

$$\begin{aligned} U &= U_s + U_v + U_c \\ &= \frac{1}{2}\int_0^{2\pi}\int_0^{L}\int_{-h_s/2}^{h_s/2}\left(\sigma_x^s\varepsilon_x^s + \sigma_\theta^s\varepsilon_\theta^s + \tau_{x\theta}^s\gamma_{x\theta}^s\right)R_s dz dx d\theta \\ &+ \frac{1}{2}\int_0^{2\pi}\int_0^{L}\int_{-h_v/2}^{h_v/2}\left(\tau_{xz}^v\gamma_{xz}^v + \tau_{\theta z}^v\gamma_{\theta z}^v\right)R_v dz dx d\theta \\ &+ \frac{1}{2}\int_0^{2\pi}\int_0^{L}\int_{-h_c/2}^{h_c/2}\left(\sigma_x^c\varepsilon_x^c + \sigma_\theta^c\varepsilon_\theta^c + \tau_{x\theta}^c\gamma_{x\theta}^c\right)R_c dz dx d\theta \end{aligned} \tag{11}$$

The kinetic energies of the sandwich shell can be given as

$$\begin{aligned} T &= T_s + T_v + T_c = \frac{\rho_s h_s}{2}\int_0^{2\pi}\int_0^{L}\left(\dot{u}_s^2 + \dot{v}_s^2 + \dot{w}^2\right)R_s dx d\theta \\ &+ \frac{\rho_v h_v}{2}\int_0^{2\pi}\int_0^{L}\dot{w}^2 R_v dx d\theta + \frac{\rho_c h_c}{2}\int_0^{2\pi}\int_0^{L}\left(\dot{u}_c^2 + \dot{v}_c^2 + \dot{w}^2\right)R_c dx d\theta \end{aligned} \tag{12}$$

where ρ_i (i=s,v,c) denotes the density of each layer.

The elastic energies stored in distributed springs of base and constraining layers can be given as

$$\begin{aligned} U_{BC} &= \frac{1}{2}\int_0^{2\pi}\left[k_{u0}u_s^2 + k_{v0}v_s^2 + k_{w0}w^2 + K_{w0}\left(\partial w/\partial x\right)\right]_{x=0} R_s d\theta \\ &+ \frac{1}{2}\int_0^{2\pi}\left[k_{u1}u_s^2 + k_{v1}v_s^2 + k_{w1}w^2 + K_{w1}\left(\partial w/\partial x\right)\right]_{x=L} R_s d\theta \\ &+ \frac{1}{2}\int_0^{2\pi}\left[k_{u0}u_c^2 + k_{v0}v_c^2 + k_{w0}w^2 + K_{w0}\left(\partial w/\partial x\right)\right]_{x=0} R_c d\theta \\ &+ \frac{1}{2}\int_0^{2\pi}\left[k_{u1}u_c^2 + k_{v1}v_c^2 + k_{w1}w^2 + K_{w1}\left(\partial w/\partial x\right)\right]_{x=L} R_c d\theta \end{aligned} \tag{13}$$

The total Lagrangian energy function of the sandwich conical shell can be written as follows.

$$L = T - U - U_{BC} \tag{14}$$

By introducing the Legendre polynomial, the displacement components can be expanded as follows.

$$\begin{aligned} u_s &= \sum_{m=1}^{N}\sum_{n=0}^{M}\phi_m(x)\left[\cos(n\theta)\tilde{U}_{mn}^s + \sin(n\theta)\bar{U}_{mn}^s\right]e^{i\omega t} \\ v_s &= \sum_{m=1}^{N}\sum_{n=0}^{M}\phi_m(x)\left[\sin(n\theta)\tilde{V}_{mn}^s + \cos(n\theta)\bar{V}_{mn}^s\right]e^{i\omega t} \\ u_c &= \sum_{m=1}^{N}\sum_{n=0}^{M}\phi_m(x)\left[\cos(n\theta)\tilde{U}_{mn}^c + \sin(n\theta)\bar{U}_{mn}^c\right]e^{i\omega t} \\ v_c &= \sum_{m=1}^{N}\sum_{n=0}^{M}\phi_m(x)\left[\sin(n\theta)\tilde{V}_{mn}^c + \cos(n\theta)\bar{V}_{mn}^c\right]e^{i\omega t} \\ w &= \sum_{m=1}^{N}\sum_{n=0}^{M}\phi_m(x)\left[\cos(n\theta)\tilde{W}_{mn} + \sin(n\theta)\bar{W}_{mn}\right]e^{i\omega t} \end{aligned} \tag{15}$$

where $\phi_m(x)$ is the m-order Legendre polynomial for the displacement function; ω is an angular frequency, t denotes time. The symbols $\tilde{U}_{mn}^s$, $\tilde{V}_{mn}^s$, $\tilde{U}_{mn}^c$, $\tilde{V}_{mn}^c$, $\tilde{W}_{mn}$, $\bar{U}_{mn}^s$, $\bar{V}_{mn}^s$, $\bar{U}_{mn}^c$, $\bar{V}_{mn}^c$ and $\bar{W}_{mn}$ unknown coefficients of the Legendre polynomials that you want to obtain;

Minimizing the above Lagrangian energy function with respect to the unknown nodal displacement components,

$$\frac{\partial L}{\partial q} = 0, \quad q = \tilde{U}_{mn}^s, \tilde{V}_{mn}^s, \tilde{U}_{mn}^c, \tilde{V}_{mn}^c, \tilde{W}_{mn}, \bar{U}_{mn}^s, \bar{V}_{mn}^s, \bar{U}_{mn}^c, \bar{V}_{mn}^c, \bar{W}_{mn} \tag{16}$$

Substituting Eq. (15) into Eq.(14), following governing equations are obtained.

$$\left(\boldsymbol{K} - \omega^2\boldsymbol{M}\right)\boldsymbol{U}_s = \boldsymbol{0} \tag{17}$$

where $\boldsymbol{U}_s$ is the nodal displacement vector, and $\boldsymbol{K}$ and $\boldsymbol{M}$ are the stiffness and mass matrices. From the above Eq.(17), the complex eigenvalues are obtained. The real part of the natural frequency ω of the sandwich shell and the modal loss factor η are defined as follows.

$$f = \frac{\sqrt{\mathrm{Re}\left(\omega^2\right)}}{2\pi} \tag{18}$$

$$\eta = \frac{\mathrm{Im}\left(\omega^2\right)}{\mathrm{Re}\left(\omega^2\right)} \tag{19}$$

III. Numerical Results

It is very important to determine the proper degree of polynomial that can simultaneously guarantee the accuracy of the solution and the computational efficiency.

Because, increasing the degree of the polynomial will reduce the computational efficiency: the computation time for the solution process will be long, and the increase of the excessive polynomial degree will result in the case that the solution does not converge and diverges. For the determination of the proper degree of a polynomial, the natural frequency convergence characteristics with increasing polynomial degree are investigated and the results are shown in Fig. 2.

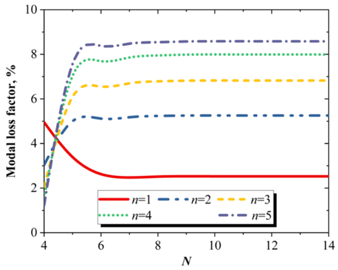
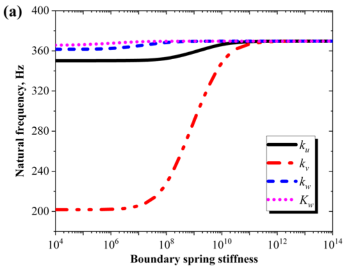


Fig.2. Convergence of modal loss factor for a sandwich conical shell.

As shown in Fig. 2, the present method can be ensured stable convergence for predicting the natural frequencies and modal loss factors of the sandwich conical shell. The boundary conditions are generalized by the introduction of an artificial spring technique, and the type of boundary conditions is selected according to the spring stiffness. The effects of spring stiffness values of elastic boundary on the natural frequencies and modal loss factors of sandwich conical and cylindrical shells are investigated in Fig. 3.



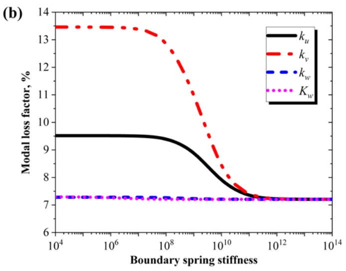


Fig.3. Convergence of natural frequency for a sandwich shells with different boundary spring stiffness;
(a) conical, (b) cylindrical.

Based on the above study, the spring stiffness values for different boundary conditions considered in this study are set as shown in Table 1, in which the symbols F, C, S and SD mean free, clamped, simply supported and shear diaphragm boundary conditions, respectively.

Table 1. Stiffness values for different boundary springs.

B.C.	Boundary spring stiffness value			
	$k_{u,}$	k_v	k_w	K_w
F	0	0	0	0
C	10^{14}	10^{14}	10^{14}	10^{14}
S	10^{14}	10^{14}	10^{14}	0
SD	0	10^{14}	10^{14}	0

Next, the free vibration results of a sandwich cylindrical shell are compared with those of literature to verify the accuracy of the proposed method. In Table 2, the natural frequencies for a sandwich cylindrical shell with various boundary conditions are compared with those of literature.

Table. 2. Comparison of natural frequencies for a sandwich cylindrical shell with various boundary conditions.

B.Cs	n	Ref.[9]	Present	Diff,%
C-C	1	873.13	873.128	0.0002
	2	821.85	821.847	0.0004
	3	764.95	764.945	0.0007
C-S	1	820.19	820.185	0.0006
	2	765.25	765.244	0.0008
	3	702.95	702.942	0.0011
C-SD	1	799.83	799.824	0.0008
	2	757.92	757.918	0.0003
	3	702.45	702.442	0.0011
C-F	1	669.29	669.283	0.0010
	2	519.66	519.647	0.0025
	3	406.55	406.536	0.0034

As observed from Table 2, the frequency results obtained by the proposed method agree well with those of the literature.

Based on the verification study of the proposed method, the effect of some parameters on the natural frequency and the modal loss factor of the sandwich cylindrical and conical shells are investigated. First, the effect of semi-vertex angle of a sandwich conical shell on the frequency parameters is considered in Table 3.

Table. 3. Frequency parameters for a sandwich conical shell with various semi-vertex angles (m=1).

α	n	B.C.			
		C-C	C-S	C-SD	S-S
$\pi/6$	1	12.915	12.549	11.745	12.544
	2	10.968	10.685	10.595	10.608
	3	9.205	8.905	8.869	8.749
	4	7.958	7.620	7.410	7.403
$\pi/4$	1	10.043	9.601	8.804	9.596
	2	8.781	8.418	8.234	8.351
	3	7.572	7.207	7.206	7.062
	4	6.708	6.312	6.215	6.105
$\pi/3$	1	7.107	6.634	6.024	6.606
	2	6.384	5.940	5.756	5.847
	3	5.703	5.248	5.244	5.081
	4	5.265	4.778	4.745	4.559

The variations of the dimensionless frequencies and the modal loss factors of a sandwich cylindrical shell with various thickness ratio h_v/h are shown in Fig. 4.

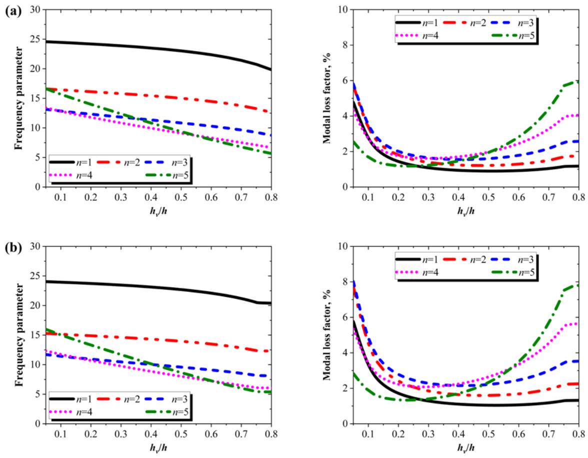


Fig.4. Variation of frequency parameters and modal loss factor for a sandwich cylindrical shell with various thickness ratios; (a) C-C, (b) C-SD.

It is clear that the frequency parameters decrease as the thickness of viscoelastic layer increases. But modal loss factors have no such change law.

IV. CONCLUSION

In this paper, a semi-analytical method is proposed to analyze the vibration and damping characteristics of sandwich cylindrical and conical shells with viscoelastic material core. The theoretical formulations of the structural model are established by using the energy principle in framework of FSDT and Donnel's shell theory. The energy of the sandwich shell is composed of that of base layer, viscoelastic core and constraining layer. The displacement components at any point of the sandwich shell are expanded by the Legendre polynomials in the meridional direction and Fourier series in the circumferential direction. Numerical examples for free vibration and damping analyses of sandwich cylindrical and conical shells with viscoelastic core are presented to verify the reliability and accuracy of the presented method. First, the free vibration analysis results of the sandwich shells obtained by the proposed method are compared with those of published literature. Finally, the effects of several parameters such as geometric dimension, material properties and boundary condition on the frequency parameter and modal loss factor of the shell are investigated.

REFERENCES

[1] Yanchun Zhai, Jianmin Su, Sen Liang. Damping properties analysis of composite sandwich doubly-curved shells. Composites Part B 2019;161:252-262.

[2] Mahsa Karimiasl, Farzad Ebrahimi. Large amplitude vibration of viscoelastically damped multiscale composite doubly curved sandwich shell with flexible core and MR layers. Thin-Walled Structures 2019;144:106128.

[3] Deepak Kumar Biswal, Sukesh Chandra Mohanty. Free vibration and damping characteristics study of doubly curved sandwich shell panels with viscoelastic core and isotropic/laminated constraining layer. European Journal of Mechanics/A Solids 2018;72:424-439.

[4] Chuanmeng Yang, Guoyong Jin, Weijian Xu and Zhigang Liu. A modified Fourier solution for free damped vibration analysis of sandwich viscoelastic-core conical shells and annular plates with arbitrary restraints. International Journal of Applied Mechanics 2016;8:1650094:1-30.

[5] Qingshan Wang, Xiaohui Cui, Bin Qin, Qian Liang, Jinyuan Tang. A semi-analytical method for vibration analysis of functionally graded (FG) sandwich doubly-curved panels and shells of revolution. International Journal of Mechanical Sciences 2017;134:479-499.

[6] N. S. Bardell, R. S. Langley, J. M. Dunsdon, G. S. Aglietti. An h-p finite element vibration analysis of open conical sandwich panels and conical sandwich frusta. Journal of Sound and Vibration 1999;226(2):345-377.

[7] Tripuresh Deb Singha, Mrutyunjay Rout, Tanmoy Bandyopadhyay, Amit Karmakar. Free vibration of rotating pretwisted FG-GRC sandwich conical shells in thermal environment using HSDT 2021;257:113144.

[8] A.H. Sofiyev, E. Osmancelebioglu. The free vibration of sandwich truncated conical shells containing functionally graded layers within the shear deformation theory. Composites Part B 2017;120:197-211.

[9] Guoyong Jin, Chuanmeng Yang, Zhigang Liu, Siyang Gao, Chunyu Zhang. A unified method for the vibration and damping analysis of constrained layer damping cylindrical shells with arbitrary boundary conditions. Composite Structures 2015;130:124-142.

[10] Nishant Kumar Sahu, Deepak Kumar Biswal, Shince V Joseph, Sukesh Chandra Mohanty. Vibration and damping analysis of doubly curved viscoelastic-FGM sandwich shell structures using FOSDT. Structures 2020;26:24-38.

[11] Nicholas Fantuzzi, Francesco Tornabene, Michele Bacciocchi, Antonio J.M. Ferreira. On the Convergence of Laminated Composite Plates of Arbitrary Shape through Finite Element Models. Journal of Composites Science 2018;16:1-50:doi:10.3390/jcs2010016

[12] Shu C, Chen W, Du H. Free vibration analysis of curvilinear quadrilateral plates by the differential quadrature method. J Comput Phys 2000;163(2):452-466.

[13] Al-Bermani FGA, Liew KM. Natural frequencies of thick arbitrary quadrilateral plates using the pb–2 Ritz method. J Sound Vib 1996;196(4):371-385.

[14] Fantuzzi N, Tornabene F. Strong formulation isogeometric analysis (SFIGA) for laminated composite arbitrarily shaped plates. Compos B Eng 2016; 96: 173–203.

[15] Bekir Bediz. A spectral-Tchebychev solution technique for determining vibrational behavior of thick plates having arbitrary geometry. Journal of Sound and Vibration 2018;432:272-289.

[16] Md. Imran Ali, M.S. Azam, V. Ranjan, J.R. Banerjee. Free vibration of sigmoid functionally graded plates using the dynamic stiffness method and the Wittrick-Williams algorithm. Computers and Structures 2021;244:106424.

[17] Subodh Kumar, Vinayak Ranjan, Prasun Jana. Free vibration analysis of thin functionally graded rectangular plates using the dynamic stiffness method. Composite Structures 2018;197:39-53.

[18] Suraj Yadav, Pramod Kumar. Free vibration analysis of an orthotropic plate by dynamic stiffness method and Wittrick-Williams algorithm. Materials Today: Proceedings 2021;47(13):4046-4051.

[19] Song Hun Kwak Kwang Hun Kim, Kyong Jin Pang, Sok Kim, Pyol Kim. Free vibration analysis of bulkhead-stiffened functionally graded open shell using a meshless method. Shock and Vibration 2022; 2022:7372167.

[20] Song Hun Kwak, Kwang Hun Kim, Jong Guk Yun, Sok Kim, Phyong Chol Ri. Free vibration analysis of laminated closed conical, cylindrical shells and annular plates with a hole using a meshfree method. Structures 2021;34:3070-3086.

[21] H. Mellouli, H. Jrad, M. Wali, F. Dammak. Free vibration analysis of FG-CNTRC shell structures using the meshfree radial point interpolation method. Computers and Mathematics with Applications 2020;79:3160-378.

[22] Song Hun Kwak, Kwang Hun Kim, Kwang Il Jong, Jaeliong Cha, U. Juhyok. A meshfree approach for free vibration analysis of ply drop-off laminated conical, cylindrical shells and annular plates. Acta Mechanica 2021;232:4775-4800.

[23] Hu shuangwei et al. Vibration analysis of closed laminated conical, cylindrical shells and annular plates using meshfree method, Engineering Analysis with Boundary Elements, 133(2021); 341-361.