

Anti Fuzzy Subalgebras and Homomorphism of CI- algebras

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Abstract

In this paper, we introduce the concept of Anti fuzzy subalgebras of CI-algebras. Also we discussed about ideals in CI-algebra under homomorphism and anti homomorphism and some of its properties. We proved that if μ and δ are anti fuzzy ideals in a CI – algebra X , then $\mu \times \delta$ is an anti fuzzy ideal in $X \times X$ and few more results in Cartesian product.

Keywords

CI-algebra, fuzzy ideal, Anti fuzzy ideal, fuzzy sub algebra, Anti fuzzy sub algebra, Homomorphism, Anti Homomorphism, Cartesian Products.

AMS Subject Classification (2000): 20N25, 03E72, 03F05, 06F35, 03G25.

1.Introduction

Y.Imai and K.Iseki introduced two classes of abstract algebras : BCK-algebras and BCI –algebras [6,7]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [4,5] Q.P.Hu and X .Li introduced a wide class of abstract BCH-algebras. They have shown that the class of BCI-algebras. J.Neggers, S.S.Ahn and H.S.Kim introduced Q-algebras which is generalization of BCK / BCI algebras and obtained several results. In K.Megalai and A.Tamilarasi introduced a class of abstract algebras : TM-algebras , which is a generalisation of Q / BCK / BCI / BCH algebras. In [9] B.L.Meng introduced the notion of a CI-algebra as a generation of a BE-algebra. The concept of fuzzification of ideals in CI-algebra have introduced by Samy.M.Mostafa[14] and the concept of anti fuzzy ideals of CI-algebra have introduced by Priya T. and Sithar Selvam P.M. [12]. R.Biswas introduced the concept of Anti fuzzy subgroups of groups[2]. Modifying his idea, in this paper we apply the idea in CI-algebras . We introduce the notion of anti fuzzy ideals, anti fuzzy subalgebras of CI-algebras and studied some of its properties under Homomorphism and Cartesian products.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1 [9] An algebraic system $(X, *, 1)$ of type $(2,0)$ is called a CI -algebra if it satisfies the following axioms.

$$1. x * x = 1 \quad (2.1)$$

$$2. 1 * x = x, \quad (2.2)$$

$$3. x * (y * z) = y * (x * z), \text{ for all } x, y, z \in X \quad (2.3)$$

In X we can define a binary operation \leq by $x \leq y$ if and only if $x * y = 1$ for all $x, y \in X$. (2.4)

Example 2.1

Let $X = \{1, 2, 3, 4\}$ be a set with a binary operation $*$ defined by the following table

*	1	2	3	4
1	1	2	3	4
2	1	1	2	4
3	1	1	1	4
4	1	2	3	1

Then $(X, *, 1)$ is a CI-algebra.

Remark : In an CI- algebra, the following identities are true:

$$4. y * ((y * x) * x) = 1. \quad (2.5)$$

$$5. (x * 1) * (y * 1) = (x * y) * 1. \quad (2.6)$$

Definition 2.2 [14] Let $(X, *, 1)$ be a CI -algebra. A non empty subset I of X is called an ideal of X if it satisfies the following conditions

$$(i) \text{ If } x \in X \text{ and } a \in I, \text{ then } x * a \in I, \text{ (i.e) } X * I \subseteq I \quad (2.7)$$

(ii) If $x \in X$ and $a, b \in I$, then $(a * (b * x)) * x \in I$.
(2.8)

Let X be a CI – algebra. Then (i) Every ideal of X contains 1.(ii) If I is an ideal of X , then $(a * x) * x \in I$ for all $a \in I$ and $x \in X$.

Definition 2.3 Let X be a non-empty set. A fuzzy subset μ of the set X is a mapping $\mu : X \rightarrow [0, 1]$.

Definition 2.4 [14] Let X be a CI – algebra. A fuzzy set μ in X is called a fuzzy ideal of X if

$$(i) \mu(x * y) \geq \mu(y), \text{ for all } x, y \in X. \quad (2.9)$$

$$(ii) \mu(x * (y * z) * z) \geq \min\{\mu(x), \mu(y)\}, \text{ for all } x, y, z \in X. \quad (2.10)$$

Definition 2.5 A fuzzy set μ of a CI-algebra X is called an anti fuzzy ideal of X , if

$$(i) \mu(x * y) \leq \mu(y), \text{ for all } x, y \in X. \quad (2.11)$$

$$(ii) \mu(x * (y * z) * z) \leq \max\{\mu(x), \mu(y)\}, \text{ for all } x, y, z \in X. \quad (2.12)$$

Definition 2.6 A non empty subset S of a CI - algebra X is said to be a sub algebra of X if $x * y \in S$ whenever $x, y \in S$.

3.Fuzzy Sub algebra and Anti Fuzzy Sub algebra of CI algebra

Definition 3.1[11] A fuzzy set μ in a CI-algebra X is called a fuzzy Sub algebra of X if

$$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}, \text{ for all } x, y \in X.$$

Definition 3.2[11] A fuzzy set μ in a CI- algebra X is called an Anti fuzzy sub algebra of X if

$$\mu(x * y) \leq \max\{\mu(x), \mu(y)\}, \text{ for all } x, y \in X.$$

Remark:

Every anti fuzzy ideal of a CI-algebra X is an anti fuzzy sub algebra if $x = y$ for any $x, y \in X$.

Theorem 3.1

If μ is an anti fuzzy sub algebra of a CI-algebra X , then $\mu(1) \leq \mu(x)$, for any $x \in X$.

Proof

Since $x * x = 1$ for any $x \in X$, then

$$\begin{aligned} \mu(1) &= \mu(x * x) \\ &\leq \max\{\mu(x), \mu(x)\} \end{aligned}$$

$$= \mu(x)$$

$$\mu(1) \leq \mu(x).$$

Definition 3.3 [11] Let μ be a fuzzy set of X . For a fixed $t \in [0, 1]$, the set $\mu^t = \{x \in X \mid \mu(x) \leq t\}$ is called the lower level subset of μ .

Clearly $\mu^{t_1} \cup \mu^{t_2} = X$ for $t \in [0, 1]$ if $t_1 < t_2$, then $\mu^{t_1} \subseteq \mu^{t_2}$.

Theorem 3.2

A fuzzy set μ of a CI – algebra X is an anti fuzzy subalgebra if and only if for every $t \in [0, 1]$, μ^t is either empty or a sub algebra of X .

Proof:

Assume that μ is an anti fuzzy sub algebra of X and $\mu^t \neq \emptyset$. Then for any $x, y \in \mu^t$, we have

$$\mu(x * y) \leq \max\{\mu(x), \mu(y)\} \leq t.$$

Therefore $x * y \in \mu^t$. Hence μ^t is a sub algebra of X .

Now Let $x, y \in X$.

Take $t = \max\{\mu(x), \mu(y)\}$. Then by assumption μ^t is a sub algebra of X implies $x * y \in \mu^t$.

Therefore $\mu(x * y) \leq t = \max\{\mu(x), \mu(y)\}$.

Hence μ is an Anti fuzzy sub algebra of X .

Theorem 3.3

Any sub algebra of a CI – algebra X can be realized as a level sub algebra of some Anti fuzzy sub algebra of X .

Proof:

Let A be a sub algebra of a given CI – algebra X and let μ be a fuzzy set in X defined by

$$\mu(x) = \begin{cases} t, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

Where $t \in [0, 1]$ is fixed. It is clear that $\mu^t = A$.

Now we prove such defined μ is an anti fuzzy sub algebra of X .

Let $x, y \in X$. If $x, y \in A$, then $x * y \in A$.

Hence $\mu(x) = \mu(y) = \mu(x * y) = t$ and

$$\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$$

If $x, y \notin A$, then $\mu(x) = \mu(y) = 0$ and

$$\mu(x * y) \leq \max\{\mu(x), \mu(y)\} = 0.$$

If at most one of $x, y \in A$, then at least one of $\mu(x)$ and $\mu(y)$ is equal to 0.

Therefore, $\max\{\mu(x), \mu(y)\} = 0$ so that $\mu(x * y) \leq 0$, which completes the proof.

Theorem 3.4

Two level sub algebras μ^s, μ^t ($s < t$) of an anti fuzzy sub algebra are equal iff there is no $x \in X$ such that $s \leq \mu(x) < t$.

Proof

Let $\mu^s = \mu^t$ for some $s < t$. If there exist $x \in X$ such that $s \leq \mu(x) < t$, then μ^t is a proper subset of μ^s , which is a contradiction.

Conversely, assume that there is no $x \in X$ such that $s \leq \mu(x) < t$.

If $x \in \mu^s$, then $\mu(x) \leq s$ and $\mu(x) \leq t$.

Since $\mu(x)$ does not lie between s and t . Thus $x \in \mu^t$, which gives

$\mu^s \subseteq \mu^t$, Also $\mu^t \subseteq \mu^s$.

Therefore $\mu^s = \mu^t$.

4. Homomorphism and Anti Homomorphism of CI- algebra

In this section, we discussed about ideals in CI- algebra under homomorphism and anti homomorphism and some of its properties.

Definition 4.1 Let $(X, *, 1)$ and $(Y, \Delta, 1')$ be CI- algebras. A mapping $f: X \rightarrow Y$ is said to be a homomorphism if $f(x * y) = f(x) \Delta f(y)$ for all $x, y \in X$.

Definition 4.2 Let $(X, *, 1)$ and $(Y, \Delta, 1')$ be CI- algebras. A mapping $f: X \rightarrow Y$ is said to be a anti homomorphism if $f(x * y) = f(y) \Delta f(x)$ for all $x, y \in X$.

Definition 4.3 Let $f: X \rightarrow X$ be an endomorphism and μ be a fuzzy set in X . We define a new fuzzy set in X by μ_f in X as $\mu_f(x) = \mu(f(x))$ for all x in X .

Definition 4.4 For any homomorphism $f: X \rightarrow Y$, the set $\{x \in X / f(x) = 1'\}$ is called the kernel of f , denoted by $\text{Ker}(f)$ and the set $\{f(x) / x \in X\}$ is called the image of f , denoted by $\text{Im}(f)$.

Theorem 4.1

Let f be an endomorphism of a CI- algebra X . If μ is an anti fuzzy ideal of X , then so is μ_f .

Proof:

$$\begin{aligned} \mu_f(x * y) &= \mu(f(x * y)) \\ &= \mu(f(x) * f(y)) \\ &\leq \mu(f(y)) = \mu_f(y), \text{ for all } x, y \in X. \end{aligned}$$

Let $x, y, z \in X$.

Then

$$\begin{aligned} \mu_f((x * (y * z)) * z) &= \mu(f((x * (y * z)) * z)) \\ &= \mu(f(x * (y * z)) * f(z)) \\ &= \mu((f(x) * f(y * z)) * f(z)) \\ &= \mu((f(x) * (f(y) * f(z))) * f(z)) \\ &\leq \max\{\mu(f(x)), \mu(f(y))\} \\ &= \max\{\mu_f(x), \mu_f(y)\} \end{aligned}$$

$$\therefore \mu_f((x * (y * z)) * z) \leq \max\{\mu_f(x), \mu_f(y)\}$$

Hence μ_f is an anti fuzzy ideal of X .

Theorem 4.2

Let $f: X \rightarrow Y$ be an epimorphism of CI- algebra. If μ_f is an anti fuzzy ideal of X , then μ is an anti fuzzy ideal of Y .

Proof:

Let $y \in Y$. Then there exists $x \in X$ such that $f(x) = y$.

Let $y_1, y_2, y_3 \in Y$.

$$\begin{aligned} \mu(y_1 \Delta y_2) &= \mu(f(x_1) \Delta f(x_2)) \\ &= \mu(f(x_1 * x_2)) \\ &= \mu_f(x_1 * x_2) \\ &\leq \mu_f(x_2) = \mu(f(x_2)) = \mu(y_2) \end{aligned}$$

$$\therefore \mu(y_1 \Delta y_2) \leq \mu(y_2)$$

Then

$$\begin{aligned} \mu((y_1 \Delta (y_2 \Delta y_3)) \Delta y_3) &= \mu([f(x_1) \Delta (f(x_2) \Delta f(x_3))] \Delta f(x_3)) \\ &= \mu([f(x_1) \Delta f(x_2 * x_3)] \Delta f(x_3)) \\ &= \mu(f[x_1 * (x_2 * x_3)] \Delta f(x_3)) \\ &= \mu(f([x_1 * (x_2 * x_3)] * x_3)) \\ &= \mu_f([x_1 * (x_2 * x_3)] * x_3) \\ &\leq \max\{\mu_f(x_1), \mu_f(x_2)\} \\ &= \max\{\mu(f(x_1)), \mu(f(x_2))\} \\ &= \max\{\mu(y_1), \mu(y_2)\} \end{aligned}$$

$$\therefore \mu((y_1 \Delta (y_2 \Delta y_3)) \Delta y_3) \leq \max\{\mu(y_1), \mu(y_2)\}$$

Hence μ is an anti fuzzy ideal of Y .

Theorem 4.3

Let $f: X \rightarrow Y$ be a homomorphism of CI- algebra. If μ is an antifuzzy ideal of Y then μ_f is an anti fuzzy ideal of X .

Proof:

Let $x, y, z \in X$.

$$\begin{aligned} \mu_f(x * y) &= \mu(f(x * y)) \\ &= \mu(f(x) \Delta f(y)) \\ &\leq \mu(f(y)) = \mu_f(y). \end{aligned}$$

$$\therefore \mu_f(x * y) \leq \mu_f(y).$$

Then

$$\begin{aligned}
\mu_f((x * (y * z)) * z) &= \mu(f((x * (y * z)) * z)) \\
&= \mu(f(x * (y * z)) \Delta f(z)) \\
&= \mu((f(x) \Delta f(y * z)) \Delta f(z)) \\
&= \mu((f(x) \Delta (f(y) \Delta f(z))) \Delta f(z)) \\
&\leq \max\{\mu(f(x)), \mu(f(y))\} \\
&= \max\{\mu_f(x), \mu_f(y)\}
\end{aligned}$$

$$\therefore \mu_f((x * (y * z)) * z) \leq \max\{\mu_f(x), \mu_f(y)\}$$

Hence μ_f is an anti fuzzy ideal of X .

Theorem 4.4 Let $(X, *, 1)$ and $(Y, \Delta, 1')$ be CI – algebras. A mapping $f: X \rightarrow Y$ is a homomorphism of CI-algebra, Then $\text{Ker}(f)$ is an ideal.

Proof:

It is clear that $1 \in \text{ker } f$.

By [14] it is enough to prove that $(x * y) * z \in \text{ker}(f) \Rightarrow x * z \in \text{ker } f$, for all $x, z \in X$ and $y \in \text{ker } f$.

Let $(x * y) * z \in \text{ker}(f)$ & $y \in \text{ker}(f)$.

Then $f((x * y) * z) = 1'$ & $f(y) = 1'$

Since $1' = f((x * y) * z)$

$$\begin{aligned}
&= f(x) \Delta f(y * z) \\
&= f(x) \Delta (f(y) \Delta f(z)) \\
&= f(x) \Delta (1' \Delta f(z)) \\
&= f(x) \Delta f(z) \\
&= f(x * z) \\
&\Rightarrow x * z \in \text{ker}(f)
\end{aligned}$$

Hence $\text{ker } f$ is an ideal.

5. Cartesian Product of Anti Fuzzy ideals of CI – algebras

In this section, we introduce the concept of Cartesian product of anti fuzzy ideals of CI-algebra.

Definition 5.1 Let μ and δ be the fuzzy sets in X . The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by

$$(\mu \times \delta)(x, y) = \min\{\mu(x), \delta(y)\}, \text{ for all } x, y \in X.$$

Definition 5.2 Let μ and δ be the anti fuzzy sets in X .

The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta)(x, y) = \max\{\mu(x), \delta(y)\}$, for all $x, y \in X$.

Theorem 5.1

If μ and δ are anti fuzzy ideals in a CI – algebra X , then $\mu \times \delta$ is an anti fuzzy ideal in $X \times X$.

Proof:

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$.

$$(\mu \times \delta)((x_1, x_2) * (y_1, y_2)) = (\mu \times \delta)(x_1 * y_1, x_2 * y_2)$$

$$\begin{aligned}
&= \max\{\mu(x_1 * y_1), \delta(x_2 * y_2)\} \\
&\leq \max\{\mu(y_1), \delta(y_2)\} \\
&= (\mu \times \delta)(y_1, y_2)
\end{aligned}$$

$$\begin{aligned}
\therefore (\mu \times \delta)((x_1, x_2) * (y_1, y_2)) &\leq (\mu \times \delta)(y_1, y_2) \\
(\mu \times \delta)\{((x_1, x_2) * ((y_1 * y_2) * (z_1, z_2))) * (z_1, z_2)\} \\
&= (\mu \times \delta)\{[(x_1, x_2) * (y_1 * z_1, y_2 * z_2)] * (z_1, z_2)\} \\
&= (\mu \times \delta)\{(x_1 * (y_1 * z_1)), x_2 * (y_2 * z_2)\} * (z_1, z_2) \\
&= (\mu \times \delta)\{(x_1 * (y_1 * z_1)) * z_1, (x_2 * (y_2 * z_2)) * z_2\} \\
&= \max\{\mu((x_1 * (y_1 * z_1)) * z_1), \delta((x_2 * (y_2 * z_2)) * z_2)\} \\
&\leq \max\{\max\{\mu(x_1), \mu(y_1)\}, \max\{\delta(x_2), \delta(y_2)\}\} \\
&= \max\{\max\{\mu(x_1), \delta(x_2)\}, \max\{\mu(y_1), \delta(y_2)\}\} \\
&= \max\{(\mu \times \delta)(x_1, x_2), (\mu \times \delta)(y_1, y_2)\} \\
\therefore (\mu \times \delta)\{((x_1, x_2) * ((y_1 * y_2) * (z_1, z_2))) * (z_1, z_2)\} \\
&\leq \max\{(\mu \times \delta)(x_1, x_2), (\mu \times \delta)(y_1, y_2)\}
\end{aligned}$$

Hence, $\mu \times \delta$ is an antifuzzy ideal in $X \times X$.

Theorem 5.2:

Let μ & δ be fuzzy sets in a CI -algebra X such that $\mu \times \delta$ is an Anti-fuzzy ideal of $X \times X$. Then

- (i) Either $\mu(1) \leq \mu(x)$ (or) $\delta(1) \leq \delta(x)$ for all $x \in X$
- (ii) If $\mu(1) \leq \mu(x)$ for all $x \in X$, then either $\delta(1) \leq \mu(x)$ (or) $\delta(1) \leq \delta(x)$
- (iii) If $\delta(1) \leq \delta(x)$ for all $x \in X$, then either $\mu(1) \leq \mu(x)$ (or) $\mu(1) \leq \delta(x)$.
- (iv) Either μ or δ is an anti fuzzy ideal of X .

Proof:

Let $\mu \times \delta$ be an anti fuzzy ideal of $X \times X$.

Therefore $(\mu \times \delta)((x_1, x_2) * (y_1, y_2)) \leq (\mu \times \delta)(y_1, y_2)$ and

$$\begin{aligned}
(\mu \times \delta)\{((x_1, x_2) * ((y_1 * y_2) * (z_1, z_2))) * (z_1, z_2)\} \\
\leq \max\{(\mu \times \delta)(x_1, x_2), (\mu \times \delta)(y_1, y_2)\}
\end{aligned}$$

for all $(x_1, x_2), (y_1, y_2)$ & $(z_1, z_2) \in X \times X$.

- (i) Suppose that $\mu(1) > \mu(x)$ and $\delta(1) > \delta(x)$ for some $x, y \in X$

$$\begin{aligned}
\text{Then } (\mu \times \delta)(x, y) &= \max\{\mu(x), \delta(y)\} \\
&< \max\{\mu(1), \delta(1)\}
\end{aligned}$$

$= (\mu \times \delta)(1, 1)$, Which is a contradiction.[12]

Therefore $\mu(1) \leq \mu(x)$ and $\delta(1) \leq \delta(x)$ for all $x \in X$.

- (ii) Assume that there exists $x, y \in X$ such that

$$\delta(1) > \mu(x) \text{ and } \delta(1) > \delta(x).$$

Then $(\mu \times \delta)(1, 1) = \max\{\mu(1), \delta(1)\} = \delta(1)$ and hence

$$(\mu \times \delta)(x, y) = \max \{ \mu(x), \delta(y) \} < \delta(1) = (\mu \times \delta)(1, 1)$$

Which is a contradiction.[12]

Hence if $\mu(1) \leq \mu(x)$ for all $x \in X$, then either $\delta(1) \leq \mu(x)$ (or) $\delta(1) \leq \delta(x)$.

Similarly, we can prove that if $\delta(1) \leq \delta(x)$ for all $x \in X$, then either $\mu(1) \leq \mu(x)$ or $\mu(1) \leq \delta(y)$, which yields (iii).

(iv) First we prove that δ is an anti fuzzy ideal of X

Since by (i) either $\mu(1) \leq \mu(x)$ (or) $\delta(1) \leq \delta(x)$ for all $x \in X$.

Assume that $\delta(1) \leq \delta(x)$, for all $x \in X$.

It follows from (iii) that either $\mu(1) \leq \mu(x)$ or $\mu(1) \leq \delta(x)$.

If $\mu(1) \leq \delta(x)$ for any $x \in X$, then

$$\delta(x) = \max \{ \mu(1), \delta(x) \} = (\mu \times \delta)(1, x)$$

$$\delta(x * y) = \max \{ \mu(1), \delta(x * y) \}$$

$$= (\mu \times \delta)(1, x * y)$$

$$= (\mu \times \delta)(1 * 1, x * y)$$

$$= (\mu \times \delta)((1, x) * (1, y))$$

$$\leq (\mu \times \delta)(1, y)$$

$$= \delta(y)$$

$$\therefore \delta(x * y) \leq \delta(y)$$

$$\delta((x * (y * z)) * z) = \max \{ \mu(1), \delta((x * (y * z)) * z) \}$$

$$= (\mu \times \delta)(1, (x * (y * z)) * z)$$

$$= (\mu \times \delta) \{ 1 * 1, (x * (y * z)) * z \}$$

$$= (\mu \times \delta) \{ (1, x * (y * z)) * (1, z) \}$$

$$= (\mu \times \delta) \{ (1 * 1, x * (y * z)) * (1, z) \}$$

$$= (\mu \times \delta) \{ [(1, x) * (1, y * z)] * (1, z) \}$$

$$= (\mu \times \delta) \{ [(1, x) * (1 * 1, y * z)] * (1, z) \}$$

$$= (\mu \times \delta) \{ [(1, x) * ((1, y) * (1, z))] * (1, z) \}$$

$$\leq \max \{ (\mu \times \delta)(1, x), (\mu \times \delta)(1, y) \}$$

$$= \max \{ \delta(x), \delta(y) \}$$

$$\delta((x * (y * z)) * z) \leq \max \{ \delta(x), \delta(y) \}$$

Hence δ is an Anti fuzzy ideal of X .

Next we will prove that μ is an anti fuzzy ideal of X .

Let $\mu(1) \leq \mu(x)$

Since by (ii), either $\delta(1) \leq \mu(x)$ or $\delta(1) \leq \delta(x)$.

Assume that $\delta(1) \leq \mu(x)$, then

$$\mu(x) = \max \{ \mu(x), \delta(1) \} = (\mu \times \delta)(x, 1)$$

$$\mu(x * y) = \max \{ \mu(x * y), \delta(1) \}$$

$$= (\mu \times \delta)(x * y, 1)$$

$$= (\mu \times \delta)(x * y, 1 * 1)$$

$$= (\mu \times \delta)((x, 1) * (y, 1))$$

$$\leq (\mu \times \delta)(y, 1)$$

$$= \mu(y)$$

$$\mu((x * (y * z)) * z) = \max \{ \mu((x * (y * z)) * z), \delta(1) \}$$

$$= (\mu \times \delta) \{ (x * (y * z)) * z, 1 \}$$

$$= (\mu \times \delta) \{ (x * (y * z)) * z, 1 * 1 \}$$

$$= (\mu \times \delta) \{ (x * (y * z), 1) * (z, 1) \}$$

$$= (\mu \times \delta) \{ (x * (y * z), 1 * 1) * (z, 1) \}$$

$$= (\mu \times \delta) \{ [(x, 1) * (y * z, 1)] * (z, 1) \}$$

$$= (\mu \times \delta) \{ [(x, 1) * (y * z, 1 * 1)] * (z, 1) \}$$

$$= (\mu \times \delta) \{ [(x, 1) * ((y, 1) * (z, 1))] * (z, 1) \}$$

$$\leq \max \{ (\mu \times \delta)(x, 1), (\mu \times \delta)(y, 1) \}$$

$$= \max \{ \mu(x), \mu(y) \}$$

$$\mu((x * (y * z)) * z) \leq \max \{ \mu(x), \mu(y) \}$$

Hence μ is an Anti fuzzy ideal of X .

Conclusion

In this article we have discussed anti fuzzy ideal, Anti fuzzy sub algebra of CI-algebras under homomorphism and Anti homomorphism, Cartesian Products. It has been observed that the CI-algebra as a generation of BE-algebra. These concepts can further be generalized.

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