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Analytical Solution for Lateral Buckling of Double Layer Grid-Walls by the Sandwich Plate Analogy Method and the Kantorovich Method

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Abstract—There is a new trend appearing in new and expanding engineering, that is the space grid structure is gradually used to construct the structural wall which is called grid-wall in this paper. Because of being subjected to load in vertical plane, the lateral buckling of grid-wall may occur. A simplified analytical method for lateral buckling of Double Layer Grid-wall(DLG-wall) of orthogonal square pyramid space grid structure with one side free and the others simply supported is put forward in this paper. In practical design, DLG-wall can be simplified to a continuum mode based on the sandwich plate analogy method and three-dimensional, platelike lattice structures is transformed into one-dimensional problem through the Kantorovich method, and then the minimum critical load formula of lateral buckling for DLG-wall is deduced, its accuracy is verified by finite element method. The comparison of different results shows that the presented method is not only very simple and convenient but also has fairly high precision. Furthermore, this method can relieve designer of the time-consuming task of building a complicated model for finite element analysis. Therefore, this practical simplified method is suitable for making plan and preliminary design for DLG-wall.

Keywords—Double layer grid-wall; lateral buckling; the sandwich plate analogy method; the Kantorovich method, finite element method

I. INTRODUCTION

The space grid structure has the advantages of large space stiffness, good mechanical performance and attractive appearance, etc, and it has been widely used in the practical engineering. The application was mainly used for horizontal span structure in the past, for instance, the roof structure or floor structure. However, as a spatial structure with good performance, its application should also be spatial, and there are no limitations for the horizontal or vertical layout scheme in architecture and structure design. In recent years, with the development of the structure and the need to adapt to the function of the building, various forms of grid structure began to be used for structural wall in large buildings and public buildings, among them, the vertical double layer grid-wall (DLG-wall) structure used in New York Javits Exhibition and Conference Center are successful examples[1].

DLG-wall is used not only in the new project but also in reconstructed project or extension project to meet the requirement of building functions, such as acquiring large architectural space in new constructions or extension constructions. However, in the case of reconstructed project or extension project, the boundary conditions of the DLG-wall are three sides of them are connected with other structure components or foundations, while the other (usually the top side) is free. Because the height of the DLG-wall is very large (H≥30 m), and the thickness of the wall can't be too large, so that the study of stability for the DLG-wall is of great importance.

Based on the background of this kind of practical projects and considering generality of problems, the boundary conditions of the vertical bearing DLG-wall are considered as one side free and others simply supported. Taking orthogonal square pyramid space grid-wall as an example, it can be simplified a continuum mode based on the sandwich plate analogy method, and then its lateral stability can be analyzed by using the Kantorovich method.

The currently analytical approaches for analyzing large repetitive lattice structures can be grouped into five classes, namely, direct method[2], discrete field method[3], periodic structure approaches[4], substitute continuum approaches[5] and plate analogy method. The number of publications on continuum modeling of repetitive lattice structures has been steadily increasing, several studies are discussed in the literature[6]-[10], such as continuum modeling of large lattice structures by A. K. Noor and M. M. Jr Martin[6], and continuum models for beam-and platelike lattice structures by T. Anne and D. R. Guido[7].

The sandwich plate analogy method is a simplified calculation method for the design of the grid structure. In 1982, S. L. Dong and X. Heng[11] proposed sandwich plate analogy method of orthogonal square pyramid space grids and adopted in design specification of double layer grid structures[12]. W. F. Zhang[13]-[14] first applied the sandwich plate analogy method to analyze the vertical dynamic response of the double layer grid structure, and the exact solution formula for the natural vibration of orthogonal square pyramid space grids was derived. N. Bai[15] applied the sandwich plate analogy method to the vertical seismic analysis of square grid structure, and the correctness of the practical formula for the vertical seismic internal force of the grid structure was verified. And then, the sandwich plate analogy method was used for static analysis, natural vibration analysis and the stability analysis of square grid structure by W. F. Zhang and W. Y. Liu[16], the accuracy of the formulas for the natural vibration the critical load of buckling were verified.

The Kantorovich method[17] is a method for solving differential equations proposed by the former Soviet scholar, Kantorovich, in 1933, which can convert variational problem of multiple integrals to ordinary differential equation. If combine it with the energy variational method, the more accurate approximate analytical solution can be obtained. Earlier, A. D. Kerr[18] successfully used the extended Kantorovich method for the problem of bending and buckling of an isotropic rectangular plate. The efficiency and accuracy of the method have also been demonstrated in the stress analysis of clamped isotropic plates[19] and clamped orthotropic plates[20], also. J. M. Ding and X. L. Su[21] have solved the displacement of the space truss under vertical load. As for the buckling problems, U. Variddhi and S. Pairod[22] finished the buckling analysis of symmetrically laminated composite plates by the extended Kantorovich method, and W. F. Zhang and K. Y. Liu[23] have studied application for the solutions of lateral buckling for rectangular plate by the Kantorovich method.

In 1998, C.X. Xu and C. T. Ding[1] have studied the stability calculation of vertical DLG-wall, with the sandwich plate analogy method, taking the DLG-wall as a sandwich plate with four side simply supported, analytical buckling equations of the sandwich plate have been derived, and the critical load of the space truss wall has been obtained. Different from above mentioned results, in this paper, the boundary conditions of the vertical bearing DLG-wall in reconstructed or extension building are considered as one side free and top side simply supported, its lateral buckling analysis is based on the sandwich plate analogy method and the Kantorovich method, and then the minimum critical load formula of lateral buckling for the DLG-wall is deduced, whose accuracy is verified by finite element method.

II. PRESENTATION OF QUESTIONS

When the orthogonal square pyramid space grid structure acts as structure wall (see Fig.1), it is subjected to in-plane load in vertical plane and then in-plane deflection may occur. As the load reaches critical load, the DLG-wall will occur out-plane deflection and twist, which is called lateral buckling for the DLG-wall. However, this problem has not been studied until now.

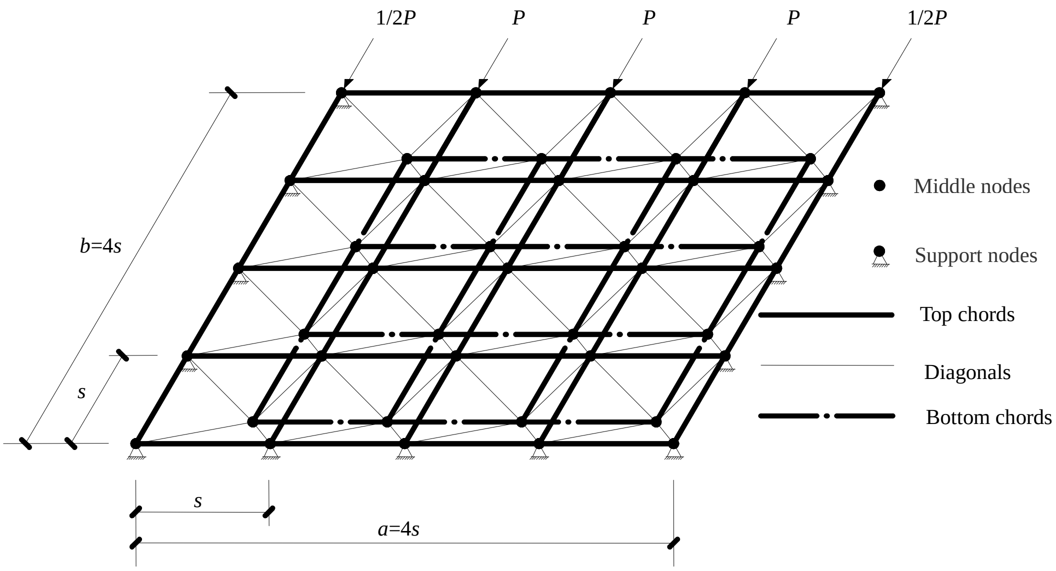

Fig.1. *Example of orthogonal square pyramid space grid-wall*

The lateral buckling analysis can be carried out by using the finite element software, however, which needs to know the detailed dimensions of structural components and has complicated process in establishing FEM model. Moreover, the analysis conducted by software is not universal for any grid-wall. For this reason, finite element method is not suitable for preliminary design, but applicable to checking the final structure design. In this paper, we intend to search for a simplified analysis method to predict the buckling strength of the space DLG-wall in the preliminary design of the grid-wall, by which critical load of lateral buckling for the DLG-wall could be obtained through several main parameters, such as the length, rigidity, etc.

Usually, it is more difficult to calculate the critical load of lateral buckling by making use of partial differential equations for orthogonal square pyramid space grids with one side free and others simply supported, when the free side is subjected to vertical load. The sandwich plate analogy method and the Kantorovich method will be introduced to solve this problem. With the Kantorovich method, the deflection function along the free side is not necessary to be chosen prior to calculate the minimum critical load of grid-wall lateral buckling. Numerical analyses on the lateral buckling behavior of the grid-wall are further performed for a various dimentions of grid-wall by finite element software ANSYS, which is found that the FEM results correspond well with the analytical solutions. So this simplified method may provide reference for the application of DLG-wall in practice.

III. THE LATERAL BUCKLING OF GLD-WALL BY THE SANDWICH PLATE ANALOGY METHOD AND THE KANTOROVICH METHOD

In this paper, we establish the sandwich plate analogy model for orthogonal square pyramid space grid, which consists of two parallel surfaces (top and bottom chord plane) and core layer (Diagonals). Double-layer grids have comparatively low transverse shear stiffness. Hence basic equations are set up with the flat plate bending theory and shear deformation is not taken into account as calculating the critical load. The displacement function ω, which, according to the relations between the stress and deformation states[12], describes the displacement pattern caused by vertical uniform load q acting on the free side of grid-wall in vertical plane, as sown in the Fig.1. In the FEM modeling, the uniform load q is translated to concentrated load P on middle support nodes and $\frac{1}{2}P$ on the two end nodes.

According to the sandwich plate analogy model, the lateral buckling of DLG-wall should fulfill the requirement of making the following total potential energy minimum

$$\Pi = U + V = \frac{1}{2}\int_0^a\int_0^b (D_x\,\omega_{xx}^2 + D_y\,\omega_{yy}^2)dxdy - \frac{1}{2}\int_0^a\int_0^b q(x)\Delta_y(x)dxdy \tag{1}$$

Where U and V denote the strain energy of the DLG-wall and the external potential energy induced by the external load, respectively. D_x and D_y, which depend on the plate dimensions and elastic constants, are called equivalent bending stiffness, and in the case of orthogonal square pyramid space grid, its expression has the form as

$$D_x = \frac{\mu_x E A_{bx} h^2}{(1+\mu_x)s}\ ;\ D_y = \frac{\mu_y E A_{by} h^2}{(1+\mu_y)s} \tag{1a}$$

$$\mu_x = \frac{A_{ax}}{A_{bx}}\ ;\ \mu_y = \frac{A_{ay}}{A_{by}} \tag{1b}$$

Where h is the height of DLG-wall, s is the size of top chord for the cell unit of single square pyramid, A_a, A_b are the areas of top chord and bottom chord, respectively.

Furthermore, in the expression of the total potential energy, ω_{xx} , ω_{yy} , ω_y denote the relations between deflection function ω and the corresponding deformation components. Their definition can be expressed as

$$\omega_{xx} = \frac{\partial^2\omega}{\partial x^2};\ \omega_{yy} = \frac{\partial^2\omega}{\partial y^2};\ \omega_y = \frac{\partial\omega}{\partial y} \tag{2}$$

$q(x)$ is vertical load applied in vertical plane. $\Delta_y(x)$ denotes the displacement function of the points lying on the vertical plane z =0 when the lateral buckling of grid-wall occurs, and it can be expressed as $\Delta_y(x) = (\omega_y)^2$ when the external load is uniform load.

Then we can rewrite the total potential energy as

$$\Pi = \frac{1}{2}\int_0^a\int_0^b (D_x\,\omega_{xx}^2 + D_y\,\omega_{yy}^2)dxdy - \frac{1}{2}\int_0^a\int_0^b q(x)\omega_y^2 dxdy \tag{3}$$

Define the deflection function as

$$\omega(x,y) = f(y)\sin(\frac{\pi x}{a}) \tag{3a}$$

Obviously this function fulfills the geometric boundary conditions for x-direction, but for y-direction we set an undetermined function $f(y)$. Here we define $q(x) = k\frac{\pi^2 D}{a^2}$ and $D = \sqrt{D_x D_y}$, where k is minimum lateral buckling coefficient of grid-wall. Introducing relations (2) and (3a) into the Eq.(3), and performing the integrals with respect to x, we arrive

$$\begin{aligned}\Pi &= \int_0^b (\frac{\pi^4 D_x}{4a^3} f^2 - \frac{1}{4}aq(f')^2 + \frac{1}{4}aD_y(f'')^2)dy \\ &= \frac{\pi^4 D_x}{4a^3} f^2 - \frac{1}{4}k\pi^2 D(f')^2 + \frac{1}{4}aD_y(f'')^2\end{aligned} \tag{4}$$

The total potential energy also could be expressed in the following form:

$$\Pi = \int_0^b F(y, f, f', f'')dy \tag{5}$$

Where

$$F = \frac{\pi^4 D_x}{4a^3} f^2 - \frac{1}{4}k\pi^2 D(f')^2 + \frac{1}{4}aD_y(f'')^2 \tag{6}$$

According to the stationary value principle of total potential energy, the necessary condition of obtaining minimum of energy functional is $\delta\Pi = 0$, i.e.

$$\begin{aligned}\delta\Pi = &\left[\frac{\partial F}{\partial f'}\delta f\right]_0^b - \left[\frac{d}{dy}\left(\frac{\partial F}{\partial f''}\right)\delta f\right]_0^b + \left[\frac{\partial F}{\partial f''}\delta f\right]_0^b \\ &+ \int_0^b\left[\frac{\partial F}{\partial f} - \frac{d}{dy}\left(\frac{\partial F}{\partial f'}\right) + \frac{d^2}{dy^2}\left(\frac{\partial F}{\partial f''}\right)\right]\delta f\,dy = 0\end{aligned} \tag{7}$$

Eq.(7) need fulfill the following differential equation:

$$\frac{\partial F}{\partial f} - \frac{d}{dy}\left(\frac{\partial F}{\partial f'}\right) + \frac{d^2}{dy^2}\left(\frac{\partial F}{\partial f''}\right) = 0 \tag{8}$$

This is called Euler differential equation of the variational problem. With the aid of Eq.(6) Eq.(8)can be rewritten as follows:

$$a^4 f^{(4)} + k\cdot k_d a^2 f'' + \pi^4 k_d^2 f = 0 \tag{9}$$

This is an ordinary differential equation, from which we can find that the Kantorovich method is semi-analytical method that transforms the partial differential equation problem into the ordinary differential equation.

In addition, Eq.(7) must fulfill the boundary conditions at $y=0$, $y=b$. Firstly, at the side of $y=0$, Eq.(7) must fulfill the natural boundary condition, i.e. bending moment $M=0$ and geometric boundary condition displacement $\omega=0$; Secondly, at the other side of $y=b$, Eq.(7) must fulfill the natural boundary condition, i.e. bending moment $M=0$ and shear force $Q=0$, as follows:

$$\begin{cases} \left[\dfrac{\partial F}{\partial f^{''}}\right]_{y=0} = 0 \\ \left[\omega(x,y)\right]_{y=0} = 0 \\ \left[\dfrac{\partial F}{\partial f^{''}}\right]_{y=b} = 0 \\ \left[\dfrac{\partial F}{\partial f^{'}} + \dfrac{d}{dy}\dfrac{\partial F}{\partial f^{''}}\right]_{y=b} = 0 \end{cases} \quad (10)$$

If the equality $\omega = 0$ would come into existence forever at the side of $y = 0$, since expression $\sin(\frac{\pi x}{a})$ can't be zero for anytime, we must set $f(y) = 0$. Introducing Eq.(6) and parameter $k_d = \sqrt{D_x / D_y}$, Eq.(10) can be transformed into

$$\begin{cases} \left[f^{''}\right]_{y=0} = 0 \\ \left[f(y)\right]_{y=0} = 0 \\ \left[f^{''}\right]_{y=b} = 0 \\ \left[k \cdot k_d \cdot \pi \cdot f^{'} + af^{'''}\right]_{y=b} = 0 \end{cases} \quad (11)$$

Let us take the buckling mode as $f(y) = e^{sy}$, then we can obtain the characteristic equation of the ordinary differential equation as

$$(\mathrm{as})^4 + k \cdot k_d \cdot \pi^2 (\mathrm{as})^2 + \pi^4 k_d^2 = 0 \quad (12)$$

First of all, the characteristic of this type of equation roots should be discussed. If there are real roots in the above equation, the discriminant must be $\Delta \ge 0$, i.e. $k \ge 2$, so the discriminant becomes

$$(as)^2 = \frac{k_d \pi^2}{2}\left(\sqrt{k^2 - 4} - \sqrt{k^2}\right) \quad (13)$$

Obviously, Eq.(12) has no real roots, so that the right-hand side of Eq.(13) will never be greater than 0 for $k > 2$, and $s = 0$ even though for $k = 2$. Defining the complex solution as $s = \pm\frac{1}{a}(\alpha + i\beta)$ and introducing this relation into Eq.(13), the real and imaginary parts can be obtained respectively, namely $\alpha = \frac{\pi}{2}\sqrt{2k_d - kk_d}$, $\beta = \frac{\pi}{2}\sqrt{2k_d - kk_d}$. Consequently, general solution of differential Eq. (12) has been deduced, the displacement function for y-direction is

$$f(y) = A^* \cosh(\frac{\alpha y}{a})\cos(\frac{\beta y}{a}) + B^* \sinh(\frac{\alpha y}{a})\sin(\frac{\beta y}{a}) + C^* \cosh(\frac{\alpha y}{a})\sin(\frac{\beta y}{a}) + D^* \sinh(\frac{\alpha y}{a})\cos(\frac{\beta y}{a}) \quad (14)$$

Introducing general solution (14) into boundary condition (11), a linear equation system consisted of four homogeneous linear equations with coefficient a_{mn} can be obtained as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{Bmatrix} A^* \\ B^* \\ C^* \\ D^* \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (15)$$

Where the coefficients of a_{mn} are listed below.

$$a_{11} = \frac{\alpha^2}{a^2} - \frac{\beta^2}{a^2}; \quad a_{12} = \frac{2\alpha\beta}{a^2}; \quad a_{13} = 0; \quad a_{14} = 0;$$

$$a_{21} = 1; \quad a_{22} = 0; \quad a_{23} = 0; \quad a_{24} = 0;$$

$$a_{31} = (-\frac{k k_d \pi^2 \beta}{a} - \frac{3\alpha^2\beta}{a} + \frac{\beta^3}{a})\cosh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a}) + (\frac{k k_d \pi^2 \alpha}{a} - \frac{3\alpha\beta^2}{a} + \frac{\alpha^3}{a})\sinh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a});$$

$$a_{32} = (\frac{k k_d \pi^2 \alpha}{a} - \frac{3\alpha\beta^2}{a} + \frac{\alpha^3}{a})\cosh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a}) + (\frac{k k_d \pi^2 \beta}{a} + \frac{3\alpha^2\beta}{a} - \frac{\beta^3}{a})\sinh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a});$$

$$a_{33} = (\frac{k k_d \pi^2 \beta}{a} + \frac{3\alpha^2\beta}{a} - \frac{\beta^3}{a})\cosh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a}) + (\frac{k k_d \pi^2 \alpha}{a} - \frac{3\alpha\beta^2}{a} + \frac{\alpha^3}{a})\sinh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a});$$

$$a_{34} = (\frac{k k_d \pi^2 \alpha}{a} - \frac{3\alpha\beta^2}{a} + \frac{\alpha^3}{a})\cosh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a}) + (-\frac{k k_d \pi^2 \beta}{a} - \frac{3\alpha^2\beta}{a} + \frac{\beta^3}{a})\sinh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a});$$

$$a_{41} = (\frac{\alpha^2}{a^2} - \frac{\beta^2}{a_2})\cosh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a}) - (\frac{2\alpha\beta}{a^2})\sinh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a});$$

$$a_{42} = (\frac{2\alpha\beta}{a^2})\cosh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a}) + (\frac{\alpha^2}{a^2} - \frac{\beta^2}{a_2})\sinh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a});$$

$$a_{43} = (\frac{\alpha^2}{a^2} - \frac{\beta^2}{a_2})\cosh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a}) + (\frac{2\alpha\beta}{a^2})\sinh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a});$$

$$a_{44} = (-\frac{2\alpha\beta}{a^2})\cosh(\frac{\alpha b}{a})\sin(\frac{\beta b}{a}) + (\frac{\alpha^2}{a^2} - \frac{\beta^2}{a_2})\sinh(\frac{\alpha b}{a})\cos(\frac{\beta b}{a})$$

Due to the absence of constant term in the right side of Eq.(15), if A^*, B^*, C^*, D^* have non zero solutions, the necessary condition is the determinant of coefficient must be zero, i.e.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = 0 \quad (16)$$

There is only one unknown quantity k in Eq.(16), so we can obtain minimal lateral buckling coefficient k for different value of a, b, D, k_d. Thereby, up to now, when the DLG-wall is subjected to uniform load in vertical plane, the formula of minimum critical load for the lateral buckling of orthogonal square pyramid space grids with one side free and others simply supported has been derived, That is

$$q_{cr} = k\frac{\pi^2 D}{a^2} \tag{17}$$

IV. VERIFICATION BY THE FINITE ELEMENT METHOD

In this paper, the minimum critical load for the lateral buckling of grid-wall is also calculated by the finite element software ANSYS to verify the accuracy of the analytical solutions. In the finite model, the element LINK8 is adopted to simulate the actual members, which is a uniaxial tension-compression element with three degrees of freedom on each node. In the process of establishing model, creating nodes firstly, and then building elements based on the nodes, last the finite element model of orthogonal square pyramid space grids can be established. In addition, the uniform load q is translated to concentrated load $P=qs$ on top middle nodes and $\frac{1}{2}P=\frac{1}{2}qs$ on top end nodes, which are on the free side, and $UX = UY = UZ = 0$ is adopted for other simply supported sides. The parameters of seven kinds of DLG-walls used in the analysis is listed in Table I, where A_a, A_b, A_c denote the area of the cross section for top chord, bottom chord and diagonals respectively (arithmetic mean if cross sections are varying along the span), and s denotes the length of top chord and h denotes the thickness of the grid-wall.

The lateral buckling for the grid-wall are performed by the Kantorovich method and finite element method respectively for each grid-wall. Table II gives the results for seven kinds of grid-wall and the errors. Fig.2 shows the comparison between Kantorocich method and finite element method (FEM). In addition, taking 30×30m and 39×30m orthogonal square pyramid space grid-walls as examples, the buckling deformation of them are obtained by ANSYS, as shown in Fig.3 and Fig.4.

TABLE I. THE PARAMETERS OF GRID-WALL

Span(*m*)	Parameters					
	Ratio of Side λ	A_a (cm^2)	A_b (cm^2)	A_c (cm^2)	*s(m)*	*h(m)*
30×30	1.0	11.44	8.41	3.64	3.0	2.0
45×45	1.0	17.17	13.08	7.90	3.0	3.0
60×60	1.0	20.90	16.80	7.97	3.0	4.0
36×30	1.2	29.91	19.20	8.51	3.0	2.3
39×30	1.3	46.80	29.91	10.7	3.0	2.3
42×30	1.4	63.80	46.8	19.2	3.0	2.3
45×30	1.5	45.40	28.60	10.70	3.0	3.0

TABLE II. THE COMPARISON OF CALCULATION RESULTS

Span(*m*)	k	Kantorovich method (*kN/m*)	Finite Element method (*kN/m*)	Error (*%*)
30×30	1.07784	1573.54	1554.40	+1.23
45×45	1.07784	2410.29	2470.20	-2.43
60×60	1.07784	3023.65	3051.62	-0.92
36×30	1.21807	3940.18	3852.40	+2.28
39×30	1.25568	5400.84	5113.9	+5.61
42×30	1.24741	6844.21	7035.27	-2.72
45×30	1.19522	6316.86	6232.77	+1.35

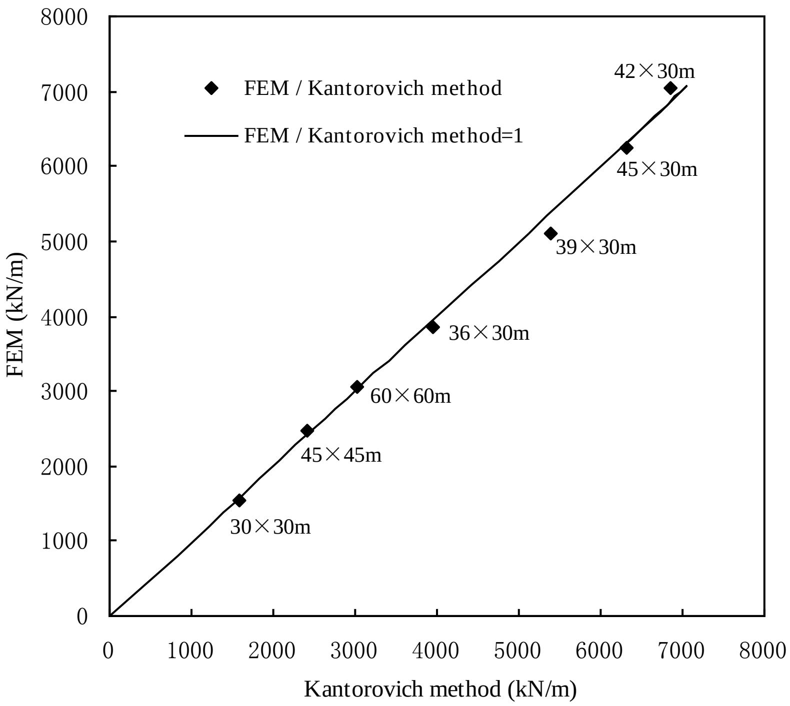

Fig.2. Comparison between Kantorocich method and finite element method

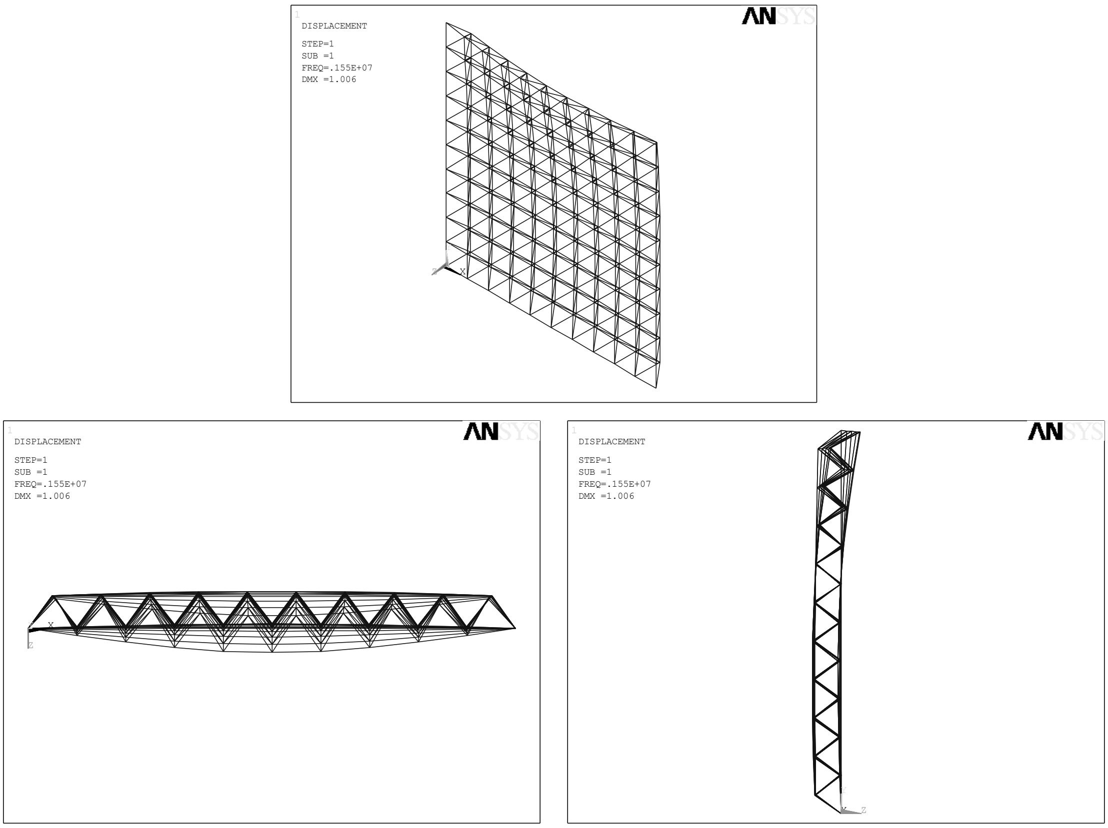

Fig.3. *Buckling deformation for 30×30m orthogonal square pyramid space grid-wall*

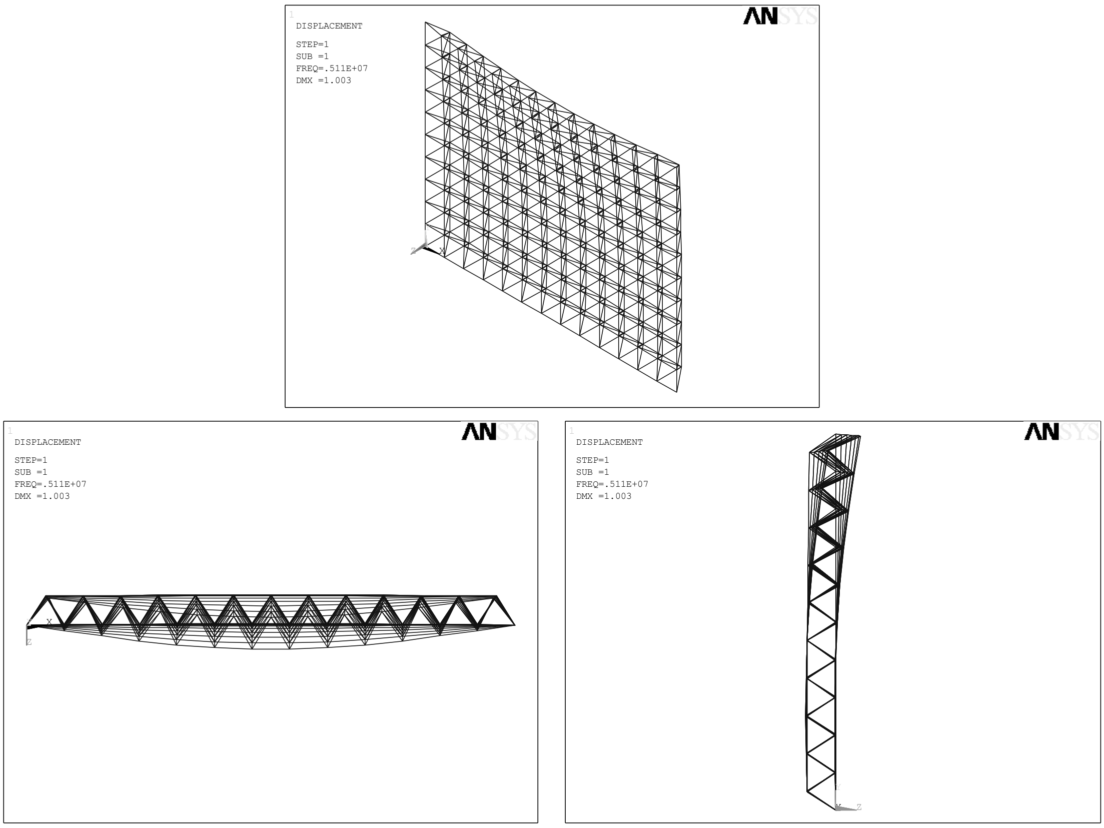

Fig.4. Buckling deformation for 39×30m orthogonal square pyramid space grid-wall

V. Conclusion

For double layer grid-wall, based on the sandwich plate analogy method and the Kantorovich method, a simplified analytical method to calculate minimum critical load of lateral buckling for DLG-walls has been put forward in this paper. It is verified through seven examples of DLG-walls by ANSYS, which shows this analytical method is not only very simple and convenient but also has fairly high precision. The most absolute errors are no more than 5%, which are acceptable from the standpoint of engineering design requirements. Thus, from what have been discussed above, it can be concluded that this simplified analytical method is very practical and suitable for making plan and preliminary design of double layer grid-walls.

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