

# Analytical Method for Estimation of Structure and Model Parameters of Single Phase Rectifier based Loads

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**Abstract-** Now a days, the interest in single phase rectifier based loads has risen, as a result of which the current and voltage distortion in low power electrical systems has increasing. In order to specify the current and voltage distortion in power systems because of nonlinear loads, it is required to know the structure and model parameters of those power electronics circuits. As the structure of a typical single-phase rectifier based load is well known, the parameters of those loads remain to be found. An algorithm for the estimation of unknown parameters of rectifier based load is given here. The obtained load and parameters model occurred due to the algorithm are compared with the lab designed rectifier based load.

**Index Terms** —rectifier, harmonics, parameter measurement.

## I. INTRODUCTION

Due to their nonlinear character, single-phase low power rectifier based loads attract recently a considerable attention by injecting non-sinusoidal currents into electrical network causing system voltage distortions that exceed the allowable limits leading to power quality problems of distribution power networks. Due to ever increasing number of those loads (computers, scanners, printers, light dimmers, battery charges, TV, DVD, audio devices, medical equipment etc.) in residential, commercial and industrial areas it is necessary to estimate and analyse higher current- and voltage harmonics in the power system and correspondingly suggest suitable solutions for increasing the power quality. For doing this it is important to develop accurate models of such nonlinear loads applicable to higher harmonic analysis of power networks. The nonlinear single-phase loads are in simplified models traditionally represented as fixed harmonic current injectors. The main drawback of this modelling, however, is possible overestimation of the net harmonic current due to the neglected effects on the phase angle cancellation between different loads, as well as the attenuation effect which is a consequence of the series system impedance along with the system voltage distortion. For this reason dedicated analyses aim presently at investigation of the total effects of higher harmonics, i.e. at the attenuation and cancellation factors in the electric distribution network .

The above mentioned problems of the nonlinear load representation in form of current injectors can be avoided if circuit models of nonlinear loads are developed and used. The circuit modelling of nonlinear loads in the existing literature is predominantly based on analytical time-domain functions of the load current and voltage. A typical equivalent circuit model of the full wave rectifier based low-power load is presented in Figure 1.

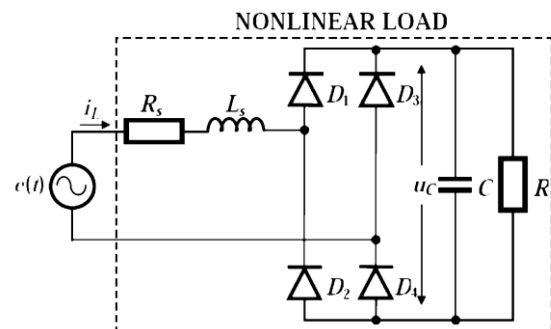


Fig.1. Typical single phase rectifier based load

Generally speaking the main components of the nonlinear load model are non-sinusoidal voltage source  $e(t)$ , along with the associated series parameters  $L_s$  and  $R_s$  representing the local rectifier choke inductor with the corresponding resistance, respectively. Behind the full wave rectifier attached to the source is located the DC-smoothing capacitor  $C$  and the equivalent load resistance  $R$ . The main assumption of this model is that the diodes of the full wave rectifier are ideal switching elements.

The model structure, the values of the circuit parameters  $R_s$ ,  $L_s$ ,  $R$  and  $C$  as well as the time dependence of the voltage source  $e(t)$  are known data prior to the model development. After its development, the model must be validated, i.e. it is necessary to compare the measured waveforms of the current  $i_L$  and voltage  $u_C$  against the corresponding simulated values. This method could be considered as a classical approach for modeling and simulation of the nonlinear rectifier loads in operation. However, the following inverse problem became recently very important: It is necessary, based on the known waveforms of the voltage source  $e(t)$  and the measured input current of the load  $i_L(t)$ , according to Figure 1, to estimate the unknown parameters of the nonlinear load

model  $R_s$ ,  $L_s$ ,  $R$  and  $C$ . Such definition of the problem is, generally speaking, completely justified by the topology of the electronic load given in Figure 1, since only the input voltage and current can be always measured.

There are very few papers dealing with the topic of parameter estimation of single-phase rectifier loads. In the existing work the analytical functions for load current and capacitor voltage are derived and subsequently based on these analytical solutions the fitness function for minimizing the difference between the referent and analytical function is defined. Finally, as an optimization result the estimation parameters are obtained. In the work a set of analytical functions for estimating the circuit parameters are given. However, no optimization for parameter estimation in the work is used.

The main drawback of the existing reference is a considerable simplification of the load structure shown in Figure 1. More precisely, the work completely neglects the resistance  $R_s$  which is a rough approximation of the general load model. Additionally, in the work, the source voltage is represented by an ideal sinusoidal function, which is a considerable simplification that could lead to inaccuracies. Furthermore, the suggested estimation method is not validated against measurements.

The procedure, suggested in [1], is more general and was validated against the corresponding measurements. The reference offers, however, analytical current- and voltage equations valid for only one half-period of the analysed time. This will be discussed in detail in the remainder of the work.

The work neglects the resistance  $R_s$  being therefore a special case of the general method suggested in this work. Additionally, the work considers the capacitor voltage constant within the parameter estimation process which is not the case within the methodology suggested here.

The original scientific contribution of this work is manifold: (a) the original objective function of the parameter estimation technique including the initial capacitor voltage as a variable, (b) the advanced algorithm for estimating the initial values of the estimation parameters, and (c) the advanced technique for obtaining an accurate initial value of the capacitor voltage at the beginning of each half-period included into the parameter estimation process.

In this work the general analytical functions of the load current and capacitor voltage valid for an arbitrary number of periods of the system voltage are presented. It will be shown that those functions are continuous, i.e. that the load current, as a basis of the estimation process, is computed in each period from the obtained capacitor voltage. Additionally, more general initial estimation parameters are established. For the optimization the Nedler's simplex method is used. This method does not require the Jacobi determinant to be computed which makes this method more robust compared to the classical derivative based optimization methods. Based on the performed estimation procedure and the obtained load parameters several laboratory measurements were completed in order to validate the suggested model and estimation algorithm.

## II. MODELING AND ANALYSIS OF RECTIFIER BASED LOADS

As already emphasized, the basic circuit model of a single-phase rectifier based load is shown in Figure 1. The typical waveforms of the ac load current  $-i_L$  and dc capacitor voltage  $-u_C$  are depicted in Figure 2. Additionally, the system voltage  $-e(t)$ , which generally does not have to be a sinusoidal signal of the fundamental frequency (50 Hz), is visualized. This figure shows two periods of the system voltage along with the other variables relevant for the presented analysis.

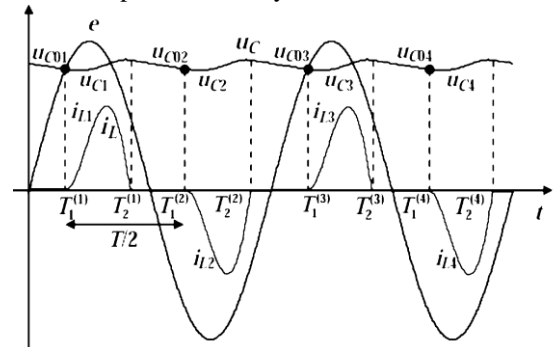


Fig. 2. Typical waveform of the ac load current  $-i_L$  and the dc capacitor voltage  $-u_C$

More precisely, the load current  $i_L$  is represented as a composition of the currents valid in each half-period  $i_{Lk}$  of the system voltage. Similar to the load current, the capacitor voltage  $u_C$  consists of a composition of the half-period solutions  $u_{Ck}$ . Thus, the following relations can be written:

$$i_L(t) = \bigcup_k i_{Lk}(t), \quad u_C(t) = \bigcup_k u_{Ck}(t), \quad k = 1, 2, \dots, k_{\max} \quad (1)$$

where  $k_{\max}$  represents the total number of the system voltage half-periods over which the load current  $i_L$  and the capacitor voltage  $u_C$  are considered. Furthermore, the corresponding moments of time at which the diodes are entering the forward range (i.e. they start conducting)  $T_1^k$  and the reverse range (i.e. they stop conducting)  $T_2^k$  are indicated in Figure 2.

Evidently, the following relations are valid in general (of course, with a reasonable assumption that the system voltage is a periodic function with the half-period  $T/2$ , where  $T=20$  ms):

$$T_1^{(k+1)} = T_1^{(k)} + T/2, \quad T_2^{(k+1)} = T_2^{(k)} + T/2 \quad (2)$$

Finally, the values of the dc capacitor voltage in the moments of time when the diodes entering the forward range  $u_{Ck}$  are also indicated.

A detailed analysis of the relevant functions is presented in Figure 3. For the reasons of completeness and generality an arbitrary  $k$ -th half-period is considered in this figure.

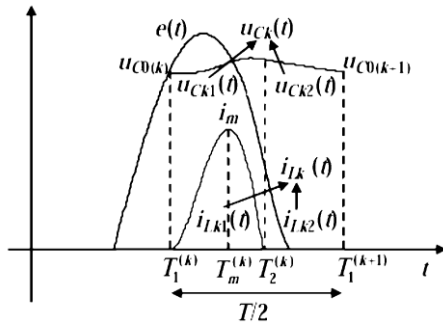


Fig. 3. Single relevant waveforms over k-th half-period: the load current –  $i_{Lk}$  and the capacitor voltage –  $u_{Ck}$

Over the chosen k-th half-period, considering the forward and reverse range of the diodes, the following equations for the load current and capacitor voltage can be written:

$$i_{Lk}(t) = \begin{cases} i_{Lk1}(t), & T_1^{(k)} \leq t \leq T_2^{(k)} \\ i_{Lk2}(t), & T_2^{(k)} \leq t \leq T_1^{(k+1)} \end{cases}$$

$$u_{Ck}(t) = \begin{cases} u_{Ck1}(t), & T_1^{(k)} \leq t \leq T_2^{(k)} \\ u_{Ck2}(t), & T_2^{(k)} \leq t \leq T_1^{(k+1)} \end{cases}$$

Analytical solutions for the load current and the capacitor voltage are obtained by solving the corresponding state-space equation along with the defined initial conditions. The state- space equations written in a compact matrix form for the both diode states over the k-th half-period read:

$$\frac{dX_{k_p}(t)}{dt} = A_p X_{k_p}(t) + B_p U(t) \tag{3}$$

$$X_{k_p}(t_{p_0}) = X_{k_{p_0}} \tag{4}$$

where  $p=1,2$  represents the diode states: 1-forward range, 2- reverse range.

### III. DETERMINATION OF THE INITIAL ESTIMATION PARAMETERS FOR THE RECTIFIER BASED LOAD

The initial values of the parameters  $R_s$ ,  $R$  and  $C$  are slightly different compared to the methodology available in [10]. At the beginning the total input power is computed ( $j_{max} = k \max l_{max}$  and  $i_{meas.}(t)$  the measured load current):

$$P = \frac{1}{j_{max}} \sum_{j=1}^{j_{max}} e(t_j) i_{meas.}(t_j)$$

Thereafter the average value of the dc capacitor voltage out of the system voltage source is estimated:

$$u_{C_{avg}} = \frac{u_{C_{min}} + u_{C_{max}}}{2}$$

where the maximum and minimum values read

$$u_{C_{max}} = e(T_{ms}^{(1)}), \quad u_{C_{min}} = e(T_{1s}^{(1)})$$

Here are the time instants  $T_{1s}^{(1)}$  and  $T_{ms}^{(1)}$  obtained from

samples of the measured rectifier current.

The effective and average value of the measured load current are computed according to the following well-known equations:

$$i_{rms} = \sqrt{\frac{1}{j_{max}} \sum_{j=1}^{j_{max}} i_{meas.}^2(t_j)}, \quad i_{avg} = \frac{1}{j_{max}} \sum_{j=1}^{j_{max}} |i_{meas.}(t_j)|$$

Based on the following equation:

$$P_R = u_{C_{avg}} i_{avg}$$

the initial value of  $R$  is computed:

$$R = \frac{P_R}{i_{avg}^2}$$

Then the initial value of  $R_s$  can be obtained as follows:

$$R_s = \frac{P - P_R}{i_{rms}^2}$$

The initial capacitance value can be obtained according to the standard value of the dc voltage ripple:

$$X_C = 0.474R \frac{u_{C_{max}} - u_{C_{min}}}{u_{C_{avg}}}, \quad C = \frac{1}{2\pi f X_C}$$

The initial inductance value reported in is based on the usage of the known dependence between the inductance and the ac current impulse, which is in general not available. Therefore, a new method for determining this parameter is suggested here. For the electric system in Figure 1 holds the following differential equation:

$$e(t) = R_s i_L(t) + L_s \frac{di_L(t)}{dt} + u_C(t) \tag{5}$$

or written in the corresponding finite difference form:

$$e(t) = R_s i_L(t) + L_s \frac{\Delta i_L(t)}{\Delta t} + u_C(t) \tag{6}$$

Equation (6) should be written for all the known measured points of the interval  $t_j \in [T_{1s}^{(1)}, T_{ms}^{(1)}]$  where the current is a monotonically increasing function:

$$e(t_j) = R_s i_{meas.}(t_j) + L_s \frac{i_{meas.}(t_j) - i_{meas.}(t_{j-1})}{\Delta t} + u_C(t_j) \tag{7}$$

where  $\Delta t$  is the time step of the interval discretization. By adding all the equations (7) up, the following is obtained:

$$\sum_{j=1}^{j_m} e(t_j) = R_s \sum_{j=1}^{j_m} i_{meas.}(t_j) + L_s \frac{i_{meas.}(T_{ms}^{(1)})}{\Delta t} + \sum_{j=1}^{j_m} u_C(t_j)$$

where according to Figure 2 and 3 the following relations holds :

$$T_{1s}^{(1)} = (j_1 - 1)\Delta t, \quad T_{ms}^{(1)} = (j_m - 1)\Delta t, \quad T_{2s}^{(1)} = (j_2 - 1)\Delta t$$

The subscript  $s$  denotes the values obtained from samples of the measured ac current. By taking into account that the average value of the dc capacitor voltage over the considered interval can be calculated as:

$$u_{C_{avg}} = \frac{1}{j_m - j_1} \sum_{j=h}^{j_m} u_C(t_j) \tag{8}$$

the initial inductance can be obtained as follows:

$$L_s = \Delta t \frac{\sum_{j=h}^{j_m} e(t_j) - R_s \sum_{j=h}^{j_m} i_{meas.}(t_j) - (j_m - j_1) u_{C_{avg}}}{i_{meas.}(T_{ms}^{(1)})}$$

Only the initial value of the capacitor voltage  $u_{C01}$  remains to be computed.

$$u_{C01} = e(T_{1s}^{(1)}) \tag{9}$$

In the mentioned references Equation (9) defines the only value of the initial capacitor voltage that holds for the entire simulation time, i.e. for all the half-periods of the load current. However, Equation (9) is accurate only if  $L_s = 0$ . This is evident from the differential equation (5). In case when  $L_s \neq 0$  (in particular when the inductance value is high) Equation (9) is not fulfilled. Furthermore, it could happen that  $\Delta iL(t)/\Delta t \gg 0$  which additionally supports the statement that Equation (9) is not in general fulfilled.

For the reasons elaborated above Equation (9) is in this work used only as the starting value of the initial capacitor voltage when the diodes start conducting in the first half-period.

#### IV. MEASUREMENTS , ESTIMATIONS AND SIMULATIONS OF RECTIFIER BASED LOAD

The suggested parameter estimation method is used to analyse a special laboratory designed rectifier based load with the known system voltage and with the known load parameters.

In Table the obtained results in the form of load parameters are presented. The initial load parameters as well as for the final load parameters obtained by using the suggested parameter estimation technique are shown.

TABLE :The initial and final rectifier load values

	R(Ω)	Rs(Ω)	Ls(mH)	C(μF)
Actual values	700	17.5	91.9	250
Calculated values	698	17.52	88	246.5
Actual values	750	17.5	91.9	150
Calculated values	746.4	18.16	86.67	151
Actual values	1000	17.5	91.9	150
Calculated values	995.9	18.89	99.34	149.4

The simulation has been performed with the load parameters obtained by using the presented estimation method.

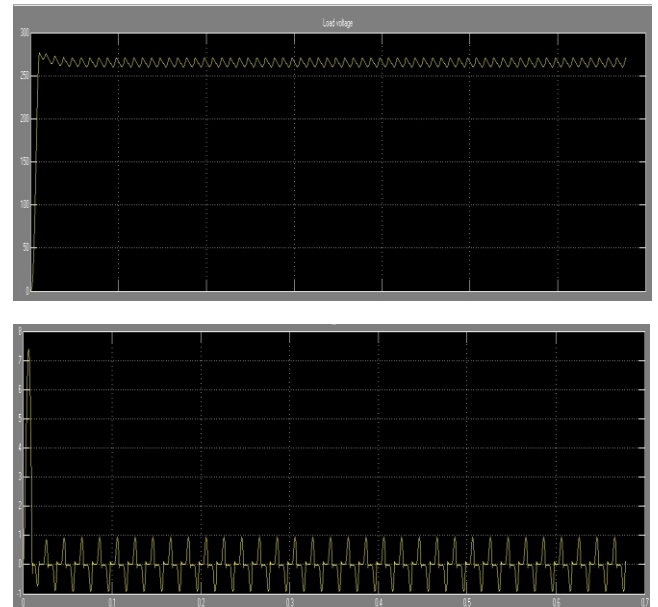


Fig.4.The simulated capacitor voltage and load current

#### CONCLUSIONS

The estimation of structure and model parameter method for determining the unknown parameters of a typical single-phase rectifier based load suitable for harmonic studies is given in detail. The algorithm proposed here is an analytical solutions for the ac load current and dc capacitor voltage with an arbitrary number of the considered half-periods of the system voltage. On the basis of the given algorithm the obtained load parameters are accurate which is confirmed when compared it with the lab designed test rectifier. Finally the simulated value of load current and capacitor voltage are obtained.

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