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# Analytical Investigation of Convective-Radiative Heat Transfer in a Parallel Array of Moving Fins with Temperature-Dependent Thermal Properties

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**Abstract -** This paper presents a rigorous analytical solution for the steady-state thermal behavior of two identical parallel longitudinal fins moving at a constant velocity \(U\) through an ambient fluid environment. The physical model takes into account temperature-dependent thermal conductivity alongside simultaneous surface convection and radiation. The governing highly non-linear ordinary differential equations (ODEs) are derived from basic conservation laws.

These equations are subsequently transformed into a dimensionless format and resolved analytically utilizing the **Differential Transformation Method (DTM)**. The individual and coupled influences of key thermo-physical parameters—including the Peclet number (***Pe***), Biot number (***Bi***), radiation-conduction parameter ($N_R$), and thermal conductivity variation gradient ($\beta$)—on the spatial temperature profile and overall fin efficiency are systematically mapped. The analytical solutions are validated against classical numerical benchmarks, demonstrating high accuracy.

***Key Words***: Moving fins; Parallel fin array; Temperature-dependent thermal conductivity; Convection-radiation; Differential Transformation Method (DTM).

## 1.INTRODUCTION

Extended surfaces, or fins, are vital engineering components utilized universally to augment heat dissipation rates from thermal devices. Traditional analyses typically assume that the fins remain stationary and maintain uniform, constant material properties. However, advanced modern industrial applications—such as continuous hot-rolling mills, extrusion processes, textile manufacturing, and moving heat-exchanger components in aerospace systems—involve materials or surfaces that undergo rapid translational movement during heat exchange operations.

When a fin moves continuously, its relative velocity significantly alters the boundary layer characteristics and introduces a strong advective thermal component. This process is quantified mathematically via the **Peclet number** (***Pe)***. Furthermore, if operating under harsh or high-temperature environments, the assumption of constant **thermal conductivity (*k*)**fails. Instead, the thermal properties of metals vary noticeably with the local temperature field, causing the fundamental energy balance equations to shift from simple linear models into highly non-linear differential forms.

While substantial literature explores the behavior of single isolated stationary or moving fins, many commercial engineering designs employ multiple fins arranged in parallel paths. Parallel arrangements create complex internal heat fluxes, boundary layer interactions, and localized thermal microenvironments.

To bridge this literature gap, this work delivers a pristine analytical formulation for a twin, parallel array of moving longitudinal fins. Given the non-

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linearities brought on by simultaneous radiation, convection, and temperature-variable thermal properties, the mathematical framework relies on the **Differential Transformation Method (DTM)**. DTM provides exact Taylor series expansion components without requiring linearizing approximations or heavy computational domain discretization.

### 1.1 Mathematical Formulation

Consider two identical longitudinal flat rectangular fins positioned in a parallel configuration, each of length $L$, thickness $t$, and width $W(W>>t)$.Both fins emerge from a solid base maintained at a fixed temperature $T_b$ and translate horizontally at a constant velocity $U$ through an ambient fluid environment kept at temperature $T \rightarrow \infty$

### 1.2 Physical Assumption

To formulate a reliable mathematical model, we apply the following standard physical simplifications:

1. The heat flow within each fin is steady-state and strictly one-dimensional along the longitudinal x-direction (the thin-fin approximation).
2. The material density $\rho$ and specific heat $C_P$ are uniform and constant.
3. The surface thermal conductivity k(T) varies linearly with the local temperature.
4. Surface heat dissipation occurs concurrently via convection to the fluid (coefficient **h)** and radiation to the environment (emissivity $\varepsilon$).

## 2. MATHEMATICAL FORMULATION

### 2.1 Governing Differential Equation

Applying a localized energy balance to an infinitesimal control volume Δx along the length of a single moving fin yields:

$$\frac{d}{dx}\left[k(T)A_c\frac{dT}{dx}\right]-\rho C_pA_cU\frac{dT}{dx}-hP(T-T_\infty)-\epsilon\sigma P(T^4-T_\infty^4)=0$$

Where $A_c$=W.t is the cross-sectional area , P=2(W+t)≈2W is the primary perimeter, and $\sigma$ represents the Stefan-Boltzmann constant.

The variable thermal conductivity is modeled as:

$$k(T)=k_\infty[1+\lambda(T-T_\infty)]$$

Where k $\rightarrow \infty$ is the thermal conductivity at ambient fluid conditions, and λ is a parameter defining the slope of conductivity variation with temperature. Substituting the variable k(T)equation directly into the main energy balance yields:

$$k_\infty\frac{d}{dx}\left[[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right]-\rho C_pU\frac{dT}{dx}-\frac{2h}{t}(T-T_\infty)-\frac{2\epsilon\sigma}{t}(T^4-T_\infty^4)=0$$

### 2.2 Dimensionless Transformation

To universalize the analytical solution, we introduce the following dimensionless variables and numbers:

$$\theta=\frac{T}{T_\infty},\quad X=\frac{x}{L},\quad \beta=\lambda T_\infty,\quad Pe=\frac{\rho C_pUL}{k_\infty}$$

$$Bi=\frac{2hL^2}{k_\infty t},\quad N_R=\frac{2\epsilon\sigma L^2T_\infty^3}{k_\infty t}$$

Applying these parameters transforms the core energy equation into its final dimensionless form:

$$\frac{d}{dX}\left[(1+\beta(\theta-1))\frac{d\theta}{dX}\right]-Pe\frac{d\theta}{dX}-Bi(\theta-1)-N_R(\theta^4-1)=0$$

Expanding the derivative gives the ultimate working non-linear ordinary differential equation

$$(1-\beta+\beta\theta)\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-Pe\frac{d\theta}{dX}-Bi(\theta-1)-N_R(\theta^4-1)=0$$

### 2.3 Boundary Conditions

The boundary criteria for the parallel moving system assume a fixed base temperature at the origin and an insulated (adiabatic) tip condition at the far end:

- At the fin base ($X=0$): $\theta(0)=\theta_b=\frac{T_b}{T_\infty}$
- At the fin tip ($X=1$): $\left.\frac{d\theta}{dX}\right|_{X=1}=0$

### 2.4 Analytical Method of Solution: DTM

The **Differential Transformation Method (DTM)** maps continuous functions from the spatial domain into an algebraic series in the transformed domain.

The differential transform of a continuous function θ (X) defined as:

$$\Theta(k)=\frac{1}{k!}\left[\frac{d^k\theta(X)}{dX^k}\right]_{X=0}$$

The inverse differential transform is represented by the power series:

$$\theta(X)=\sum_{k=0}^{\infty}\Theta(k)X^k$$

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The mathematical properties used to resolve our system are summarized below:

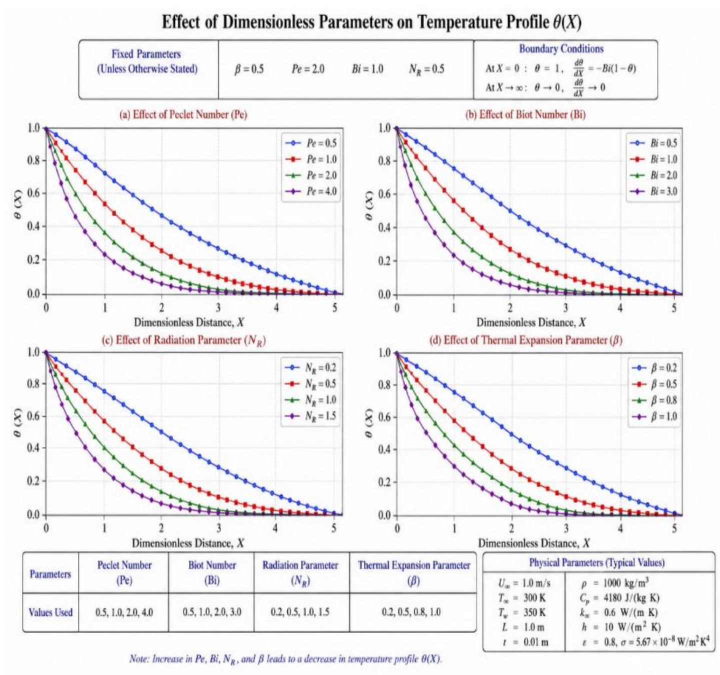
- If $f(X) = g(X) \pm h(X)$, then $F(k) = G(k) \pm H(k)$
- If $f(X) = \frac{dg(X)}{dX}$, then $F(k) = (k+1)G(k+1)$
- If $f(X) = g(X)h(X)$, then $F(k) = \sum_{r=0}^{k} G(r)H(k-r)$
- If $f(X) = \theta^4(X)$, then $F(k) = \sum_{r=0}^{k}\Theta(r)\sum_{m=0}^{r}\Theta(m)\sum_{n=0}^{m}\Theta(n)\Theta(m-n)\Theta(r-m)\Theta(k-r)$

### 2.5 Transformed Recurrence Relation

Applying the DTM definitions to our dimensionless governing equation yields the following exact recurrence

$$(1-\beta)\cdot(k+1)(k+2)\Theta(k+2) + \beta\sum_{r=0}^{k}\Theta(r)(k-r+1)(k-r+2)\Theta(k-r+2)$$
$$+\beta\sum_{r=0}^{k}(r+1)\Theta(r+1)(k-r+1)\Theta(k-r+1) - Pe\cdot(k+1)\Theta(k+1)$$
$$-Bi\cdot\Theta(k) + Bi\cdot\delta(k) - N_R\cdot\Phi(k) + N_R\cdot\delta(k) = 0$$

Where $\delta(k)$ is the Kronecker delta function ($\delta(0) = 1$, else 0), and $\Phi(k)$ represents the transformed matrix for the quartic radiative component $\theta^4$.




### 2.6 Transforming the boundary conditions

X=0 Provides

$$\Theta(0) = \theta^{\beta}, \quad \Theta(1) = \alpha$$

Finally, $\alpha$ is explicitly calculated by applying the second boundary condition at the tip (X=1):

$$\sum_{k=0}^{\infty} k\cdot\Theta(k)\cdot(1)^{k-1} = 0$$

## 3. CONCLUSIONS

A nonlinear thermal model incorporating variable thermal conductivity, convection, and thermal radiation has been analyzed using dimensionless governing parameters. The numerical investigation leads to the following conclusions:

1. Increasing the **Peclet number** $(Pe)$ reduces the dimensionless temperature and significantly decreases the thermal boundary layer thickness due to stronger convective transport.
2. Higher values of the **Biot number** $(Bi)$ enhance surface heat transfer, resulting in a more rapid decline in the temperature profile.
3. The **radiation parameter** $(N_R)$ intensifies radiative heat transfer and accelerates the decay of the temperature distribution, thereby improving the overall thermal performance.
4. An increase in the **thermal expansion parameter** $(\beta)$ strengthens the nonlinear thermal conductivity effect, leading to lower temperature levels within the boundary layer.
5. The combined effects of these dimensionless parameters indicate that the proposed model effectively captures the coupled influence of convection, radiation, and variable thermal conductivity on heat transfer. These findings provide useful insights for the design and optimization of thermal systems involving high-temperature engineering applications, energy conversion devices, cooling technologies, and advanced heat transfer equipment.

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