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Analytical Investigation of a Deterministic Inventory Model using the Cubic Demand Rate and the Exponential Distribution as Deterioration Rate

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***Abstract*:- This research looks into an inventory model for decomposing items or products with a cubic demand over time, with an exponential deterioration rate. In the model, shortages are allowed. It also shows that the cubic demand function is convex and yields the best result. A three-dimensional graphical representation of this model's convexity is shown. An illustration is created to double-check the model. The ideal solution was put through a sensitivity analysis with respect to the main parameters, and the results were presented.**

Keywords: Deterioration, cubic demand, Shortages, Total inventory cost.

INTRODUCTION:

Inventory is defined as the stock of items kept on hand to ensure a trade or business's efficient and smooth operation. It also contributes to the company's growth. Manufacturers, schools, farms, hospitals, and higher education institutions all rely on it. Both merchants and wholesalers must keep their inventory of things or products at a minimum. It is made up of several diverging conditions that can be turned into models. These circumstances could include deterministic demand, demand changes over time, degradation, and so forth.

The mathematical model is distributed into two kinds, i.e., Deterministic and Stochastic models. Deterministic and stochastic mathematical models are the two types of models available. The demand rate is constant in the Deterministic model. In inventory, demand is quite crucial. The demand rate was expected to remain constant in the basic inventory model. However, this isn't always possible.

For many inventory products, like fashionable items, dairy goods, electric items, fruits and vegetables, and so on, the hypothesis of not changing demand rate is not appropriate; the demand rate may be dependent on time, stock, and on price. Fruits, vegetables, medications, dairy products, and other items have a finite shelf life. They deteriorate over time. Items or things that deteriorate are referred to as degrading items. The inventory system is plagued by issues caused by the deterioration of items or products.

The change in demand rate could also be linear, i.e., demand might increase or decrease linearly with time. We employ a linear polynomial as a demand function for linear demand. It should occasionally be a significant change in demand, i.e., demand is rapidly increasing with relevance time. A biquadratic polynomial demand function can be utilized to achieve this increment. Biquadratic demand can help you reduce inventory costs and grow your business.

In 1963, Ghare and Schrader noted that some products deteriorate by a negative exponential function concerning time. This observation gives the differential equation of inventory products with deterioration. An inventory mathematical model for degrading products with exponentially deteriorating demand was proposed by Liang- Yuh Ouyang, Kun-Shan WU, and Mei-Chuan CHENG (2005). Alfares (2007) looked at the inventory policy for an item with a demand rate dependent on the stock level and a storage time dependent on the holding cost. In 2008, Ajanta Roy created an mathematical model for decaying items in which demand was depends on price. He'd also added a time-based holding cost. The case in which degrading rate follows Weibull distributions, C. K. Tripathy* and U. Mishra (2010) devised a listing model. A list model for decaying products was presented by R. Amutha and Dr. E. Chandrasekaran (2012). Demand was supposed to be a straight line in this model. The cost of holding is calculated as a linear function of time. Dr. Ravish Kumar Yadav and Ms. PratibhaYadav(2013) demonstrate the production model with assumption of cubic demand rate. R. Venkateswarlu and R. Mohan (2014) created a list model for deteriorating products constructed on the hypothesis that demand is a quadratic function of time. This concept was created to lower total inventory costs (TIC). Garima Sharma and Bhawna Vyas (2018) suggested a inventory model for decaying products with a Weibull distribution. Demand is measured to be a one degree function of time, with shortages permitted and partially backlogged. Ganesh Kumar, Sunita, and Ramesh Inaniyan (2020) proposed a listing model with a time-dependent demand rate employing various factors. It is supposed that the demand rate is a three degree polynomial of your time.

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The demand rate is treated as a cubic polynomial of time in this paper, and the deterioration rate fluctuates with time. The cost of the order is believed to be constant and does not fluctuate over time. This model's convexity is checked using a three-dimensional graphical representation. To validate the model, an illustration is also created. The ideal solution has been subjected to a sensitivity analysis concerning essential factors, and the results are displayed.

NORMS AND REPRESENTATIONS:

The mathematical model is explained using the norms and representations listed below.

Norms

1. a, b, c, and d are constant.
2. The demand rate $f(t)$ at time t is supposed as $f(t) = \mathrm{a} + \mathrm{bt} + \mathrm{ct}^2 + \mathrm{dt}^3$;a, b, c, d is constant.
3. Replenishment takes place.
4. Shortages are permitted.
5. $\theta(t) = \theta e^{-\theta t}$ Denotes the deterioration rate.

Representations

C_{SC}	Shortage cost per unit per unit time.
C_{OC}	Ordering cost per order.
C_{DC}	Deterioration cost.
C_{HC}	Holding Cost.
W	The maximum inventory level for each ordering cycle.
S	Shortage level for each cycle.
Q	The order quantity ($Q = W + s$).
$Q(t)$	Inventory level at time t.
t_1	Time at which shortages start.
T	The total length of each ordering cycle.
TC	Total inventory cost over the period (0, T).

MATHEMATICAL FORMULATION:

The graph below (Figure 1) shows how inventory changes over time. The ideal order quantity, Q, and the total optimal inventory cost, TC, are shown in this diagram.

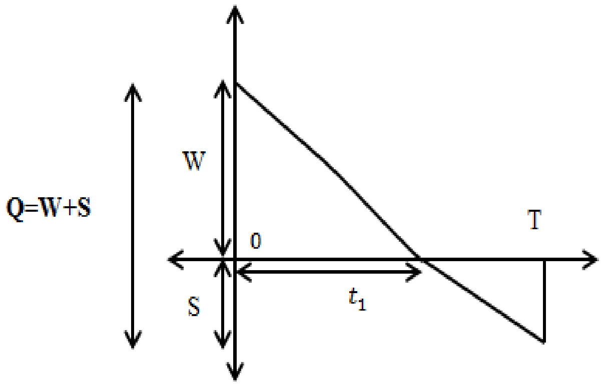

Figure 1: Inventory level (Q) vs time

At t=0, the inventory level is extreme. Following that, the inventory level lowers over the time range [0, t] until falling to zero at t=t_1. Further, at $t = t_1$ shortages occur during the time interval $[t_1, T]$.

Now till the shortages are allowed at interval $[0, t_1]$, the differential equation is given by:

$$\frac{dQ_1(t)}{dt} + \theta e^{-\theta t} Q_1(t) = -(a + bt + ct^2 + dt^3); 0 \le t \le t_1 \tag{1}$$

And during the interval $[t_1, T]$, the shortage occurs, so the differential equation is given by:-

$$\frac{dQ_2(t)}{dt} + \theta e^{-\theta t} Q_2(t) = -(a + bt + ct^2 + dt^3); \mathrm{t}_1 \le \mathrm{t} \le \mathrm{T} \tag{2}$$

With the boundary conditions: t=0, Q(0)=W

T=t_1; Q(t_1)=0

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T=T; Q(T)=S

Now, by solving the above equations (1), we get:

$$Q_1(t)=\Big[a(t_1-t)+\frac{b}{2}(t_1^2-t^2)+\frac{c}{3}(t_1^3-t^3)+\frac{d}{4}(t_1^4-t^4)+\frac{a\theta}{2}(t_1^2-t^2)+\frac{b\theta}{3}(t_1^3-t^3)+\frac{c\theta}{4}(t_1^4-t^4)+\frac{d\theta}{5}(t_1^5-t^5)-\frac{a\theta^2}{6}(t_1^3-t^3)-\frac{b\theta^2}{8}(t_1^4-t^4)-\frac{c\theta^2}{10}(t_1^5-t^5)-\frac{d\theta^2}{12}(t_1^6-t^6)-\Big(a\theta(t_1t-t^2)+\frac{b\theta}{2}(t_1^2t-t^3)+\frac{c\theta}{3}(t_1^3t-t^4)+\frac{d\theta}{4}(t_1^4t-t^5)+\frac{a\theta^2}{2}(t_1^2t-t^3)+\frac{b\theta^2}{3}(t_1^3t-t^4)+\frac{c\theta^2}{4}(t_1^4t-t^5)+\frac{d\theta^2}{5}(t_1^5t-t^6)-\frac{a\theta^3}{6}(t_1^3t-t^4)-\frac{b\theta^3}{8}(t_1^4t-t^5)-\frac{c\theta^3}{10}(t_1^5t-t^6)-\frac{d\theta^3}{12}(t_1^6t-t^7)\Big)+\Big(\frac{a\theta^2}{2}(t_1t^2-t^3)+\frac{b\theta^2}{4}(t_1^2t^2-t^4)+\frac{c\theta^2}{6}(t_1^3t^2-t^5)+\frac{d\theta^2}{8}(t_1^4t^2-t^6)+\frac{a\theta^3}{4}(t_1^2t^2-t^4)+\frac{b\theta^3}{6}(t_1^3t^2-t^5)+\frac{c\theta^3}{8}(t_1^4t^2-t^6)+\frac{d\theta^3}{10}(t_1^5t^2-t^7)-\frac{a\theta^4}{12}(t_1^3t^2-t^5)-\frac{b\theta^4}{16}(t_1^4t^2-t^6)-\frac{c\theta^4}{20}(t_1^5t^2-t^7)-\frac{d\theta^4}{24}(t_1^6t^2-t^8)\Big)\Big]$$

By solving equation (2), we get:

$$Q_2(t)=a(t_1-t)+\frac{b}{2}(t_1^2-t^2)+\frac{c}{3}(t_1^3-t^3)+\frac{d}{4}(t_1^4-t^4) \tag{3}$$

Now, at t=0, the maximum inventory level for each cycle is given by:

Q(0)=W, t=0

$$W=Q_1(0)=at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}+\frac{dt_1^4}{4}+\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}$$

And at t=T, the maximum amount of cubic demand per cycle is given by:

t=T, $Q_2(t)= -S$

$$S=-\left(a(t_1-T)+\frac{b}{2}(t_1^2-T^2)+\frac{c}{3}(t_1^3-T^3)+\frac{d}{4}(t_1^4-T^4)\right)$$

Now, the order quantity per cycle is:

$$Q=W+S=at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}+\frac{dt_1^4}{4}+\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}-\left(a(t_1-T)+\frac{b}{2}(t_1^2-T^2)+\frac{c}{3}(t_1^3-T^3)+\frac{d}{4}(t_1^4-T^4)\right)$$

Holding cost per unit per unit time is given by: (5)

$$\text{Holding Cost per cycle}=C_{HC}\int_0^{t_1}Q_1(t)dt$$

$$\text{Holding Cost per cycle}=C_{HC}\left[\frac{at_1^2}{2}+\left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^3+\left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^4+\left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^5+\left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^6+\left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^7+\left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^8-\frac{d\theta^4}{96}t_1^9\right]$$

Shortages cost per unit per unit time is given by:

$$\text{Shortage Cost per cycle}=(-)\,C_{SC}\int_{t_1}^{T}Q_2(t) \tag{6}$$

$$\text{Shortage Cost per cycle}=-C_{SC}\left[a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+b\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+c\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)+d\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\right]$$

Ordering cost per order is given by: (7)

$$\text{Ordering cost per order}=C_{OC}$$

Now, the deteriorating cost is given by: (8)

$$\text{Cost due to Deterioration}=C_{DC}\left[W-\int_0^{t_1}Q(t)dt\right]$$

$$\text{Cost due to Deterioration}=C_{DC}\left[\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}\right]$$

Therefore, the total cost per unit time per unit cycle is given by:

(9)

$$\mathrm{TC} = \frac{1}{\mathrm{T}}(\text{Holding Cost per cycle} + \text{Shortage Cost per cycle} + \text{Ordering cost per cycle} + \text{Cost due to Deterioration})$$

$$\mathrm{TC} = \frac{1}{T}\left\{C_{HC}\left[\frac{at_1^2}{2} + \left(\frac{b}{3} + \frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4} + \frac{b\theta}{8} - \frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5} + \frac{c\theta}{10} - \frac{5b\theta^2}{30} + \frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12} - \frac{5c\theta^2}{36} + \frac{5b\theta^3}{72} - \frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42} + \frac{5c\theta^3}{84} - \frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96} - \frac{c\theta^4}{96}\right)t_1^8 - \frac{d\theta^4}{96}t_1^9\right] - C_{SC}\left[a\left(t_1T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right)\right] + \mathrm{C_{OC}} + C_{DC}\left[\frac{a\theta t_1^2}{2} + \frac{b\theta t_1^3}{3} + \frac{c\theta t_1^4}{4} + \frac{d\theta t_1^5}{5} - \frac{a\theta^2t_1^3}{6} - \frac{b\theta^2t_1^4}{8} - \frac{c\theta^2t_1^5}{10} - \frac{d\theta^2t_1^6}{12}\right]\right\}$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{\mathrm{d(TC)}}{\mathrm{dT}} = -\frac{1}{T^2}\left\{C_{HC}\left[\frac{at_1^2}{2} + \left(\frac{b}{3} + \frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4} + \frac{b\theta}{8} - \frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5} + \frac{c\theta}{10} - \frac{5b\theta^2}{30} + \frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12} - \frac{5c\theta^2}{36} + \frac{5b\theta^3}{72} - \frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42} + \frac{5c\theta^3}{84} - \frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96} - \frac{c\theta^4}{96}\right)t_1^8 - \frac{d\theta^4}{96}t_1^9\right] + C_{SC}\left[a\left(t_1T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right)\right] + \mathrm{C_{OC}} + C_{DC}\left[\frac{a\theta t_1^2}{2} + \frac{b\theta t_1^3}{3} + \frac{c\theta t_1^4}{4} + \frac{d\theta t_1^5}{5} - \frac{a\theta^2t_1^3}{6} - \frac{b\theta^2t_1^4}{8} - \frac{c\theta^2t_1^5}{10} - \frac{d\theta^2t_1^6}{12}\right]\right\} - \frac{1}{T}\left[\mathrm{C_{SC}}\left(a(t - b\left(\frac{t^2}{2} - \frac{T^2}{2}\right) + c\left(\frac{t^3}{3} - \frac{T^3}{3}\right) + d\left(\frac{t^4}{4} - \frac{T^4}{4}\right)\right)\right] \quad (10)$$

$$\frac{\mathrm{d(TC)}}{\mathrm{dt_1}} = \frac{1}{\mathrm{T}}\left\{C_{HC}\left[\frac{at_1}{2} + 3\left(\frac{b}{3} + \frac{a\theta}{6}\right)t_1^2 + 4\left(\frac{c}{4} + \frac{b\theta}{8} - \frac{5a\theta^2}{24}\right)t_1^3 + 5\left(\frac{d}{5} + \frac{c\theta}{10} - \frac{5b\theta^2}{30} + \frac{5a\theta^3}{60}\right)t_1^4 + 6\left(\frac{d\theta}{12} - \frac{5c\theta^2}{36} + \frac{5b\theta^3}{72} - \frac{a\theta^4}{72}\right)t_1^5 + 7\left(\frac{5d\theta^2}{42} + \frac{5c\theta^3}{84} - \frac{b\theta^4}{84}\right)t_1^6 - 8\left(\frac{5d\theta^3}{96} - \frac{c\theta^4}{96}\right)t_1^7 - 9\frac{d\theta^4}{96}t_1^8\right] - C_{SC}[a(T-1) + b(t_1T - t_1^2) + c(t_1^2T - t_1^3) + d(t_1^3T - t_1^4)] + C_{DC}\left[a\theta t_1 + b\theta t_1^2 + c\theta t_1^3 + d\theta t_1^4 - \frac{a\theta^2t_1^2}{2} - \frac{b\theta^2t_1^3}{2} - \frac{c\theta^2t_1^4}{2} - \frac{d\theta^2t_1^5}{2}\right]\right\} \quad (11)$$

We get the optimal values of t_1 and T by solving equations (11) & (12) by using MAPLE 15.

(12)

NUMERICAL ILLUSTRATION:

Now check the optimality of the model with the help of numerical illustration and solve the illustration with the help of Maple 15.

To explain the model numerically, assume the following parameters of the inventory system are:

$$a = 15, b = 5, c = 3, d = 1, C_{HC} = 4, C_{SC} = 15, C_{OC} = 100, C_{DC} = 10, \theta = 0.005$$

Under the above-given parameters, by using Maple 15 get the optimal shortage value t_1 =1.965520277 per unit time and the optimal length of the ordering cycle is T=2.169637827 unit time. The total inventory cost is TC=171.7030613

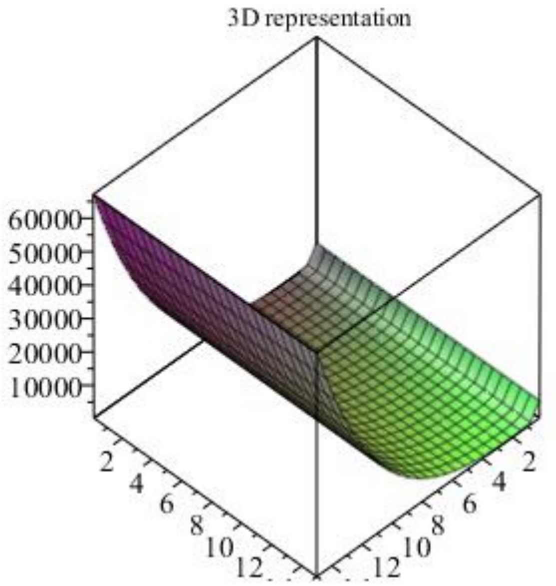

Figure 2: 3-Dimensional Graphical representation of TC of Inventory Model

SENSITIVITY ANALYSIS:

Here we study the effect of changes in the model parameters such that $a, b, c, d, C_{HS}, C_{CS}, C_{OC}, C_{DC}$, $and\ \theta$.The outcome is given in the below table:

Parameter	% change	Change in T	t_1	Q	TC
a	+20%	1.0160680850	1.0889027267	18.18963695	252.5956185
	+10%	1.7077281320	1.159175496	20.45041909	241.0584430
	-10%	1.9920037314	1.185750586	27.07175271	218.2594875
	-20%	2.044515532	1.816860712	48.10762535	158.6757269
b	+20%	2.126896788	1.915704782	60.49513233	176.6141174
	+10%	2.1547467362	1.957865123	60.37356030	172.6973260
	-10%	2.191291732	1.990975590	60.25658514	169.1540634
	-20%	2.213117867	2.016789921	59.04959300	166.5729344
c	+20%	2.113447263	1.902173202	59.46007555	176.0471621
	+10%	2.140927371	1.933085200	59.90343378	173.9257361
	-10%	2.199662771	1.999598585	60.82422853	201.3702879
	-20%	2.210247264	2.006920108	61.55764241	228.8856832
d	+20%	2.112338659	1.906072384	58.51067445	175.0532139
	+10%	2.199954826	1.926720236	60.76614908	172.7088979
	-10%	2.200425661	1.993271308	61.71324677	169.5107908
	-20%	2.236482331	2.034550137	62.55851032	167.9186105
C_{HC}	+20%	2.011399454	1.738349860	52.72142939	190.6949737
	+10%	2.166399602	1.9346398922	58.90606113	172.7767587
	-10%	2.260306526	2.091810959	65.10123636	160.0418475
	-20%	2.361669905	2.229717027	70.74288999	146.5633663
C_{SC}	+20%	2.2915508098	2.1824406245	68.55092132	255.0259308
	+10%	2.235118114	2.066042302	63.75989053	230.6794186
	-10%	2.092877649	2.03703989	59.82044284	216.7695858
	-20%	2.030681482	1.933548455	53.59939368	195.9338203
C_{OC}	+20%	2.208712799	1.999529473	62.35879326	180.8380976
	+10%	2.189558638	1.982895452	61.37187552	176.2909535
	-10%	2.148869472	1.947321081	59.31515212	167.0719555
	-20%	2.128345692	1.928054384	58.95488632	159.6773670
C_{DC}	+20%	2.117510096	1.912520935	58.25023377	171.9721423
	+10%	.9995693954	.2251108959	18.74167160	229.6118473
	-10%	.9994046753	.2248812972	18.73771531	229.6078397
	-20%	2.171770536	1.968525070	60.46662157	171.4330541
θ	+20%	.9996592125	.2252363333	18.74423028	229.6139733
	+10%	.9996493226	.2251252345	18.74482032	229.6147822
	-10%	.9994010197	.2248760819	18.73742754	229.6077803
	-20%	.9993151458	.2247562686	18.73516556	229.6057199

- With the increase in a,d TC increase, and Q decrease.
- With the increase in b, TC and Q increases.
- With the increase in c, TC, and Q decreases.
- If C_{HC} (Holding Cost) increases, TC increases, and Q will decrease.
- If C_{SC} (Shortage Cost), C_{OC} (Ordering Cost) increases, Q and TC will increases.
- If θ increases, Q, and TC increase.

On the basis of supposition of cubic demand and exponential distribution as deterioration function different cases are arise:

CASE1: **Study of Model in case of cubic demand and exponential distribution with variable Holding Cost**

In this case we find out the Total Inventory Cost and verify with the help of numerical illustration

$$HC = \int_0^{t_1} (A + Bt)\, Q_1(t) dt$$

With the help of equation (3), we get

$$HC = A\left[\frac{at_1^2}{2} + \left(\frac{b}{3} + \frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4} + \frac{b\theta}{8} - \frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5} + \frac{c\theta}{10} - \frac{5b\theta^2}{30} + \frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12} - \frac{5c\theta^2}{36} + \frac{5b\theta^3}{72} - \frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42} + \frac{5c\theta^3}{84} - \frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96} - \frac{c\theta^4}{96}\right)t_1^8 - \frac{d\theta^4}{96}t_1^9\right] + B\left[\frac{at_1^3}{6} + \left(\frac{b}{8} + \frac{a\theta}{24} - \frac{a\theta^2}{8}\right)t_1^4 + \left(\frac{c}{10} + \frac{b\theta}{30} - \frac{b\theta^2}{10} - \frac{a\theta^2}{40}\right)t_1^5 + \left(\frac{d}{12} + \frac{c\theta}{36} - \frac{c\theta^2}{12} - \frac{b\theta^2}{48} + \frac{7a\theta^3}{144}\right)t_1^6 + \left(\frac{d\theta}{42} - \frac{d\theta^2}{14} - \frac{c\theta^2}{56} + \frac{7b\theta^3}{168} - \frac{a\theta^4}{112}\right)t_1^7 + \left(-\frac{d\theta^2}{64} + \frac{7c\theta^3}{192} - \frac{b\theta^4}{128}\right)t_1^8 + \left(\frac{7d\theta^3}{216} - \frac{c\theta^4}{144}\right)t_1^9 - \frac{d\theta^4}{160}t_1^{10}\right]$$

$$TC = \frac{1}{T}(HC + SC + OC + DC)$$

$$TC = \frac{1}{T}\left\{A\left[\frac{at_1^2}{2} + \left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^8 - \frac{d\theta^4}{96}t_1^9\right] + B\left[\frac{at_1^3}{6} + \left(\frac{b}{8}+\frac{a\theta}{24}-\frac{a\theta^2}{8}\right)t_1^4 + \left(\frac{c}{10}+\frac{b\theta}{30}-\frac{b\theta^2}{10}-\frac{a\theta^2}{40}\right)t_1^5 + \left(\frac{d}{12}+\frac{c\theta}{36}-\frac{c\theta^2}{12}-\frac{b\theta^2}{48}+\frac{7a\theta^3}{144}\right)t_1^6 + \left(\frac{d\theta}{42}-\frac{d\theta^2}{14}-\frac{c\theta^2}{56}+\frac{7b\theta^3}{168}-\frac{a\theta^4}{112}\right)t_1^7 + \left(-\frac{d\theta^2}{64}+\frac{7c\theta^3}{192}-\frac{b\theta^4}{128}\right)t_1^8 + \left(\frac{7d\theta^3}{216}-\frac{c\theta^4}{144}\right)t_1^9 - \frac{d\theta^4}{160}t_1^{10}\right] - C_{SC}\left[a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\right] + C_{DC}\left[\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}\right] + C_{OC}\right\}$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{\mathrm{d(TC)}}{\mathrm{dT}} = -\frac{1}{T^2}\left\{A\left[\frac{at_1^2}{2} + \left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^8 - \frac{d\theta^4}{96}t_1^9\right] + B\left[\frac{at_1^3}{6} + \left(\frac{b}{8}+\frac{a\theta}{24}-\frac{a\theta^2}{8}\right)t_1^4 + \left(\frac{c}{10}+\frac{b\theta}{30}-\frac{b\theta^2}{10}-\frac{a\theta^2}{40}\right)t_1^5 + \left(\frac{d}{12}+\frac{c\theta}{36}-\frac{c\theta^2}{12}-\frac{b\theta^2}{48}+\frac{7a\theta^3}{144}\right)t_1^6 + \left(\frac{d\theta}{42}-\frac{d\theta^2}{14}-\frac{c\theta^2}{56}+\frac{7b\theta^3}{168}-\frac{a\theta^4}{112}\right)t_1^7 + \left(-\frac{d\theta^2}{64}+\frac{7c\theta^3}{192}-\frac{b\theta^4}{128}\right)t_1^8 + \left(\frac{7d\theta^3}{216}-\frac{c\theta^4}{144}\right)t_1^9 - \frac{d\theta^4}{160}t_1^{10}\right] - C_{SC}\left[a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\right] + C_{DC}\left[\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}\right] + C_{OC}\right\} - \frac{1}{T}\left\{\mathrm{C_{SC}}\left[a(t-T) + b\left(\frac{t^2}{2}-\frac{T^2}{2}\right) + c\left(\frac{t^3}{3}-\frac{T^3}{3}\right) + d\left(\frac{t^4}{4}-\frac{T^4}{4}\right)\right]\right\}$$

$$\frac{\mathrm{d(TC)}}{\mathrm{dt_1}} = \frac{1}{\mathrm{T}}\left\{A\left[\frac{at_1}{2} + 3\left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^2 + 4\left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^3 + 5\left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^4 + 6\left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^5 + 7\left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^6 + 8\left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^7 + 9\left(-\frac{d\theta^4}{96}\right)t_1^8\right] + B\left[\frac{at_1^2}{2} + 4\left(\frac{b}{8}+\frac{a\theta}{24}-\frac{a\theta^2}{8}\right)t_1^3 + 5\left(\frac{c}{10}+\frac{b\theta}{30}-\frac{b\theta^2}{10}-\frac{a\theta^2}{40}\right)t_1^4 + 6\left(\frac{d}{12}+\frac{c\theta}{36}-\frac{c\theta^2}{12}-\frac{b\theta^2}{48}+\frac{7a\theta^3}{144}\right)t_1^5 + 7\left(\frac{d\theta}{42}-\frac{d\theta^2}{14}-\frac{c\theta^2}{56}+\frac{7b\theta^3}{168}-\frac{a\theta^4}{112}\right)t_1^8 + 8\left(-\frac{d\theta^2}{64}+\frac{7c\theta^3}{192}-\frac{b\theta^4}{128}\right)t_1^7 + 9\left(\frac{7d\theta^3}{216}-\frac{c\theta^4}{144}\right)t_1^8 + 10\left(-\frac{d\theta^4}{160}\right)t_1^9\right] - C_{SC}[a(T-1) + b(t_1T-t_1^2) + c(t_1^2T-t_1^3) + d(t_1^3T-t_1^4)] + C_{DC}\left[a\theta t_1 + b\theta t_1^2 + c\theta t_1^3 + d\theta t_1^4 - \frac{a\theta^2t_1^2}{2} - \frac{b\theta^2t_1^3}{2} - \frac{c\theta^2t_1^4}{2} - \frac{d\theta^2t_1^5}{2}\right]\right\}$$

NUMERICAL ILLUSTRATION:

Now check the optimality of the model with the help of numerical illustration and solve the illustration with the help of Maple 15.

To explain the model numerically, assume the following parameters of the inventory system are:

$$a = 15, b = 5, c = 3, d = 1, C_{SC} = 15, C_{OC} = 100, C_{DC} = 10, \theta = 0.005, A = 3, B = 1$$

Under the above-given parameters, by using Maple 15 get the optimal shortage value $t_1 = 1.946300858$ per unit time and the optimal length of the ordering cycle is $T = 2.144001938$ unit time. The total inventory cost is $TC = 163.8867454$

CASE 2: Study of Model in case of cubic demand and exponential distribution with Variable Ordering Cost

$$OC = \frac{\mathrm{C_{OC}}}{T}$$

$$TC = \frac{1}{T}\left\{C_{HC}\left[\frac{at_1^2}{2} + \left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^8 + \left(-\frac{d\theta^4}{96}\right)t_1^9\right] - C_{SC}\left[a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\right] + \frac{\mathrm{C_{OC}}}{T} + C_{DC}\left[\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}\right]\right\}$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{\mathrm{d(TC)}}{\mathrm{dT}} = -\frac{1}{T^2}\left\{C_{HC}\left[\frac{at_1^2}{2} + \left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^3 + \left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^4 + \left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^5 + \left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^6 + \left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^7 + \left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^8 + \left(-\frac{d\theta^4}{96}\right)t_1^9\right] + C_{SC}\left[a\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\right] + \mathrm{C_{OC}} + C_{DC}\left[\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}+\frac{c\theta t_1^4}{4}+\frac{d\theta t_1^5}{5}-\frac{a\theta^2t_1^3}{6}-\frac{b\theta^2t_1^4}{8}-\frac{c\theta^2t_1^5}{10}-\frac{d\theta^2t_1^6}{12}\right]\right\} - \frac{1}{T}\left\{\mathrm{C_{SC}}\left[a(t-T) + b\left(\frac{t^2}{2}-\frac{T^2}{2}\right) + c\left(\frac{t^3}{3}-\frac{T^3}{3}\right) + d\left(\frac{t^4}{4}-\frac{T^4}{4}\right) - \frac{\mathrm{C_{OC}}}{T^2}\right]\right\}$$

$$\frac{\mathrm{d(TC)}}{\mathrm{dt_1}} = \frac{1}{\mathrm{T}}\left\{C_{HC}\left[\frac{at_1}{2} + 3\left(\frac{b}{3}+\frac{a\theta}{6}\right)t_1^2 + 4\left(\frac{c}{4}+\frac{b\theta}{8}-\frac{5a\theta^2}{24}\right)t_1^3 + 5\left(\frac{d}{5}+\frac{c\theta}{10}-\frac{5b\theta^2}{30}+\frac{5a\theta^3}{60}\right)t_1^4 + 6\left(\frac{d\theta}{12}-\frac{5c\theta^2}{36}+\frac{5b\theta^3}{72}-\frac{a\theta^4}{72}\right)t_1^5 + 7\left(-\frac{5d\theta^2}{42}+\frac{5c\theta^3}{84}-\frac{b\theta^4}{84}\right)t_1^6 + 8\left(\frac{5d\theta^3}{96}-\frac{c\theta^4}{96}\right)t_1^7 + 9\left(-\frac{d\theta^4}{96}\right)t_1^8\right] - C_{SC}[a(T-1) + b(t_1T-t_1^2) + c(t_1^2T-t_1^3) + d(t_1^3T-t_1^4)] + C_{DC}\left[a\theta t_1 + b\theta t_1^2 + c\theta t_1^3 + d\theta t_1^4 - \frac{a\theta^2t_1^2}{2} - \frac{b\theta^2t_1^3}{2} - \frac{c\theta^2t_1^4}{2} - \frac{d\theta^2t_1^5}{2}\right]\right\}$$

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NUMERICAL ILLUSTRATION:

Now check the optimality of the model with the help of numerical illustration and solve the illustration with the help of Maple 15.

To explain the model numerically, assume the following parameters of the inventory system are:

$$a = 30, b = 5, c = 3, d = 1, C_{SC} = 15, C_{OC} = 100, C_{DC} = 10, \theta = 0.005$$

Under the above-given parameters, by using Maple 15 get the optimal shortage value $t_1 = 2.575399869$ per unit time and the optimal length of the ordering cycle is $T = 2.683196227$ unit time. The total inventory cost is $TC = 183.6104201$

CONCLUSION:

The sensitivity analysis found that in the case of cubic demand (a function of time), the model predicts that the rate of deterioration will alter over time. It can be shown that parameters a and b are directly proportional to TC, whereas the parameters c, is inversely proportional to TC. It also demonstrates that this technique can be used to determine the total inventory cost. This also shows that Total Inventory Cost minimizes in case of variable holding cost. Finally, a graphical depiction is used to verify the model. The obtained findings determine the model's stability. This model can be expanded or replaced with a different demandrate.

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