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# Analytical Evaluation of the Influence of Cement Paste Ring in Minimizing Rebar Corrosion in RC Structures

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***Abstract*-The bond between concrete and steel reinforcement at the ITZ is normally very porous, hence it promotes corrosion due to ingress of harmful chemicals. The porous zone is weak because of insufficient compaction and accumulation of bleeding water. It is proposed that proper compaction can be achieved by rebar shaking technique which leaves steel bars enveloped by cement paste ring (CPR). From experimental tests and numerical modeling studies, the CPR has proved to reduce porosity at ITZ between rebar and concrete due to presence of fine particles and good compaction, adhesion of concrete paste on the rebar and prolonged curing. Similarly The CPR absorbs corrosion pressure more than the bulk concrete. Internal radial pressure caused by corrosion products create uniform expansive stresses at the interface surface between steel and concrete that results in a radial displacement at the surface of the rust layer. Analytical model has been used to study the behavior and mechanisms of corrosion pressure, interface pressure between rebar and rust materials and the related displacement of the CPR and concrete cover. Rebar in CPR and concrete cover is considered as a thick walled cylinder, bi-layered and coaxial components in contact subjected to uniform pressure at its inner surface, which represents expansion caused by corrosion products. The results of the proposed model is presented graphically like: Interface Corrosion Pressure in Relation to Thickness; Contact Pressure versus Displacement of Bulk Concrete; Pressure change ratio versus modulus ratio relationship and Cracking Pressure versus ratio of cover to diameter. The thickness of CPR has shown to have considerable effect on counteracting corrosion pressure, accommodating corrosion products and minimizing cracks of concrete cover**.

***Keywords—Cracks, Cement Paste Ring, Corrosion, Cover, ITZ***

## I. INTRODUCTION

Corrosion of reinforcement is one of the most obvious consequence of lack of durability of reinforced concrete structures (Yeomans, 2004). Corrosion of the steel reinforcement leads to concrete cracking, delaminating or spalling of the concrete cover, reduction of concrete and reinforcement cross sections, loss of bond between the reinforcement and concrete, reduction in strength (flexural, shear, etc.) and ductility. Therefore the safety and serviceability of reinforced concrete structures are reduced, and their service lives shortened, consequently the need to refurbish them increase (Lounis and Amleh, 2004, Cabrera, 1996). To combat consequences of corrosion and hence extend the life span of reinforced concrete (R.C) structures, extensive studies, experience and innovation have been amalgamated which led to develop a number of corrosion control methods i.e. electrical, chemicals, biological and mechanical techniques Chung (2001) Zolfagharifard 2011, Jonkers and Schlangen 2008), (Maki, 2012). These methods of corrosion control generally experience limitations on the aspects of economical, sustainability, environmental as well as their effectiveness and acceptability. These challenges of corrosion control methods are common in developed world and they are hardly adopted in developing countries due to economic and technological levels. Core objective of these corrosion control methods is to improve bond or Interfacial Transition Zone (ITZ), between rebar and concrete

The bond between concrete and steel reinforcement at the ITZ is normally very porous, hence it promotes corrosion due to ingress of harmful chemicals. The porous zone is weak because of insufficient compaction, accumulation of bleeding water (higher effective w/c ratio) hence a more open - porous structure (Weiss et al., 2009; Munns et al, 2010). Therefore improving compaction, porosity and bleeding at ITZ will minimize corrosion. It is proposed that proper compaction can be achieved by rebar shaking technique. Rebar Shaking is a process of turning rebars into a vibrator (Bennet et al., 2003). This practice leaves steel bars enveloped by cement paste in other words; it creates a cement paste ring around rebar as shown in Figure 1. The cement paste ring (CPR) is synonyms of Concrete Paste Ring, in reality this zone is comprised of cement, fine sand, fine particles of aggregates. From experimental tests and numerical modeling studies of the CPR the following findings can be realized

i. CPR has proved to reduce porosity at ITZ between rebar and concrete due to presence of fine particles and good compaction, adhesion of concrete paste on the rebar and prolonged curing.

ii. The Zone adjacent to CPR has lower porosity than CPR itself because this zone has higher presence of cement, relatively fine aggregates, optimal w/c ratio and good compaction.
iii. The Cement Paste Ring reduces the cracking pressure toward bulk concrete hence increasing the possibility of minimizing cover delamination. This is because Cement Paste Ring cracks easily hence accommodating rusting materials.
iv. The position/location of failure where Tensile stresses are higher than tensile strength of concrete is closer for bulk concrete ( concrete that his normally vibrated) than for CPR surrounded rebar
v. The CPR absorbs corrosion pressure more than the bulk concrete, the stress contours are not continuous which suggest discontinuation of the crack propagation
vi. The strength factor shows that the damage due to cracks is less in CPR than in bulk concrete. Even though the damage is less in CPR, but still delamination of concrete cover could be violent if measures like rehabilitation are not taken timely.

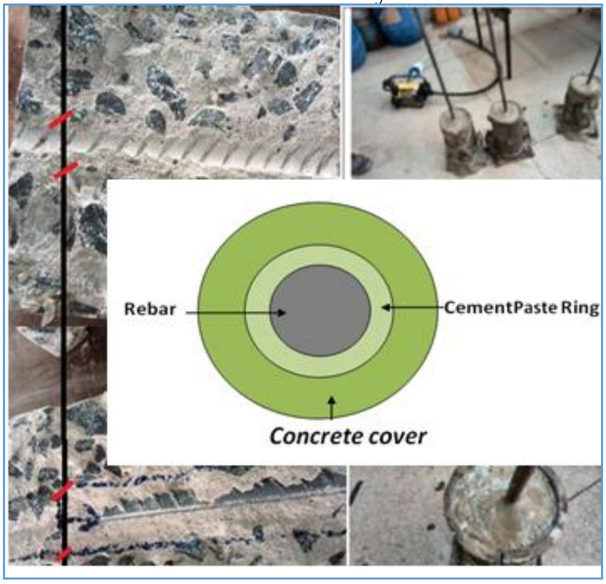

Figure 1: Sample Preparation and schematic diagram of cement paste ring.

### *A. Experimental Model and Hypothesis*

Rebar Shaking is a process of turning rebars into a vibrator (Bennet et al., 2003). This practice leaves steel bars enveloped by cement paste in other words; it creates a paste ring around rebar as shown in Figure 1(a). The cement paste ring (CPR) is synonyms of Concrete Paste Ring. Since in reality this zone is comprised of cement, fine sand, fine particles of aggregates and water. Since the Young modulus

## II. MATHEMATICAL MODELLING

In reinforced concrete, when rust is formed, depending on the level of oxidation, the volume increase due to rebar corrosion which may be up to about 6.5 times the original iron volume because of the formation of various corrosion products (Liu, 1996, Liu and Weyers, 1998). High volume corrosion products exert tensile forces and spalling of the concrete cover may occur. Corrosion cracking and pressure have widely been studied by (Wang and Liu 2008, Ahmad, 2003, Yuan and Ji 2009, Li *et al*., 2008, Zheng *et al.*, 2005, Tamer 2007, Chang *et al*., 2010). Internal radial pressure caused by corrosion products would create uniform expansive stresses at the interface surface between steel and concrete that results in a radial displacement at the surface of the rust layer. Models have been proposed to predict the corrosion pressure, interface pressure between rebar and rust materials, bond behavior at the steel-concrete interface due to corrosion of reinforcing steel, which can be divided into three categories: empirical, analytical and numerical models. In the models, concrete around a corroding reinforcing bar is considered as a thick-walled cylinder subjected to uniform pressure at its inner surface, which represents expansion caused by corrosion products. The pressure leads to formation of radial cracks near the inner surface of the cylinder (Leonid et al. 2013).The cylinder in these models were divided into two zones a partially cracked inner cylinder and an un-cracked outer one. The CPR zone in our study is regarded as cracked zone. Cracks in the inner cylinder are taken into account by gradually reducing its tangential stiffness along the radial direction. Bhargavaa et al. 2005, proposed an analytical model for predicting the time required for concrete cover cracking and the weight loss of reinforcement bars due to rebar corrosion. Liu and Weyers 1998 presented a model for estimating the time to cracking of concrete cover based on the experiments and various corrosion rates. Pantazopoulou and Papoulia (2001), proposed a numerical model to study the implications of concrete cover cracking due to reinforcement corrosion and provided time estimates for the cover cracking over corroded rebar. Liu and Weyers (1998) introduced formula for rust production and focused on detailed modeling of cracked concrete. Hua-Peng and Nan (2012) presented a model of evolution of concrete cover cracking due to reinforcement corrosion, based on the thick-walled cylinder model for the concrete cover and the cohesive crack model for the cracked concrete. Martin-Perez (1999) extended the thick wall formula, but did not study the cracked part of concrete in detail. He introduced compatibility conditions but did not properly model rust compaction, and the model gives short times to cover cracking. Other proposed models were presented by (Liu, Molina, Andrade 1993, Morinaga 1988), most are based on the hypothesis of a thick-walled cylinder with the wall thickness equal to that of the concrete cover, with different approaches to the non- linear behavior of concrete after cracking

## III. DEVELOPMENT OF A CLOSED FORM SOLUTION

### *A. Assumptions*

To simplify the development of mathematical modeling especially on superimposition of various expressions, assumptions were made, as follows:

i. The corrosion products are formed uniformly around the steel reinforcing bar which results in uniform expansive stresses around the steel bar.
ii. The volume expansion caused by corrosion creates strain only in concrete (i.e. strain in steel is neglected). This assumption is reasonable because

the Young’s modulus of steel is about one order of magnitude higher than that concrete, and hence steel deformation would be small enough to be neglected compared with concrete deformation.

iii. Corrosion process is spatially uniform around the reinforcement resulting in the spatially uniform buildup of rust products over the reinforcement thereby resulting in the uniform radial steel–concrete interface pressure due to expansive rust products.

iv. Host concrete and CPR is Continuous, homogeneous, Isotropic and Linear elastic (CHILE)

v. The combination of rebar, CPR and bulk concrete (the rest of concrete after CPR) becomes a perfect thick walled cylinder.

vi. Because of its gripping capacity, axial movement and forces and concrete’s roughness effects are assumed to be negligible.

vii. High bond strength no shear that affects CPR

viii. No temperature that influences the result

ix. The corrosion products are incompressible

x. The analysis will be a plane strain condition

xi. The corrosion process around the reinforcement bar is uniform and can be realized through a radial displacement

xii. No other stresses apart from the expansion of rust materials and concrete confinement are considered

xiii. No porous zone between rebar and CPR

xiv. Corrosion products can be diffused and accommodated within the open radial cracks and hence reduce the magnitude of corrosion pressure

Once Corrosion products, CPR and bulk concrete come into contact they behave as one body, having same deformations in magnitude but with different stiffness. The model will make use of the boundary conditions; initial corrosion pressure and deformations of CPR-Bulk concrete due to the applied stresses. Because the internal expansive corrosion pressure and external confined stress of concrete can be calculated from conditions of equilibrium, the problem can be said to be statically determinate. Figure 2 shows the typical model

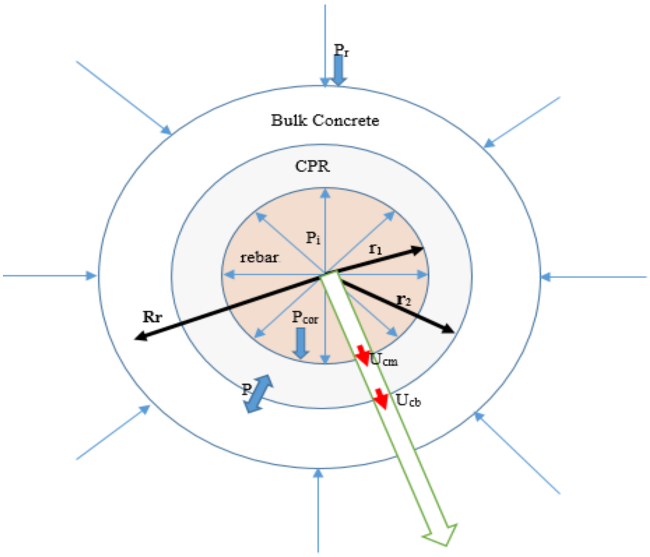

Figure 2: Rebar modelled as thick-walled cylinder in CPR subjected to Internal Pressure (Pi), reaction corrosion pressure (Pcor) and external loading (pr) (hydrostatic)

### B. *Evaluation of CPR-Bulk Concrete as pressurized thick walled cylinder*

The Rebar within CPR Bulk concrete relation are modelled as pressurized thick wall cylinders initially with no external loading. When the rebar corrode in a CPR it causes a corrosion pressure which become equivalent to an internal pressure (Pi) and corrosion pressure ($P_{cor}$) which in this research is the reaction pressure of internal pressure after meeting with the inner surface of CPR. The model is adopting thick-walled cylinder which is subjected to an external load ($P_r$). This external load is actually a confined pressure of concrete as shown in Figure 2. The cylinder can be regarded also as the interference of bi-layer thick-walled cylinder. Also the problem can be assumed as coaxial components in contact. The additional cement paste ring is assumed to absorb stress hence reducing hoop stresses which are causing damage (Urade et al. 2014)

Based on static equilibrium of the loading conditions in Figure 2, the material properties and geometric compatibility, Popov (1990) gave the radial displacements at any point on the cylinder as:

$$u = \frac{(1+\nu)(1-2\nu)}{E} \bullet \frac{p_i r_1^2 - p_r r_2^2}{r_2^2 - r_1^2} r_r + \frac{1+\nu}{E} \bullet \frac{(p_i - p_r) r_1^2 r_2^2}{r_2^2 - r_1^2} \bullet \frac{1}{r_r} \qquad (1)$$

Based on Equation (1) displacements of the CPR (from inner wall) with rebar interface (expansive - outwards) can be derived assuming plane strain conditions. In Equation 2 depicts that there is no porous zone for corrosion products to be accommodated, therefore the thickness of corrosion materials has the direct impact to the CPR zone, and however there must be the influence of corrosion pressure and interface pressure between CPR and corrosion materials and interface pressure of CPR and bulk concrete. The displacement of the inner wall of the CPR is equivalent to the thickness of corrosion material. The thickness of rust according to tests by Liu and Weyer (1998), is between 10 and 100 µm thick. This thickness is a strong function of concrete porosity

$$u_{cm} = \frac{(1+\nu_{cpr})(1-2\nu_{cpr})}{E_{cpr}} \bullet \frac{p_{cor} r_1^2 - p r_2^2}{r_2^2 - r_1^2} r_1 + \frac{1+\nu_{cpr}}{E_{cpr}} \bullet \frac{(p_{cor} - p) r_1^2 r_2^2}{r_2^2 - r_1^2} \bullet \frac{1}{r_1} \qquad (2)$$

Similarly, the displacement at the CPR-bulk interface (Ucb) (inwards) is given by Equation (3) inwards because of the presence of confined stress from the bulk concrete, this confined stress can be realized through interface pressure as well. The inward displacement is a realization of the presence of confined stress, and in reality the displacement will move outward

$$-u_{cb} = \frac{(1+\nu_{cpr})(1-2\nu_{cpr})}{E_{cpr}} \bullet \frac{p_{cor} r_1^2 - p r_2^2}{r_2^2 - r_1^2} r_2 + \frac{1+\nu_{cpr}}{E_{cpr}} \bullet \frac{(p_{cor} - p) r_1^2 r_2^2}{r_2^2 - r_1^2} \bullet \frac{1}{r_2} \qquad (3)$$

For an unsupported long hole in an infinite medium (concrete) subjected to external confined stress conditions defined by a given K-ratio, (Figure 3), Kirsch in 1898 (Brady and Brown 1993) gives the radial displacement (urR) in Equation 4.

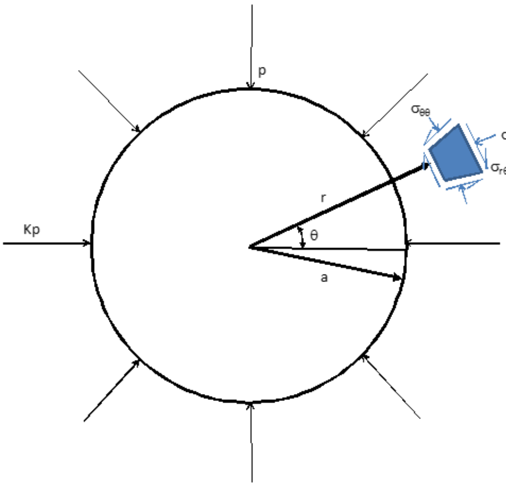

Figure 3: Long hole in an infinite concrete medium with external confinement

$$u_b = \frac{pa^2}{4G_b r}\left[(1+K) - (1-K)\left\{4(1-\nu) - \frac{a^2}{r^2}\right\}\cos 2\theta\right] \qquad (4)$$

At the interference of CPR and bulk concrete (cover) can be taken r = a and hydrostatic conditions K=1; Equation 4 simplifies to:

$$u_b = \frac{pr_2}{2G_b} \qquad (5)$$

The shear modulus of the bulk concrete $G_b$ is given by

$$G_b = \frac{E_b}{2(1+\nu_b)} \qquad (6)$$

By replacing $G_b$ in Equation (5) with Equation (6) the radial displacement at the excavation wall (inwards -ve) becomes

$$-u_b = \frac{pr_2(1+\nu_b)}{E_b} \qquad (7)$$

For compatibility and continuity reasons, the displacement at interface of CPR-Bulk concrete, must be equal to the displacement at the bulk concrete thus:

$$u_b = u_{cb} \qquad (8)$$

Corrosion pressure Pcor at CPR can be expressed in terms of the Bulk - CPR interface pressure (P) (Equation 9)

$$p_{cor} = \frac{\left(\frac{1+\nu_b}{E_b} + \frac{1+\nu_{cpr}}{E_{cpr}(r_2^2 - r_1^2)}\right)\left[(1-2\nu_{cpr})r_2^2 + r_1^2\right]}{\frac{1+\nu_{cpr}}{E_{cpr}(r_2^2 - r_1^2)}\left[(1-2\nu_{cpr})r_1^2 + r_1^2\right]} p \qquad (9)$$

Thus, the displacement at the CPR-rebar interface Equation 2 can be expressed in terms of the Corrosion pressure after making P the subject in Equation 10 as:

$$u_{cm=}\left[\frac{(1+\nu_{cpr})(1-2\nu_{cpr})}{E_{cpr}} \frac{\left(r_2^2 - \frac{r_2^2}{\frac{\left\{\frac{1+\nu_b}{E_b} + \frac{1+\nu_{cpr}}{E_{cpr}(r_2^2 - r_1^2)}\langle 1-2\nu_{cpr} r_2^2 + r_1^2\rangle\right\}}{\frac{1+\nu_{cpr}}{E_{cpr}(r_2^2 - r_1^2)}\left[(1-2\nu_{cpr} r_1^2 + r_1^2)\right]}}\right)}{r_2^2 - r_1^2} r_1 + \frac{(1+\nu_{cpr})}{E_{cpr}} \frac{\left(1 - \frac{1}{\frac{\left\{\frac{1+\nu_b}{E_b} + \frac{1+\nu_{cpr}}{E_{cpr}(r_2^2 - r_1^2)}\langle 1-2\nu_{cpr} r_2^2 + r_1^2\rangle\right\}}{\frac{1+\nu_{cpr}}{E_{cpr}(r_2^2 - r_1^2)}\left[(1-2\nu_{cpr} r_1^2 + r_1^2)\right]}}\right) r_1^2 r_2^2}{r_2^2 - r_1^2} \frac{1}{r_1}\right] p_{cor} \qquad (10)$$

### C. *Pressure Changes*

It is hypothesized that the corrosion pressure from rebar is affected by loading effect of a member especially if a member is in compression. The loading effect of a member is ignored in the calculations but confined stress of concrete which is less than corrosion is considered. Corrosion pressure may be accelerated in advance smoothly depending on the status of the confined stress from the surrounding concrete. This confined stress is influenced by the tensile strength of the CPR tensile strength and bulk concrete tensile strength; tensile strength of the CPR is ignored as it is assumed as the cracked zone. Also on the other hand cracking pressure (Bhargava et al. 2006, Tamer and Khaled 2007) can also be considered as confined stress or cracking pressure, as shown in Equation 27

The corrosion pressure is the pressure at the interface of rust materials and CPR. The status of confined pressure which is influenced by thickness of the cover, size of rebar and tensile strength of the concrete, pr by +□pr influence the acceleration of the spalling of the cover with equivalent corrosion pressure (pcor) changes by +□pcor. The stress change in the concrete is a result of continuing corrosion and reduction of the diameter of the rebar.

Because the acceleration of corrosion pressure may be influenced by the resistance of confined pressure then the relation (□Pcor/□Pr) can be proposed, but any of these pressure changes are also influenced by the ratio of the Moduli ratio of the bulk concrete and the CPR, (Eb/Ecpr), i.e. stiffness of CPR and bulk concrete

It is assumed that an increase in confined pressure (□pr) will result in increase of reaction pressure which is assumed to stabilize the displacement of the bulk concrete and CPR (i.e. □Ub and □Ucpr). Because corrosion is not expected to be compressed, the tendency for the corrosion materials to be increased by displacement (□Ucm) will result in the interface of CPR-rebar pressure increase (□pcor).

Obert and Duvall (1967) showed that the sum of the radial and tangential stresses on a thick wall cylinder loaded internally and externally is independent of position, and hence gives a uniform and constant strain in the axial direction. Thus, by assuming a thick wall cylinder with uniformly distributed axial stress (pz), it is possible to produce a total axial strain without introducing distortion in sections normal to the axis of the cylinder. By assuming axial restraint, a plane strain condition can be achieved (Popov, 1978) (Figure 4).

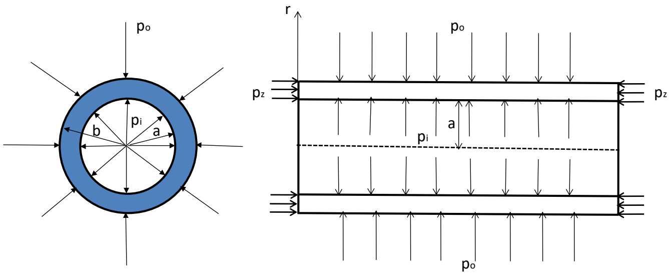

Figure 4: Thick wall cylinder under Triaxial loading showing notations

Hence, the displacement at a point in a pressurized cylinder subjected to external and axial and surface loading are given as

$$u = \frac{1+\nu}{E}\left[\frac{p_i a^2 - p_o b^2}{b^2 - a^2}(1-2\nu)r - \frac{a^2 b^2 (p_o - p_i)}{(b^2 - a^2)}\right] - \nu\varepsilon_z r \qquad (11)$$

$$w = -\frac{1}{E}\left[p_z + 2\nu\frac{p_z a^2 - p_o b^2}{b^2 - a^2}\right]z \qquad (12)$$

For a CPR, under stress changes Equation 11 can be written for the rust materials-CPR interface;

$$\Delta u_{cm} = \frac{(1+\nu_{cpr})(1-2\nu_{cpr})}{E_{cpr}}\frac{\Delta p_{cor} r_1^2 - \Delta p r_2^2}{r_2^2 - r_1^2} r_1 +$$

$$\frac{(1+\nu_{cpr})}{E_{cpr}}\frac{(\Delta p_{cor} - \Delta p) r_1^2 r_2^2}{r_2^2 - r_1^2}\frac{1}{r_1^2} - \nu_g \varepsilon_z \qquad (13)$$

$$\varepsilon_z = \frac{-p_z}{E_b} - \frac{2\nu_b}{E_b}\left(\frac{\Delta p r_2^2 - \Delta p_r r_r^2}{r_r^2 - r_2^2}\right)$$

For a CPR pz and □z are zero and as rr approaches infinity i.e., outside the bulk concrete (when the crack on the cover become visible), Equation 12 simplifies to:

$$\Delta u_{cm} = \frac{(1+\nu_{cpr})(1-2\nu_{cpr})}{E_{cpr}}\frac{\Delta p_{cor} r_1^2 - \Delta p r_2^2}{r_2^2 - r_1^2} r_1 + \frac{(1+\nu_{cpr})}{E_{cpr}}\frac{(\Delta p_{cor} - \Delta p) r_1^2 r_2^2}{r_2^2 - r_1^2}\frac{1}{r_1^2} \qquad (14)$$

Equation 14 is the thickness, i.e., displacement of corrosion products at the inner wall of the CPR is identical to Equation 2. Thus, the changes in displacements at the rust materials- CPR interface (ΔUcpr) and in the CPR- bulk concrete (ΔUb) interface, are given by Equations 15 and 16 as follows:

$$-\Delta u_{cpr} = \frac{(1+\nu_{cpr})(1-2\nu_{cpr})}{E_{cpr}}\frac{\Delta p_{cor} r_1^2 - \Delta p r_2^2}{r_2^2 - r_1^2} r_2 + \frac{(1+\nu_{cpr})}{E_{cpr}}\frac{(\Delta p_{cr} - \Delta p) r_1^2 r_2^2}{r_2^2 - r_1^2}\frac{1}{r_2} \qquad (15)$$

$$-\Delta u_b = \frac{(1+\nu_r)(1-2\nu_r)}{E_r}\frac{\Delta p r_2^2 - \Delta p_r r_r^2}{r_r^2 - r_2^2} r_2 + \frac{(1+\nu_b)}{E_b}\frac{(\Delta p - \Delta p_r) r_2^2 r_r^2}{r_r^2 - r_2^2}\frac{1}{r_2} \qquad (16)$$

As $r_r$ approaches infinity in Equation 16,

$$-\Delta u_b = \frac{r_2}{E_b}\left[\Delta p(1+\nu_b) - 2\Delta p_r(1-\nu_b^2)\right] \qquad (17)$$

For displacement compatibility (interference fit of two cylinders or multi layered cylinder) at the CPR-Bulk interface

$$\Delta u_{cpr} = \Delta u_b \tag{18}$$

From Equation 18, the interface corrosion pressure at the wall of CPR ($\Delta p_{cor}$) can be determined with respect to the CPR-bulk concrete interface pressure changes ($\Delta P$) (Equation 19).

$$\Delta p = \frac{\dfrac{(1+v_{cpr})r_1^2 r_2 \Delta p_{cor}}{E_{cpr}(r_2^2 - r_1^2)}\left\{(1-2v_{cpr}) + \dfrac{1}{r_2}\right\} + \dfrac{2\Delta p_r(1-v_b^2)}{E_b}}{\dfrac{(1+v_{cpr})}{E_{cpr}} \bullet \dfrac{r_2}{(r_2^2 - r_1^2)}\left\{(1-2v_{cpr})r_2 + \dfrac{r_1^2}{r_2}\right\} + \dfrac{(1+v_b)}{E_b}} \tag{19}$$

Equation 19 is the bulk-CPR interface pressure change for a situation where the bulk concrete react as a result of positive external confined stress change and the CPR surfaces tend to react to the internal corrosion pressure. For relaxed conditions, as assumed by Kaiser and Yazic (1992) in grout –bolt relation, □pcor is greatly minimized and is considered negligible, when the CPR is cracked and allows the rust materials to be absorbed, and Equation 19 simplifies to Equation 20 which was also obtained by Kaiser et al. (1992a):

$$\Delta p = \frac{\dfrac{2\Delta p_r(1-v_b^2)}{E_b}}{\dfrac{(1+v_{cpr})}{E_{cpr}} \bullet \dfrac{r_2}{(r_2^2 - r_1^2)}\left\{(1-2v_{cpr})r_2 + \dfrac{r_1^2}{r_2}\right\} + \dfrac{(1+v_b)}{E_b}} \tag{20}$$

Equation 19 contains 3 unknowns ($\Delta p$, $\Delta p_r$ and $\Delta p_{cor}$). It is necessary to express Equation 19 in only two unknowns and preferably in terms of $\Delta p_r$ and $\Delta p_{cor}$. For compatibility;

$$\Delta u_{cm} = \Delta u_{cpr} \tag{21}$$

Hence:

$$\Delta p = \frac{\Delta p_{cor}\left[(1-2v_{cpr})r_1^2 r_2 + r_1^2 r_2 - (1-2v_{cpr})r_1^3 - r_1 r_2^2\right]}{(1-2v_{cpr})r_2^3 + r_1^2 r_2 - (1-2v_{cpr})r_2^2 r_1 - r_1 r_2^2}$$

$$\Delta p = \chi \Delta p_{cor} \tag{22}$$

Where

$$\chi = \frac{(1-2v_{cpr})r_1^2 r_2 + r_1^2 r_2 - (1-2v_{cpr})r_1^3 - r_1 r_2^2}{(1-2v_{cpr})r_2^3 + r_1^2 r_2 - (1-2v_{cpr})r_2^2 r_1 - r_1 r_2^2} \tag{23}$$

Thus equation 22 can be substituted into Equation 19 to replace $\Delta p$ with the following result:

$$\Delta p_{cor} = \frac{\dfrac{2(1-v_b^2)\Delta p_r}{E_b}}{\chi\left[\left(\dfrac{1+v_{cpr}}{E_{cpr}} \bullet \dfrac{r_2}{(r_2^2 - r_1^2)}\right)\left\{(1-2v_{cpr})r_2 + \dfrac{r_1^2}{r_2}\right\} + \dfrac{(1+v_b)}{E_b}\right] - \left[\dfrac{(1+v_{cpr})r_1^2 r_2}{E_{cpr}(r_2^2 - r_1^2)}\left\{(1-2v_{cpr}) + \dfrac{1}{r_2}\right\}\right]}$$

………………… (24)

$$\frac{\Delta p_{cor}}{\Delta p_r} = \frac{2(1-v_b^2)}{\chi\left[\dfrac{E_b}{E_{cpr}}\left(\dfrac{(1+v_{cpr})r_2}{r_2^2 - r_1^2}\right)\left\{(1-2v_{cpr})r_2 + \dfrac{r_1^3}{r_2}\right\} + 1 + v_b\right] - \left[\dfrac{E_b}{E_{cpr}}\dfrac{(1+v_{cpr})r_1^2 r_2}{(r_2^2 - r_1^2)}\left\{(1-2v_{cpr}) + \dfrac{1}{r_2}\right\}\right]} \tag{25}$$

Let

$$\alpha = \left[\left(\frac{E_b(1+v_{cpr})}{E_{cpr}} \bullet \frac{r_2}{(r_2^2 - r_1^2)}\right)\left\{(1-2v_{cpr})r_2 + \frac{r_1^2}{r_2}\right\} + 1 + v_b\right] \quad and$$

$$\beta = \left[\frac{E_b(1+v_{cpr})r_1^2 r_2}{E_{cpr}(r_2^2 - r_1^2)}\left\{(1-2v_{cpr}) + \frac{1}{r_2}\right\}\right]$$

Hence Equation 24 can be rewritten as:

$$\frac{\Delta p_{cor}}{\Delta p_r} = \frac{2(1-v_b^2)}{\alpha\chi - \beta} \tag{26}$$

$$p_{cr} = \frac{2Cf_{ct}}{D} \tag{27}$$

Note that Pr, is equivalent to cracking pressure stated on equation 27.

## IV. ANALYSIS AND DISCUSSION

The results of the proposed expressions can be presented graphically for discussion. The Bulk Concrete Properties can be taken as: Compressive strength of concrete, $f_{cu}$ was assumed to be 25Mpa; Poisson ratio of concrete, $V_b = 0.2$; Concrete density, $\rho_c$ = 2400Kg/m$^3$. Various mechanical properties of concrete are usually correlated to the compressive strength of concrete, a parameter which can be easily measured in practice. From ACI 318-05 the concrete tensile strength and modulus of elasticity is related to compressive strength as:

$$f_t = 0.53\sqrt{fcu} \tag{28}$$

$$E_c = 4600\sqrt{fcu} \tag{29}$$

Then from equations (28) and (29) the tensile strength, $f_t$ and the modulus of elasticity, $E_b$ are found to be 2.65Mpa and 23Gpa respectively. On the other hand, Cement Paste Ring properties are Ecpr is 20GPa, $V_{cpr}$ is 0.3 (Haecker et al. 2005). The diameter of rebar is taken to be 16mm, thickness of CPR is taken to be from 1mm to 10mm, practicable maximum thickness that can be produced by rebar shaker.

Equation (24) which was simplified to Equation (26) is plotted in Figure 5

### A. *Interface Corrosion Pressure in Relation to Thickness*

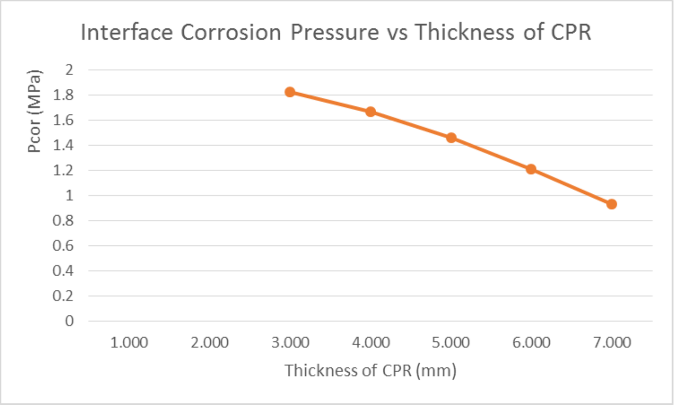

Figure 6: Contact Pressure versus Displacement of Bulk Concrete

The contact pressure between CPR and bulk concrete is increasing as the corrosion pressure increases. The displacement outward will increase as the contact pressure increases as shown in Figure 6. Experimental results have shown that, although at the early stages of crack propagation several cracks appear, by the end of the test there is a single crack that finally breaks the cover on the weakest side of the concrete element. When this crack appears, the internal stresses relax, which stops the propagation of other internal cracks (Ioannis and Burgoyane, 2011).

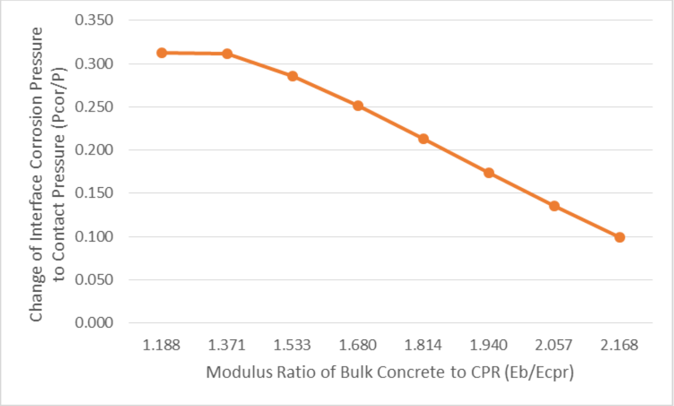

Figure 7: Pressure change ratio – modulus ratio relationship

The Figure 7 shows the ratio of corrosion pressure to the contact pressure and the ratio of bulk concrete modulus to the modulus of Cement Paste Ring. The modulus of bulk concrete is higher than that of cement paste ring. The nature of graph is common and is similar as the one proposed by Kaiser et al (1992a). The ratio and relation of stiffness of the concrete and the steel bolt is an important parameter. It deemed meaningful to present a plot of the stress change ratio as a function of the modulus ratio of the rock to the support (Nose 1993, Yazic et al, 1992). This has been presented in Figure 7. In the graph it is clear that the lower the modulus ratio, the greater the stress change ratio can be expected. At a given confined stress, the bulk modulus has considerable influence on acceleration of corrosion hence the interface corrosion pressure change in CPR. The radial pressure at the interface between the steel rebar and the concrete cover reaches highest value well before the cracks occur at the cover surface, drops rapidly when concrete becomes completely cracked through the cover Hu-Peng and Nan (2012), the bond between rebar and CPR may be reduced or increased depending on the confined pressure, amount of corrosion products and the size of crack width that can accommodate the rust materials.

Similar types of graphs were shown by (Du et al., 2006) (see Figure 8) through experiment and Finite element analysis of the effects of radial expansion of corroded reinforcement. Graphs compared the cracking pressure with the ratio of cover to diameter through Experiment done by Williamsons, Clark and Morinaga. The modified graphs shows the presence of CPR reduced cracking pressure

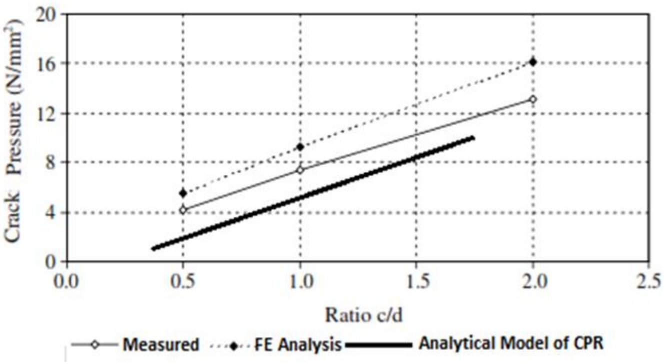

Figure: 8 Cracking Pressure versus ratio of c/d (Modified graphs after Du et al.2006)

## V CONCLUSION AND RECOMMENDATIONS

From the study carried out, the following are the conclusions.

i. Analytical model solution based on classical theory mechanics developed by assuming the pressurized thick-walled vessel shows resemblances of the mechanisms of corrosion in concrete structures, and perfectly describe the behavior of the interaction of reinforcement, CPR and concrete cover.
ii. The thickness of CPR has significant effects on counteracting corrosion pressure as well as accommodating corrosion products. Likewise the diameter of reinforcement and thickness of concrete cover are corresponding to the corrosion pressures and damage of concrete cover.
iii. The CPR have shown significant benefits towards reducing corrosion and minimizing corrosion pressure as well as damage of concrete cover.

It is recommended that further research be carried out needed towards improvement and practicality of these techniques in reinforced concrete structures

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