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Analytical and Numerical Study for Mass Loading Effect on Natural Frequencies of Aluminum Alloy Propped Cantilever Beam

Ayush Kumar[a], Anuj Yadav[b] and Hirendra Singh[c]
[a,c]Department of Mechanical Engineering, Naraina College of Engineering and Technology, Kanpur - 208020, India
[b]Department of Mechanical Engineering, National Institute of Technology, Patna - 800005, India

***Abstract*— A lot of attention has been received to determine the natural frequencies of a continuous system. Engineers are more interested in evaluating the dynamic behavior of a beam since it is one of the most essential continuous structures. The objective of the article is to study the impact of mass loading on the natural frequency of aluminum alloy propped cantilever beam. Analytical and numerical methods are used to model the dynamic behavior of the beam mass system. An Euler-Bernoulli beam of fixed-hinged end conditions carrying a point mass at an arbitrary position has been considered for analytical studies. The non-dimensional frequency equation for beam fixed-hinged end conditions is obtained by satisfying the free vibration equations of motion and by applying the corresponding end and compatibility conditions. This boundary-value problem is solved by a dual frame of reference. Simulation of the problem is done by using ANSYS and the validation of the proposed analytical method is demonstrated utilizing simulated data. It has been observed that the natural frequencies of the beam are influenced by the mass position and its magnitude.**

***Keywords*— ANSYS; Propped cantilever beam; MATLAB; Eigenfrequency**

1. INTRODUCTION

Free vibration of combined structures is a popular issue in today's world. This issue affects a wide range of engineering design disciplines. The most of the research in this field focuses on the discovery and application of methods for determining the combined system's natural frequencies. Rao [1], the basic governing equation for free vibration for simple structure like Euler-Bernoulli beam is available in reference book. Low [2], this study presents a comparative review of the Eigen-frequency analysis for an Euler-Bernoulli beam with a point mass at any position. Mermertas and Erol [3] examined the Eigenfrequency equation of a cracked cantilever beam with attached point mass. Theoretical investigation of the cracked beam was carried out. The governing equation of the cracked beam for free vibrations is developed from the elements of the Euler-Bernoulli beam. Naguleswaran [4], for the free vibrations of a beam carrying two point masses, an empirical approach based on the classical beam eigenvalue method is presented in this article. Öz and Özkaya [5] presented the Euler-Bernoulli beam with masses at various positions is considered in this research. For various boundary conditions, natural frequencies for free transverse vibrations are studied. Kotambkar [6], the impact of mass loading caused by an accelerometer on the natural frequency of a beam in a free-free end conditions was studied in this paper. Prashant et al. [7], experimental modal analysis of cantilever beam was carried out to obtain natural frequencies, modal damping, and mode shapes. Yadav and Singh [8, 9] investigated the mass attachment impact on vibrational frequencies of cantilever and simply-supported magnesium alloy beams. Caker and Sanliturk [10] and Ren et al. [11] presented a new approach for removing mass loading effects of transducers from calculated FRFs, based on the Sherman–Morrison concept. Any physical problem can be solved with help of exact and approximate methods but most of the problems are solved by an approximate method due to its complexity. In this paper, the modal analysis of the beam-mass system has been examined by approximate analytical and numerical methods.

2. ANALYTICAL SOLUTION

An Euler-Bernoulli beam of length (L) 240 mm, fixed-hinged end conditions with attaching a point mass (m) of 23 gm at an arbitrary position have been considered for analytical studies. It is an aluminum alloy beam having a dimension of 240×12×6 mm and beam's material properties like as Young's modulus 71GPa and density is 2770 Kg/m^3. A dual frame of reference has been used to solve this beam-mass system (Fig. 1) problem. By satisfying the free vibration equations of motion and applying the related end and compatibility conditions, the non-dimensional frequency equation for beam fixed-hinged end conditions has been obtained.

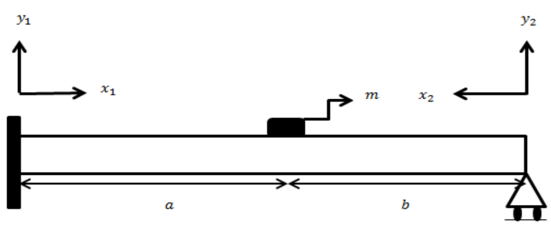

Fig. 1. Propped cantilever beam with point mass

The equation of motion for free vibration of a uniform beam is given as

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

The technique of separation of variable has been used to find the free-vibration solution.

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$y(x,t) = Y(x).T(t)$ (2)

The characteristic function $Y(x)$ of the beam is defined as

$$Y(x) = A_1 \cosh\beta x + B_1 \sinh\beta x + C_1 \cos\beta x + D_1 \sin\beta x \quad (3)$$

Where, $\beta^4 = \left(\omega^2 \frac{\rho A}{EI}\right)$

Where ω is the natural frequency.

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} \quad (4)$$

The dual frames of references have been taken to solve the problem.

$$Y_1(x_1) = A_1 \cosh\beta x_1 + B_1 \sinh\beta x_1 + C_1 \cos\beta x_1 + D_1 \sin\beta x_1 \quad (5)$$

$$Y_2(x_2) = A_2 \cosh\beta x_2 + B_2 \sinh\beta x_2 + C_2 \cos\beta x_2 + D_2 \sin\beta x_2 \quad (6)$$

The fixed end coordinate of the beam ($x_1 = 0_{at\ x=0}$) and at hinged end coordinate ($x_2 = 0_{at\ x=L}$), the fixed end transverse displacement $y_1(x,t)$ satisfies boundary conditions ($y_1 = 0$ and $y_1' = 0$), $y_1(x,t)$ can be expressed as

$$Y_1(x_1) = (cosh\beta x_1 - cos\beta x_1)A_1 + (sinh\beta x_1 - sin\beta x_1)B_1 \quad (7)$$

The hinged end transverse displacement $y_2(x,t)$ satisfies boundary conditions ($y_2 = 0$ and $y_2'' = 0$), $y_2(x,t)$ can be expressed as

$$Y_2(x_2) = -(sinh\beta x_2)B_2 - (sin\beta x_2)D_2 \quad (8)$$

For further solution, four additional compatibility equations have been used

Transverse displacement equation at $x_1 = a$ or $x_2 = b$

$$y_1(x_1)_{at\ x_1=a} = y_2(x_2)_{at\ x_2=b} \quad (9)$$

$$(cosh\beta a - cos\beta a)A_1 + (sinh\beta a - sin\beta a)B_1 + (sinh\beta b)B_2 + (sin\beta b)D_2 = 0 \quad (10)$$

Slope equation at $x_1 = a$ or $x_2 = b$

$$y_1'(x_1)_{at\ x_1=a} = -y_2'(x_2)_{at\ x_2=b} \quad (11)$$

$$(sinh\beta a + sin\beta a)A_1 + (cosh\beta a - cos\beta a)B_1 - (cosh\beta b)B_2 - (cos\beta b)D_2 = 0 \quad (12)$$

Moment equation at $x_1 = a$ or $x_2 = b$

$$y_1''(x_1)_{at\ x_1=a} = y_2''(x_2)_{at\ x_2=b} \quad (13)$$

$$(cosh\beta a + cos\beta a)A_1 + (sinh\beta a + sin\beta a)B_1 + (sinh\beta b)B_2 - (sin\beta b)D_2 = 0 \quad (14)$$

Shear force at $x_1 = a$ or $x_2 = b$

$$y_1'''(x_1)_{at\ x_1=a} + y_2'''(x_2)_{at\ x_2=b} + m.\ddot{y}_1(x_1)_{at\ x_1=a} = 0 \quad (15)$$

$$(sinh\beta a - sin\beta a)A_1 + (cosh\beta a + cos\beta a)B_1 - (cosh\beta b)B_2 + (cos\beta b)\,D_2 + \left(\frac{m\beta}{\rho A}\right)\{(cosh\beta a - cos\beta a)A_1 + (sinh\beta a - sin\beta a)B_1\} = 0 \quad (16)$$

Let us assume

Dimensionless mass location parameter $(\alpha) = \frac{a}{L}$

$M_1 = cosh\beta L\alpha - cos\beta L\alpha$

$M_2 = sinh\beta L\alpha - sin\beta L\alpha$

$M_3 = cosh\beta L\alpha + cos\beta L\alpha$

$M_4 = sinh\beta L\alpha + sin\beta L\alpha$

$N_1 = cosh\beta L(1-\alpha)$

$N_2 = sinh\beta L(1-\alpha)$

$N_3 = cos\beta L(1-\alpha)$

$N_4 = sin\beta L(1-\alpha)$

$\gamma = \frac{m\beta}{\rho A}$

The resulting dimensionless frequency equation in determinant form has been obtained by substituting the equation number (10), (12), (14), and (16).

$$\begin{vmatrix} M_1 & M_2 & N_2 & N_4 \\ M_4 & M_1 & -N_1 & -N_3 \\ M_3 & M_4 & N_2 & -N_4 \\ M_2 + M_1.\gamma & M_3 + M_2.\gamma & -N_1 & N_3 \end{vmatrix} = 0 \quad (17)$$

The values of βL for different vibrational modes have been found out by solving the equation (17) in MATLAB software. It is observed that the values of βL in mode1, 2, and 3 change with respect to the point mass attachment locations. The natural frequencies of the beam-mass system in different modes are obtained by Eq. (4). The changes in the value of βL of different modes with respect to the dimensionless mass location parameter (α) have been represented in Table 1, Fig. 2, Fig. 3, and Fig. 4.

Table 1. Changes in the value of βL of different modes with respect to α.

SI. No.	α	βL		
		Mode I	Mode II	Mode III
1	0.1	3.9179	6.9296	9.5235
2	0.2	3.8266	6.2564	9.0369
3	0.3	3.6220	6.0833	9.7016
4	0.4	3.4188	6.4076	10.1869
5	0.5	3.2925	6.9264	9.3976
6	0.6	3.2599	6.9929	9.5153
7	0.7	3.3302	6.5543	10.2035
8	0.8	3.5159	6.2467	9.5917
9	0.9	3.7846	6.4669	9.1929
10	1	3.9266	7.0685	10.2102

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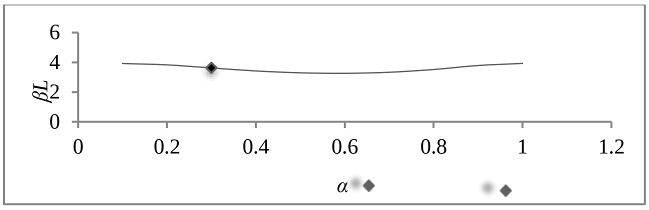

Fig. 2. Impacts of mass attachment on the natural frequency of the first mode.

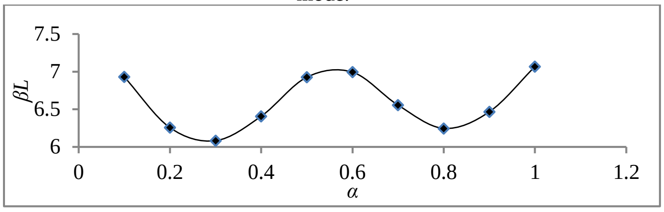

Fig. 3. Impacts of mass attachment on the natural frequency of the second mode.

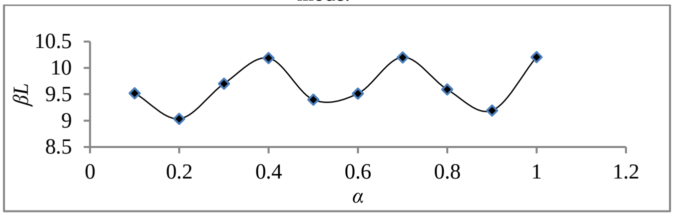

Fig. 4. Impacts of mass attachment on the natural frequency of the third mode.

3. NUMERICAL SOLUTION

Modal analysis module of ANSYS workbench software has been used for numerical modal analysis of a beam-mass system. A prismatic beam model (Fig. 5) of dimension 240×12×6 mm has been developed by the design modular of ANSYS. The appropriate boundary conditions are imposed to make the beam model propped cantilever. A point mass of 23 gm is attached at the desired location on the beam. The major assumption of finite element method is to discretized continuous structure in finite number of elements. Now the model has been discretized (Fig. 6) with Hex20 element, the total elements and nodes number are 10000 and 49581 respectively. Finally, the natural frequencies (Hz) of a propped cantilever beam with an attached point mass at multiple desirable locations are obtained in different bending modes. The results have obtained from an analytical and numerical approach, which has been compared in Table 2, 3 and 4.

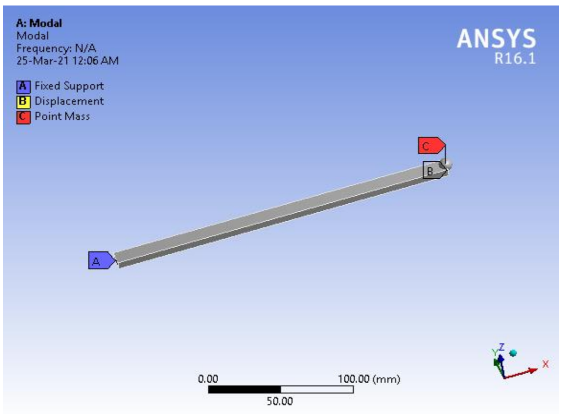

Fig. 5. Beam model

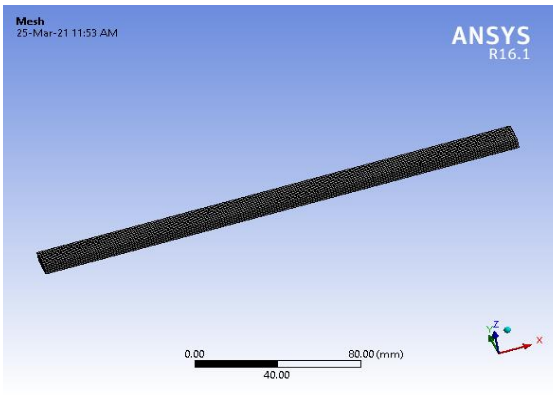

Fig. 6. Mesh model

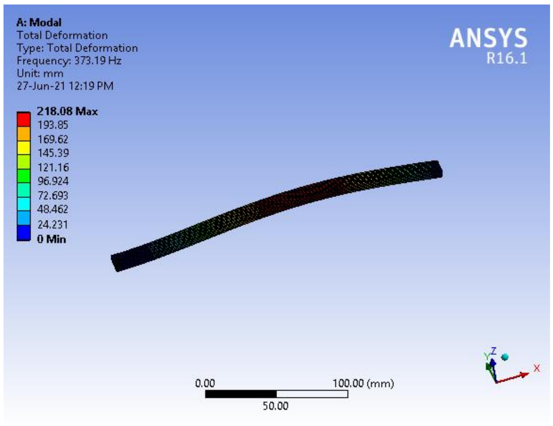

Fig. 7. Mode Shape

Table 2. Vibrational frequencies of a propped cantilever beam-mass system for Mode I.

SI. No.	α	Mode I	
		ANSYS	MATLAB
1	0.1	371.94	371.92
2	0.2	354.74	354.78
3	0.3	317.87	317.86
4	0.4	283.26	283.19
5	0.5	262.69	262.66
6	0.6	257.48	257.48
7	0.7	268.61	268.71
8	0.8	299.28	299.51
9	0.9	346.62	347.04
10	1	373.19	373.57

Table 3. Vibrational frequencies of a propped cantilever beam-mass system for Mode II.

SI. No.	α	Mode II	
		ANSYS	MATLAB
1	0.1	1154.00	1163.48
2	0.2	939.58	948.40
3	0.3	890.08	896.64
4	0.4	987.22	994.79
5	0.5	1152.40	1162.41
6	0.6	1175.80	1184.83
7	0.7	1034.40	1040.87
8	0.8	939.55	945.46
9	0.9	1003.30	1013.29
10	1	1200.00	1210.59

Table 4. Vibrational frequencies of a propped cantilever beam-mass system for Mode III.

SI. No.	α	Mode III	
		ANSYS	MATLAB
1	0.1	2112.90	2197.53
2	0.2	1937.60	1978.71
3	0.3	2235.90	2280.49
4	0.4	2459.20	2514.36
5	0.5	2099.80	2139.82
6	0.6	2147.40	2193.75
7	0.7	2464.40	2522.56
8	0.8	2189.20	2229.12
9	0.9	2001.10	2047.61
10	1	2326.10	2525.87

4. CONCLUSIONS

The effects of a point mass on the frequencies of a propped cantilever beam are investigated. For the first three fundamental frequencies, a single point mass is placed at ten evenly spaced different positions on the beam, and its influence on frequencies is plotted. It is found that the vibrational frequency of this beam is influenced by the point mass location and its magnitude. The addition of point mass reduces the natural frequencies of this beam but, if the mass is placed at a node of the modes then its frequencies do not change. The node positions for Mode II and Mode III are $\alpha = 0.5$ and $\alpha = 0.333, 0.667$ respectively. The simulated results of the numerical approach are similar to analytical results.

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