Analysis of Turbo Machinery Rotor to Determine Critical Speed

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Abstract

Rotating machines including machinery tools, industrial turbo machinery and aircraft gas turbine engines are commonly used in the industry. In practice the rotor carries several components such as gears, disks, flywheels etc. and it has several critical speed corresponding to the bending natural frequencies,. For most of the rotors, it is the fundamental mode which falls in the running speed zone and there are several methods of calculation of the first critical speed.

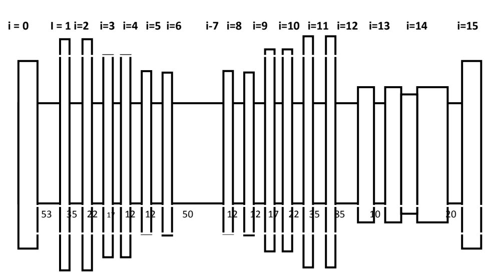
1. Introduction

This paper deals with the calculation of natural frequencies by various methods. The methods includes Dunkerley method, Rayleighs method, Myklestad method, and Macaulay method. Dunkerley method gives a lower bound value while the Rayleighs method gives an upper bond value. Therefore the exact value where the critical speed lies is well established by their two method taken together.

Natural frequencies of the turbo machinery rotor has been calculated by Myklestad method and Rayleighs method. Myklestad method consists of transfer matrix and point matrix.

2. Turbo Machinery Rotor

Turbo machinery rotor with number of disc is shown in the figure.



Specification of Low Pressure Turbine Rotor

Lengths: Overall length = 5650 mm, L1 = 530 mm (Spacing between i=0 and i=1), similarly L2=350mm, L3=220mm, L4=170mm, L5=120mm, L6=120mm, L7=500mm, L8=120mm, L9=120mm, L10=170mm, L11=220mm, L12=350mm, L13=850mm, L14=100mm, L15=200mm

Thickness of each i^{th} node							
i=0	i=1	i=2	i=3	i=4	i=5	i=6	i=7
17cm	10cm	10cm	7cm	4cm	4cm	4cm	4cm
i=8	i=9	i=10	i=11	i=12	i=13	i=14	i=15
4cm	4cm	7cm	10cm	10cm	10cm	34cm	15cm

Diameter of the rotor d = 68.4 cm

Calculation of masses of disks:

m1=[∏/4(0.31)2+∏/4(0.932 - 0.312)/2]x0.10x7750 = 292.47 kgs

m3=[∏/4(0.285)2+∏/4(0.6952 - 0.2852)/2]x0.07x7750 = 120.20 kgs

m4=[∏/4(0.24)2+∏/4(0.5852 - 0.242)/2]x0.05x7750 = 60.84 kgs

m5=[∏/4(0.24)2+∏/4(0.512 - 0.242)/2]x0.05x7750 = 48.34 kgs

m6=[∏/4(0.22)2+∏/4(0.4752 - 0.222)/2]x0.05x7750 = 41.69 kgs

m13=[∏/4(0.83)2x0.10x7750] = 389.77 kgs

m14=[∏/4(0.85)2x0.3x7750 = 1324 kgs

m2=m12=m11=m1= 292.47 kgs

m10=m3=120.20kgs

m4=m9=60.84 kgs

m5=m8=48.34 kgs

m6=m7=41.69 kgs

m13=389.77 kgs

m14=1324 kgs

Moment of Inertia: I = ∏/64 d4 =∏/64 (0.6843)4 = 0.01076 m4

Elasticity of Rotor: E = 200 x 109 N/m2

3. Using Myklestad Method

Transfer Matrix and Point Matrix of Myklestad method

$$TansferMatrix = \begin{bmatrix} 1 & l & l^2/2EI & l^3/6EI \\ 0 & 1 & l/EI & l^2/2EI \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Po\,int\ Matrix = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ mp^2 & 0 & 0 & 1 \end{bmatrix}$$

The overall transfer matrix [U] of the rotor product of transfer matrix and point matrix

$$\{S\}^L_{i=15} = [U]\{S\}^R_{i=0}$$

$$\begin{bmatrix} -w \\ \theta \\ My \\ Vz \end{bmatrix}^L_{i=15} = \begin{bmatrix} U11 & U12 & U13 & U14 \\ U21 & U22 & U23 & U24 \\ U31 & U32 & U33 & U34 \\ U41 & U42 & U43 & U44 \end{bmatrix} \begin{bmatrix} -w \\ \theta \\ My \\ Vz \end{bmatrix}^R_{i=0}$$

w = State vector containing deflection
Ɵ = slope
My = Blending Moment
Vz = Shear Force
P = Natural Frequency
i=Shaft Element of the length li and mass mi

$$\{S\}^L_{i=15} = [T]_{15}[P]_{14}[T]_{14}\cdots\cdots[P]_2[T]_2[P]_1[T]_1\{S\}^R_{i=0}$$

$$[U] = [T]_{15}[P]_{14}[T]_{14}\cdots\cdots[P]_2[T]_2[P]_1[T]_1[\ ]$$

$$T_{15} = \begin{bmatrix} 1 & l_{15} & l_{15}^2/2EI & l_{15}^3/6EI \\ 0 & 1 & l_{15}/EI & l_{15}^2/2EI \\ 0 & 0 & 1 & l_{15} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.2 & 9.29x10^{-12} & 6.19x10^{-13} \\ 0 & 1 & 9.29x10^{-11} & 9.29x10^{-12} \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{14} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ mp^2 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 132\,p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{14} = \begin{bmatrix} 1 & 0.1 & 2.32x10^{-12} & 7.44x10^{-14} \\ 0 & 1 & 4.64x10^{-11} & 2.32x10^{-12} \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{13} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 390\,p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{13} = \begin{bmatrix} 1 & 0.85 & 1.67x10^{-10} & 4.7x10^{-11} \\ 0 & 1 & 3.94x10^{-10} & 1.67x10^{-11} \\ 0 & 0 & 1 & 0.85 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 239\,p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} 1 & 0.35 & 2.845x10^{-11} & 3.32x10^{-12} \\ 0 & 1 & 1.626x10^{-10} & 2.845x10^{-11} \\ 0 & 0 & 1 & 0.35 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 293\,p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{11} = \begin{bmatrix} 1 & 0.22 & 1.124x10^{-11} & 8.245x10^{-13} \\ 0 & 1 & 1.022x10^{-10} & 1.124x10^{-11} \\ 0 & 0 & 1 & 0.22 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 120\,p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{10} = \begin{bmatrix} 1 & 0.17 & 6.71x10^{-12} & 3.80x10^{-13} \\ 0 & 1 & 7.84x10^{-11} & 6.713x10^{-12} \\ 0 & 0 & 1 & 0.17 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 60.84p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_9 = \begin{bmatrix} 1 & 0.12 & 3.345x10^{-12} & 1.33x10^{-13} \\ 0 & 1 & 5.575x10^{-11} & 3.345x10^{-12} \\ 0 & 0 & 1 & 0.12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 48.34p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_8 = \begin{bmatrix} 1 & 0.12 & 3.345x10^{-12} & 1.3382x10^{-13} \\ 0 & 1 & 5.57x10^{-11} & 3.34x10^{-12} \\ 0 & 0 & 1 & 0.12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 41.6p^2 & 0 & 0 & 1 \end{bmatrix}$$

$$T_7 = \begin{bmatrix} 1 & 0.5 & 5.8x10^{-11} & 9.68x10^{-12} \\ 0 & 1 & 2.32x10^{-10} & 5.8x10^{-11} \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since P12=P1, T12=T2, P11=P2, T11=T3, P10=P3, T10=T4, P9=P4, P8=P5, T9=T5

Applying Boundary condition for simply supported w = 0, My=0

$$\begin{bmatrix} -w \\ \theta \\ My \\ Vz \end{bmatrix}_{i=15}^{L} = \begin{bmatrix} U11 & U12 & U13 & U14 \\ U21 & U22 & U23 & U24 \\ U31 & U32 & U33 & U34 \\ U41 & U42 & U43 & U44 \end{bmatrix} \begin{bmatrix} -w \\ \theta \\ My \\ Vz \end{bmatrix}_{i=0}^{R}$$

Above equation reduces to

$$0 = \begin{bmatrix} U11 & U13 \\ U31 & U33 \end{bmatrix}$$

(U11xU33)-(U31xU13) = 0

The final form of the equation, after multiplying all the matrixes and applying the boundry condition is

20.78p12 + 2211.9p10 + 11.29p8 +2.26p6 -2355p8 = 0

20.78p12 + 2212p10 -2343.7p8 +2.2218p6 = 0

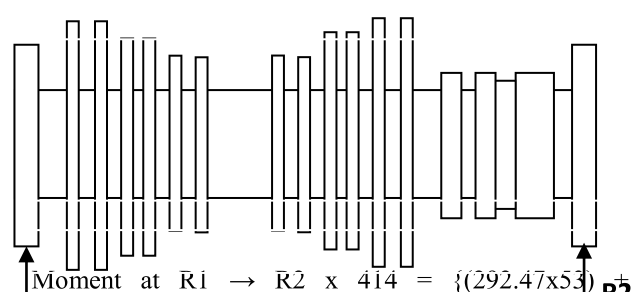
20.78p6 + 2212p4 -2343.7p2 +2.2218 = 0

Solving this equation the natural frequency are

P1= 236.36 rps, P2= 365.23 rps, P3= 125.36 rps, P4= 251.36 rps, P5= 132.36 rps, P6= 166.32 rps

4. Using Macaulay's Method

The Macaulay's method for the Low Pressure Turbine



Moment at R1 → R2 x 414 = {(292.47x53) + (292.47x88) + (120x110) + (60.84x127) + (48.34x139) + (41.69x151) + (41.69x120) + (48.34x213)+ (60.84x225) + (120x242) + (292.47x264) + (292.47x299) + (389.77x384) + (1324x394) }

R1=2349.2 Kg and R2 = 1076.18 Kg

EId2y/dx2 = 2349.2x | -1324(x-20) | -389.77(x-80) | -292.17(x-30) | -292.17(x-115) | -292.47(x-150) | -120(x-172) | -60.84(x-189) | -48.34 (x-201) | -41.69(x-213) | -41.69(x-263) | -48.34(x-275) | -60.84(x-287) | -120(x-304) | -292.47(x-326) | -292.47(x-361)

Intergrating the above equation twice , equation reduces

EIy = 391.53x3+c1x+c2 | -220.66(x-20)3 | -64.96(x-30)3 | -48.69(x-115)3 | -48.69(x-150)3 | -20(x-172)3 | -10.14(x-189)3 | -8.056(x-201)3 | -6.94(x-263)3 | -8.05(x-275)3 | -10.14(x-287)3 | -20(x-304)3 | -48.69(x-326)3 | -48.69(x-361)3 |

Applying Boundary condition at x = 0, y=0, at x=414, y=0
Constant c2 = -5617859642 and c1 = -5135123.33, E= 2x107 N/cm2, I=3.14/64d4 =1076356.3 cm4

Deflections at w1, where x1 = 53 cm : ∂1= -1.23511x10-4 cm
Deflections at w2, where x2 = 88 cm : ∂2= -1.7652x10-4 cm
Deflections at w3, where x3 = 110 cm : ∂3= -1.8504x10-4 cm
Deflections at w4, where x4 = 127 cm : ∂4= -2.1283x10-4 cm
Deflections at w5, where x5 = 139 cm : ∂5= -2.1637x10-4 cm
Deflections at w6, where x6 = 151 cm : ∂6= -3.4919x10-4 cm
Deflections at w7, where x7 = 201 cm : ∂7= -1.9354x10-4 cm
Deflections at w8, where x8 = 213 cm : ∂8= -2.1935x10-4 cm
Deflections at w9, where x9 = 225 cm : ∂9= -2.1458x10-4 cm
Deflections at w10, where x10 = 242 cm : ∂10= -2.0321x10-4 cm
Deflections at w11, where x11 = 264 cm : ∂11= -1.8948x10-4 cm
Deflections at w12, where x12 = 299 cm : ∂12= -1.5156x10-4 cm
Deflections at w13, where x13 = 384 cm : ∂13= -4.263x10-4 cm
Deflections at w14, where x14 = 394 cm : ∂14= -2.850x10-4 cm

∑∂= -2.5063 x 10-3 cm

∑∂= 0.025063 mm

Critical speed Nc = 945 / [√(∂1+∂2+-----+∂14)]

Nc= 945/(√0.025063)

Nc=5969.18 rpm

Natural frequency by Macaulay's Method of the Low Pressure Turbine Rotor is 5969.18 rpm

5. Using Reyleigh Method

w2 = g [(∑My)/(∑My2)] where M=Mass in Kg, y=deflection, w=Natural frequency speed in rpm

i th Node	Mass M (Kg)	Deflection y (cm)	My (Kg cm)	My2 (Kg cm^2)
1	292.47	1.2351 x 10-4	0.036123	4.46162 x 10-6
2	292.47	-1.7652 x 10-4	0.051626	9.11316 x 10-6
3	120	-1.8504 x 10-4	0.022048	4.10877 x 10-6
4	60.84	-2.1283 x 10-4	0.012948	2.75584 x 10-6
5	48.54	-2.1637 x 10-4	0.010502	2.27244 x 10-6
6	41.69	-3.4919 x 10-4	0.014557	5.08341 x 10-6
7	41.69	-1.9359 x 10-4	0.008070	1.56241 x 10-6
8	48.54	-2.1935 x 10-4	0.010647	2.33547 x 10-6
9	60.84	-2.1458 x 10-4	0.013055	2.80135 x 10-6
10	120	-2.0321 x 10-4	0.024385	4.95531 x 10-6
11	292.47	-1.8948 x 10-4	0.055402	1.04949 x 10-5
12	292.47	-1.5156 x 10-4	0.044326	6.71816 x 10-6
13	389.77	-4.263 x 10-5	0.016615	7.08335 x 10-7
14	1324	-2.850 x 10-5	0.037740	1.07577 x 10-6
		∑∂ = 0.025063 mm	∑My = 0.3582	∑My2 = 5.8447x10-5

w2 = g [(∑My)/(∑My2)] = 2451.97 rpm

The natural frequency of the Low Pressure Turbine Rotor is 2451.97 rpm

6. Using Dunkerley's Method

This method gives the lower bound natural frequency of the low pressure turbine rotor. The dunkerley method is helpful only when natural frequency of each rotor (disk) is known
1/w2 = 1/w12 + 1/w22 + -----------+1/w142

w = 2536.23 rpm

7. Conclusion

The Natural frequency of the low pressure turbine is evaluated through various methods. Myklestad, Macaulay, Rayleigh and Dunkerley methods are used for calculation of critical speed of rotor. Out of these methods Rayleigh, Macaulay and Dunkerlay method gives the lower and upper bound values (i.e. frequencies) compared to Myklestad method. Myklestad method give the natural frequency of the rotor exact using transfer and point matrix concept.
Hence, this paper proves that for evaluating the natural frequency of the turbine rotor for the best result Myklestad method is used.

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8. References

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