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Analysis of the Effect of the Homogenization Methods on the Shear Correction Factor of Functionally Graded Beams

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***Abstract*—This paper presents analysis of the effect of the homogenization methods on the shear correction factor of functionally graded beams. It is based on the first-order shear deformation beam theory in which the effective material properties of functionally graded beams are estimated by the homogenization schemes of Hill and Hashin-Shtrikman. The material property is assumed to be isotropic and varies through the beam depth according to a power-law form. Numerical results shows that the shear correction factor not only depends on material properties, but also on the homogenization schemes.**

Keywords*— *Functionally graded beam; Shear correction factor; Homogenization method.

I. INTRODUCTION

Functionally Graded Material (FGM) is an advanced composite material whose properties vary continuously to avoid stress concentrations at interfaces found in laminated composites ([1]). The studies on behaviors of functionally graded (FG) beams rapidly become an active subject with many researches over the recent years. Many approaches have been used to study static, vibration and buckling behaviors of FG beams, only several references cited here ([2-7]). The FG beams based on the first-order shear deformation theory (FSDT) have been widely applied by its simplificity in calculation and programming. However it requires a convenient value of the shear correction factor. In practice, this coefficient has been taken by the five-sixth one as homogeneous beams. The contribution on this topic for FSDT beams can be found in [7-11]. Moreover, the estimation of effective elastic properties of functionally graded materials has been studied by many authors in which continuous models are most used, among Voigt's approximation [12]. The works of [13-14] showed that the Mori-Tanaka's scheme is convenient for estimating the effective properties of FGM in the matrix-inclusion region while the self-consistent is for the area having interconnected phases. Many more approximations of the effective elastic properties can be found within the literature ([15]). In continuous model, practically, FG beams are first homogenized with their effective moduli such as Young's modulus, Poisson's ratio, etc, and then the effective properties of the FG beams will be derived from homogeneous beam theories. The analysis of the effect of homogenization methods on the shear correction factor of FG beams is not investigated.

The objective of this paper is to study the effect of the effective material properties on the shear correction factor of FG beams. Theory is based on the first-order shear deformation beam theory within which the effective moduli of the beam will be calculated by the Voigt and Reuss models, and Hashin-Shtrikman's estimates. The material elastic properties are supposed to be isotropic at each point and vary through the beam depth according to the power-law form.

II. THEORETICAL FORMULATION

Consider a FG beam having the thickness *h*, section $b \times h$ and length *L* as Fig. 1. The beam is made of a functionally graded material constituted by a mixture of two constituents whose material properties vary through the beam depth according to the volume fractions of the constituents. The material elastic properties are supposed to be isotropic at each point, the gravity is not taken into account.

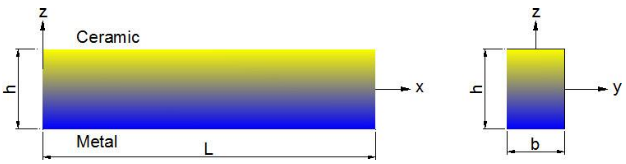

Fig. 1. Geometry of a functionally graded beam of ceramic-metal mixture.

A. Shear correction factor of FSDT FG beams

The axial strain and stress of the FSDT are related by the constitutive equation:

$$\sigma_{xx}(x,z) = E(z)\left[\varepsilon^0(x) + z\chi(x)\right] \tag{1}$$

where *E(z)* is Young's modulus at location *z*, ε^0, χ are the axial strain and curvature of the beam, respectively. These strains are related with the axial displacement *u* and rotation θ of the beam as follows: $\varepsilon^0 = u_{,x}$, $\chi = \theta_{,x}$ where the comma indicates partial differentiation with respect to the coordinate subscript that follows. Moreover, the stress resultants (*N*, *M*) are associated to the axial stress σ_{xx} by the global constitutive relations:

$$N(x) = A_{11}\varepsilon^0 + B_{11}\chi \tag{2a}$$

$$M(x) = B_{11}\varepsilon^0 + D_{11}\chi \tag{2b}$$

where A_{11}, B_{11}, D_{11} are the stiffnesses of FG beams defined by:

$$(A_{11}, B_{11}, D_{11}) = \int_{-h/2}^{h/2} (1, z, z^2) E(z)\, dz \tag{3}$$

Substituting (2a) and (2b) into (1) leads to:

$$\sigma_{xx}(x,z) = E(z)\left[(a_{11} + z b_{11}) N + (b_{11} + z d_{11}) M\right] \tag{4}$$

where $a_{11} = D_{11}\left(\bar{D}_{11} A_{11}\right)^{-1}$, $b_{11} = -B_{11}\left(\bar{D}_{11} A_{11}\right)^{-1}$, $d_{11} = \left(\bar{D}_{11}\right)^{-1}$ and $\bar{D}_{11} = D_{11} - \frac{B_{11}^2}{A_{11}}$. Moreover, it is known that the transverse shear stress can be calculated from the equilibrium equation by taking in account (4), that leads to:

$$\sigma_{xz}(x,z) = R(z) Q(x) \tag{5}$$

where

$$R(z) = -b_{11} A_{11z}(z) - d_{11} B_{11z}(z)$$
$$A_{11z}(z) = \int_{-h/2}^{z} E(\xi) d\xi, \; B_{11z}(z) = \int_{-h/2}^{z} \xi E(\xi) d\xi \tag{6}$$

Moreover, it is well-known that the FSDT beams require an convenient shear correction value to calculate the transverse shear force. In the present study, a transverse shear stiffness of the FG beams is obtained by considering the balance of the shear deformation energy and then a shear correction coefficient is derived. Based on the previous works of the author in [7,10], the expression of the improved transverse shear stiffness is obtained:

$$A^s = \left[\int_{-h/2}^{h/2} \frac{\left[b_{11} A_{11z}(z) + d_{11} B_{11z}(z)\right]^2}{G(z)} dz\right]^{-1} \tag{7}$$

where $G(z)=E(z)/2(1+\nu(z))$ is the transverse shear modulus at location z. A shear correction coefficient can be found as:

$$k^s = \frac{A^s}{\int_{-h/2}^{h/2} G(z)\, dz} \tag{8}$$

This coefficient is equal to 5/6 for homogeneous beam and a priori varies with respect to the material variation, material contrast and different homogenization methods.

B. *Effective material properties of FG beams*

It is well-known that the approximations of Voigt and Reuss based on the Hill's principle ([12]) are the simplest ones for estimating the effective elastic properties of heterogeneous composite materials in which Voigt's rule is most used for calculating the effective moduli of the FGM. The mixture of two ceramic-metal materials according to the Voigt and Reuss models through the beam depth are respectively given by:

$$E(z) = (E_c - E_m) V_c(z) + E_m \tag{9a}$$

$$E(z) = \frac{E_c E_m}{E_m V_c(z) + E_c\left(1 - V_c(z)\right)} \tag{9b}$$

where E_c, E_m are Young's modulus of the constituents, V_c is the volumic fraction of ceramic which is given by the power-law distribution as follows:

$$V_c(z) = \left(\frac{2z + h}{2h}\right)^p \tag{10}$$

where p the material parameter which is positive. The material distribution under consideration is plotted in Fig. 2.

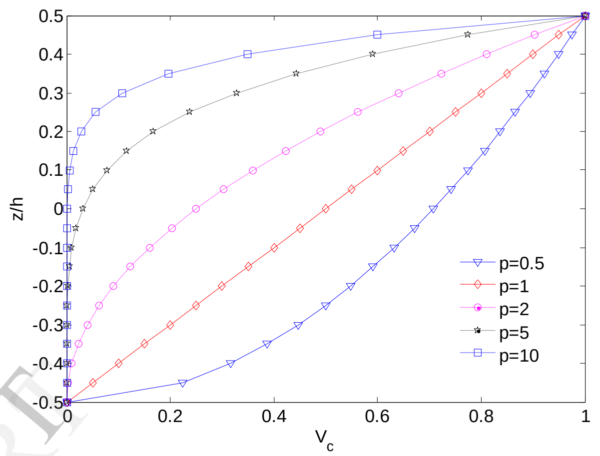

Fig. 2. Variation of the volumic fraction of ceramic through the beam depth.

Alternatively, the effective moduli of FG beams can be derived from the Hashin-Shtrikman's bounds ([16]) which are given by:

$$\frac{K_l - K_m}{K_c - K_m} = \frac{V_c}{1 + (1 - V_c)\dfrac{K_c - K_m}{K_m + 4\mu_m/3}}, \quad \frac{\mu_l - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + (1 - V_c)\dfrac{\mu_c - \mu_m}{\mu_m + f_m}} \tag{11a}$$

$$\frac{K_u - K_m}{K_c - K_m} = \frac{V_c}{1 + (1 - V_c)\dfrac{K_c - K_m}{K_m + 4\mu_c/3}}, \quad \frac{\mu_u - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + (1 - V_c)\dfrac{\mu_c - \mu_m}{\mu_m + f_c}} \tag{11b}$$

where K_m, K_c and μ_m, μ_c are the bulk and shear moduli of the metal and ceramic, respectively; K_l, K_u and μ_l, μ_u are the lower and upper bounds of bulk and shear moduli, respectively; $f_\alpha = \mu_\alpha(9K_\alpha + 8\mu_\alpha)/(6(K_\alpha + 2\mu_\alpha))$ with $\alpha = m,c$. It is noted that the relation (11a) is also known as the Mori-Tanaka's estimate ([16]). The effective moduli are finally obtained as: $E = 9K\mu/(3K + \mu)$, $\nu = (3K - 2\mu)/2(3K + \mu)$.

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III. NUMERICAL RESULTS AND DISCUSSIONS

A number of numerical examples will be carried out in this section to verify the accuracy of the present study and to investigate the effect of homogenization schemes on the shear correction factor of FG beams. It is assumed that the Poisson's ratio is constant through the beam depth. The effect of the power-law index *p*, material contrast $n= E_c/E_m$ and homogenization methods on the shear correction factor is studied.

TABLE I. COMPARISON OF THE SHEAR CORRECTION FACROR

p	Method	n			
		2	**6**	**10**	**20**
0.5	Present (VM)	0.8401	0.8458	0.8471	0.8478
	Present (RM)	0.8320	0.7848	0.7465	0.6846
	Present (UHS)	0.8371	0.8370	0.8360	0.8353
	Present (LHS)	0.8359	0.8119	0.7846	0.7328
	[7]	0.8402	0.8458	0.8471	0.8479
2	Present (VM)	0.8095	0.7662	0.7563	0.7580
	Present (RM)	0.8023	0.7219	0.6799	0.6239
	Present (UHS)	0.8058	0.7401	0.7137	0.6960
	Present (LHS)	0.8045	0.7211	0.6727	0.6058
	[7]	0.8095	0.7662	0.7563	0.7580
5	Present (VM)	0.7891	0.6641	0.5919	0.5043
	Present (RM)	0.7980	0.7335	0.7026	0.6617
	Present (UHS)	0.7923	0.6779	0.6053	0.5043
	Present (LHS)	0.7938	0.7067	0.6619	0.6026
	[7]	0.7891	0.6643	0.5923	0.5046
10	Present (VM)	0.7990	0.6746	0.5861	0.4521
	Present (RM)	0.8097	0.7678	0.7474	0.7194
	Present (UHS)	0.8034	0.7025	0.6255	0.4983
	Present (LHS)	0.8053	0.7406	0.7063	0.6596
	[7]	0.7990	0.6746	0.5861	0.4521

Table I presents the shear correction coefficients derived from Voigt model (VM), Reuss model (RM), lower bound of Hashin-Shtrikman (LHS) and upper bound of Hashin-Shtrikman (UHS) . They are compared to results obtained from [7]. It can be seen that the present results using VM are similar with ones of [7] while there are differences with other models. It proves that the shear coefficients not only depend on the power-law index and material contrast, but also on homogenization methods for estimating the effective material properties of FG beams.

Furthermore, the effects of the power-law index *p*, material contrast *n* and homogenization schemes on the shear correction factor are plotted in Figs. 3, 4 and 5. Four interaction-curves observed in Fig. 3 show that for a specific value of *p* (*p*>3), highest curve is RM and lowest one is VM, and inversely for *p*<3. Figure 5 displays the variation of the shear coefficient with respect to the ratio of Young's modulus for various homogenization schemes. It can be seen that the difference of the bounds of the shear correction factor increases with the material contrast, that confirms that the homogenization schemes based on the variational principles are suitable for small contrast of materials. This figure also shows that the Hashin-Shtrikman's bounds give the approximation more accurate than the Hill's ones.

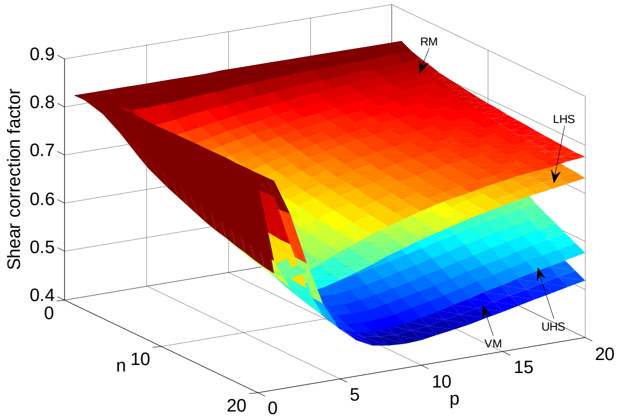

Fig. 3. Variation of the shear correction factor with respect to the power-law index and material contrast.

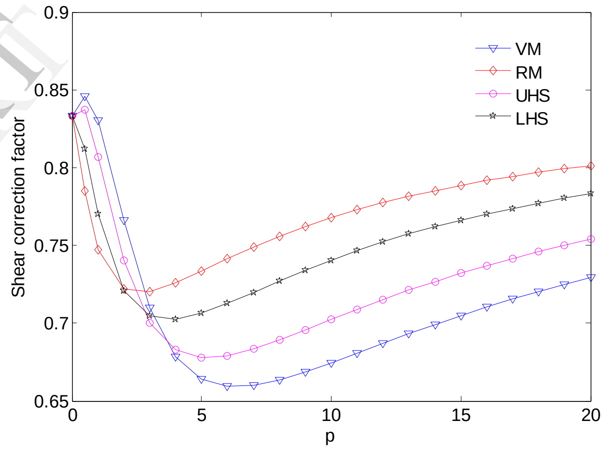

Fig. 4. Variation of the shear correction factor with respect to the power-law index with *n*=6.

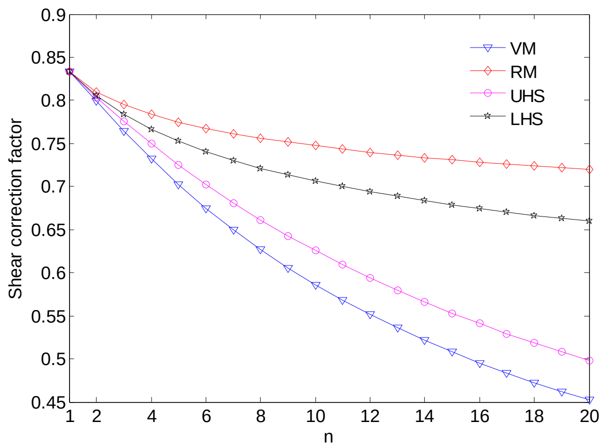

Fig. 5. Variation of the shear correction factor with respect to the material contrast with *p*=10.

IV. CONCLUSIONS

This paper analyzed the effects of the effective material properties on the shear correction factor of FG beams. It is based on the first-order shear deformation beam theory within which the effective moduli of the beam are calculated by the Voigt and Reuss models, and Hashin-Shtrikman's estimates. The material elastic properties are supposed to be isotropic and varying through the beam depth according to the power-law form. Numerical results showed that the shear correction factor not only depends on the material distribution, material contrast but also on homogenization methods. The approximation schemes based on bounds are not accurate anymore for high material contrasts and the Hashin-Shtrikman's bounds give the approximation more accurate than Hill's ones.

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