Analysis of Smart Antisymmetric Composite Laminated Plates using HSDT

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Abstract: **Smart structures are the structures where in piezoelectric layers are attached to elastic layers in patches or in distributed form. Among many types of smart or adaptive materials, piezoelectric materials are primarily use because, theoretical analysis of this material has been so well developed. In this paper an analytical procedure is developed for investigating the bending characteristics of smart material plates subjected to electromechanical loading based on higher order shear deformation theory. The solutions are obtained using Navier's method for anti-symmetric angle ply composite laminated plates attached with piezoelectric layer with a specific type of simply supported boundary conditions. The results obtained in this paper have been validated with authors.**

Keywords: Higher order theory; piezoelectric material; Navier's method.

I. INTRODUCTION

Smart materials also called as a new class of structures has been produced by sensors and actuators to perform self-monitoring and self-controlling system. Sensing and actuating the structure could be applied in many engineering applications such as aircraft structures, large space structures, satellites, automotive industries, sports goods, medical devices etc. by incorporating smart structures with piezoelectric devices. Y. L. Zhou [1] has investigated static and dynamic analysis of composite laminated plates attached with piezoelectric layer using three different finite element models based on generalized laminate plate theory of Reddy. Taotao Zhang et al [2] have studied the bending behavior of piezoelectric curved actuator with generally graded properties for one of the piezoelectric parameter. Two piezoelectric actuators considered by Shiuh-Chuan Her et al [3] in their investigation which are symmetrically surface bonded on a cross-ply composite laminate. They have used the plate theory for solving a simply supported composite plate subjected to bending moment. For validation purpose they have compared their analytical solution with finite element solution. M. Tahani et al [4] have analyzed analytically flexural behavior of piezolaminated rectangular plates with specific boundary conditions based on extended Kantorovich method. They have compared the results with other investigators and also results obtained for plates with admissible boundary conditions by Navier and Levy methods. K. M. Liew et al [5] presented meshfree formulation based on first order shear deformation theory for static analysis of laminated composite beams and plates with bonded piezoelectric layers. They have taken piezoelectric stiffness into account in model and derived the formulation based on variation principle. They have found from the investigation that actuator patches bonded on composite laminated plate are significant in deflection control. Alden C. Cook et al [6] considered two model problems for multiscale analysis procedure. In their first model problem they have considered a simply-supported sandwich plate consisting of a piezoceramic fibre and bottom surfaces. Where, as second model concerns a cantilever graphite substrate with segmented piezoceramic fiber composite extension actuators attached to its top. Iskandar Al-Thani Mahmood et al [7] have controlled the shape control analysis of piezoelectric composite laminated plate by adopting a finite element model. They have modeled actuators and sensors as additional layers which are either bonded or embedded to composite laminated plate. W.Q. Chen et al [8] have considered static and dynamic analysis of simply supported angle-ply composite laminates in cylindrical bending. Based on three dimensional exact elasticity equations they have employed a state-space approach which is effective in analyzing laminated structures. Nilanjan Mallik et al [9] have investigated performance of piezoelectric fibre reinforced composite material as the distributed actuator for smart composite laminated plates. They have performed the investigation for finding the exact solutions for static analysis of simply supported symmetric and anti-symmetric cross-ply laminated plates integrated with a layer of PFRC material. Osama J Aldraihem et al [10] have obtained the analytical solutions for bending analysis of anti-symmetric angle-ply composite laminated plates with thickness-shear piezoelectric layers. They have investigated the effects of composite and piezoelectric ply angle on the laminate deflection. They have observed from their results that increasing the ply-angle has always magnifies the deflection. J Shiva Kumar et al [11] has analyzed a simply supported cross-ply elastic substrate plates integrated with a layer of PFRC material. They have employed a Galerkin procedure for deriving nonlinear algebraic governing equations. S. M. Shiyekar et al [12] presented an analytical solution for cross ply composite laminates attached with PFRC actuator under bi-directional bending. They have obtained equations of equilibrium using principle of minimum potential energy and compared the results with exact solution. Based on first order shear deformation theory Liew et al [13] has presented an efficient meshfree formulation for static analysis of composite laminated plates and beams with integrated piezoelectric layers. They have taken piezoelectric stiffness into account and derived the formulation from variational principle. They found from results, actuator patches bonded on high strain regions have significant role in controlling deflections of laminated composite plates. An analytical solution for cross-ply composite laminates integrated with piezoelectric fiber-reinforced composite (PFRC) actuators under bidirectional bending is presented by Kant et al [14]. A higher order shear and normal deformation theory (HOSNT12) is used by them to analyze smart materials subjected to electromechanical loading. Based on Hamilton's principle and finite element methods, linear response of piezothermoelastic plate has outlined by Fariborz Heidary et al [17]. They presented numerical results for a piezolaminated plate subjected to thermomechanical loadings. With use of electric potential difference across piezo layers, vibrations can be suppressed on piezolaminated composite plate.

II. FORMULATION

In formulating the higher order shear deformation theory, a rectangular plate of 0 ≤ x ≤ a; 0 ≤ y ≤ b bonded with piezoelectric layer is considered.

In order to approximate 3D-elasticity plate problem to a 2D one, the displacement components u (x, y, z, t), v(x, y, z, t) and w (x, y, z, t) at any point in the plate are expanded in terms of the thickness coordinate. The displacement field which assumes w (x, y, z) constant through the plate thickness and thus setting $\varepsilon_z = 0$ is expressed as [12]:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \quad \ldots\ldots (1)$$

Where the parameters u_0, v_0 and w_o denote the displacements of a point (x, y) on the midplane. The functions θ_x, θ_y are rotations of the normal to the midplane about y and x–axes, respectively.

Piezoelectric coupling involving mechanical and electrical excitation can be expressed in terms of stresses and strains as [13]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{54} & Q_{55} \end{bmatrix}^L \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}^L - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \end{bmatrix}^L \begin{Bmatrix} -\dfrac{\partial \xi(x,y,z)}{\partial x} \\ -\dfrac{\partial \xi(x,y,z)}{\partial y} \\ -\dfrac{\partial \xi(x,y,z)}{\partial z} \end{Bmatrix}^L \quad \ldots\ldots (2)$$

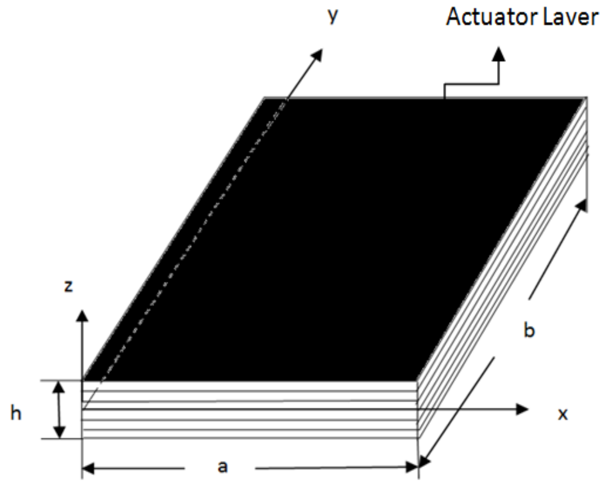

Fig. 1. Composite laminated plate attached with piezoelectric layer

Where σ, Q, ε, e, and E are stress vector, elastic constant matrix, strain vector, piezoelectric constant matrix and electric field intensity vector respectively. When piezoelectric constant matrix [e] is unavailable it can be expressed in piezoelectric strain constant matrix [d] as [13]:

$$[e] = [Q]\,[d]^t$$

Where

$$[d]^t = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \end{bmatrix} \quad \ldots\ldots (3)$$

The governing equations of displacement model will be derived using the principle of virtual work as [15]:

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \quad \ldots\ldots (4)$$

The virtual work statement shown in Eq. (4), integrating through the thickness of laminate, the in-plane and transverse force and moment resultant relations in the form of matrix obtained as:

$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ B^t & | & D_b & | & 0 \\ 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \in_0 \\ \in_0^* \\ -- \\ K \\ K^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \quad \text{...... (5)}$$

Equating the coefficients of each of virtual displacements $\delta u_0, \delta v_0, \delta w_0, \delta\theta_x, \delta\theta_y, \delta u_0^*, \delta v_0^*, \delta\theta_x^*, \delta\theta_y^*$ to zero, the equations of motion are obtained. These Equations are expressed in terms of displacements $u_0, v_0, w_0, \theta_x, \theta_y, u_0^*, v_0^*, \theta_x^*, \theta_y^*$ by substituting for the force and moment resultants.

Boundary conditions for simply supported angle-ply antisymmetric composite laminated plates attached with piezoelectric layer are:

At edges x = 0 and x = a

$u_0 = 0, w_o = 0, \theta_y = 0, N_{xy} = 0, M_x = 0, u_0^* = 0, \theta_y^* = 0, M_x^* = 0, N_{xy}^* = 0, \xi = 0$

At edges y = 0 and y = b

$v_0 = 0, w_o = 0, \theta_x = 0, N_{xy} = 0, M_y = 0, v_0^* = 0, \theta_x^* = 0, M_y^* = 0, N_{xy}^* = 0, \xi = 0$

The displacements at the mid plane will be defined to satisfy the above boundary conditions. These displacements will be substituted in governing equations to obtain the equations in terms of A, B, D parameters. The obtained equations will be solved to find the behavior of the laminated composite plates.

III. RESULTS AND DISCUSSIONS

The material properties of graphite/epoxy used for each orthotropic layer of the substrate are [12]:

$$\frac{E_1}{E_2} = 25, \ \frac{G_{12}}{E_2} = 0.5, \ \frac{G_{23}}{E_2} = 0.2, \ E_2 = E_3 = 10^6 \ N/cm^2$$
$$G_{12} = G_{13} \ and \ \mu_{12} = \mu_{23} = \mu_{13} = 0.25$$

Material properties for PFRC layer are [12]:

C_{11} = 32.6 GPa, $C_{12} = C_{21}$ = 4.3 GPa; $C_{13} = C_{31}$ = 4.76 GPa; $C_{22} = C_{33}$ = 7.2 GPa; C_{23} = 3.85 GPa; C_{44} = 1.05 GPa; $C_{55} = C_{66}$ = 1.29 GPa; e_{31} = - 6.76 C/m^2; $g_{11} = g_{22}$ = 0.037E - 9 C/V m; g_{33} = 10.64E - 9 C/V m.

Effect of piezoelectric layer with applied electric voltage at top of the actuator has been described in Fig. 2. It is seen from the figure, the actuating effect has been negligible at the mid thickness of the plate this may be because of effect of load has been decreasing from top to bottom of the plate. Variation of normal stresses σ_x and σ_y against thickness of composite laminate plate attached with actuator and sensor layers for antisymmetric angle ply laminates is shown in Fig. 3 & 4. It has been observed from figures that the top layer is effected maximum compared to other layers, this may be because of the load is applied at top of sensor layer. Fig. 5 explains the variation of transverse shear stress (τ_{xy}) against piezoelectric composite laminate plate thickness for antisymmetric angle ply laminates. The percentage variation of transverse shear stress (τ_{xy}) results with Y. L. Zhou [1] is less than 10%. Variation of transverse shear stresses (τ_{yz}, τ_{xz}) against thickness of piezoelectric composite laminate for antisymmetric angle ply laminates is shown in Figures 6 & 7.

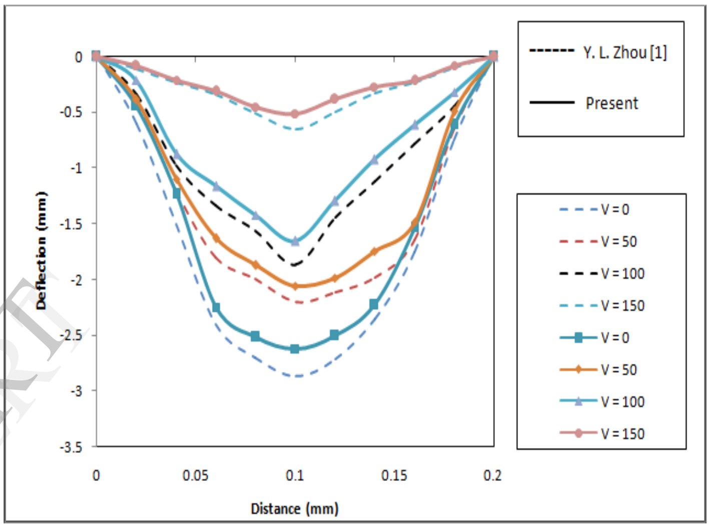

Fig. 2. Variation of displacement (ω) against thickness of piezoelectric layer with applied voltages at top of actuator

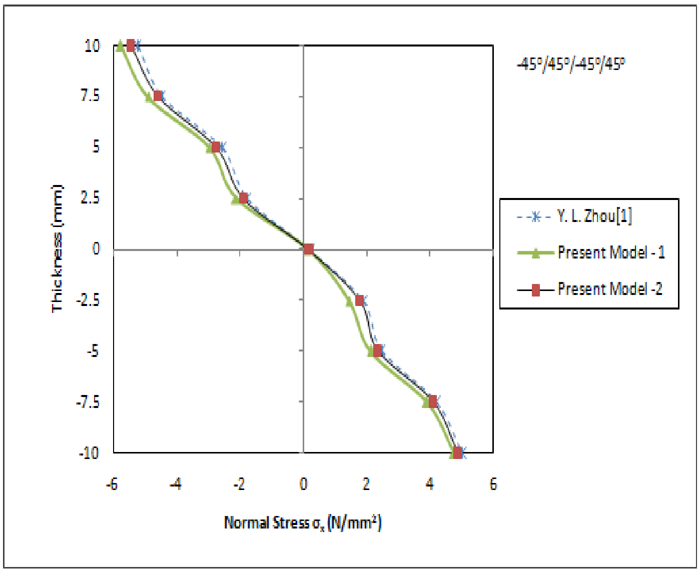

Fig. 3. Variation of normal stress (σ_x) against thickness of piezoelectric composite laminate for antisymmetric angle ply laminates.

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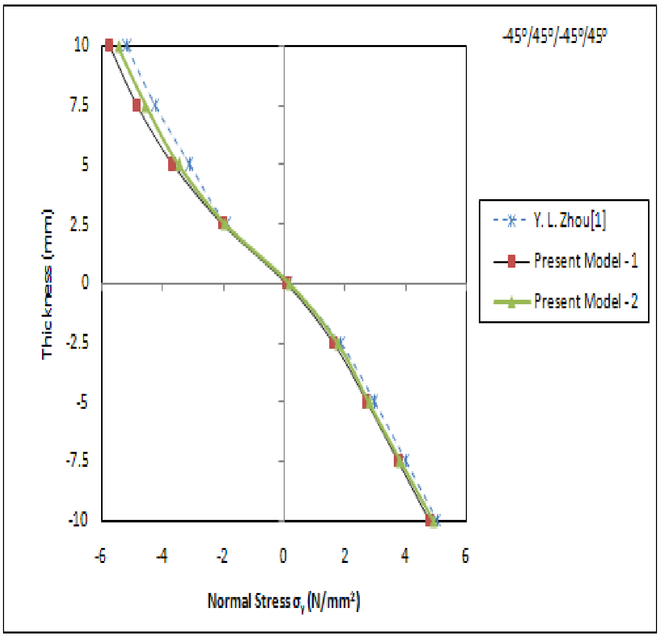

Fig. 4. Variation of normal stress (σ_y) against thickness of piezoelectric composite laminate for antisymmetric angle ply laminates.

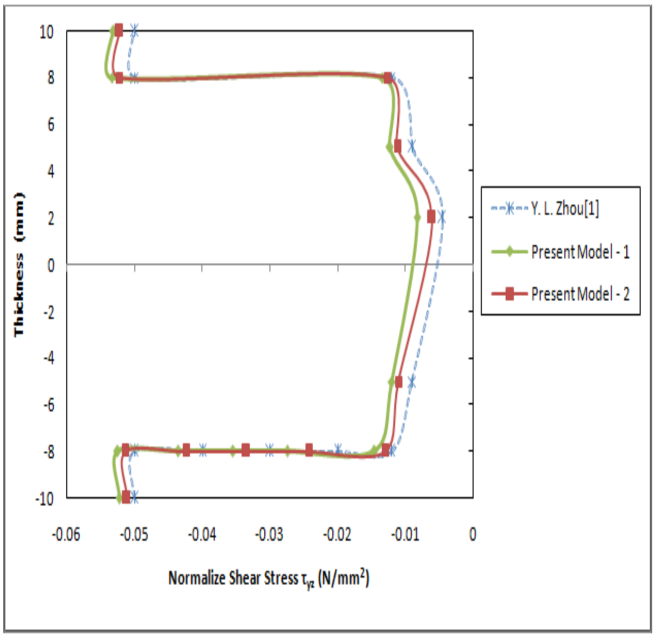

Fig. 6. Variation of transverse shear stress (τ_{yz}) against thickness of piezoelectric composite laminate for antisymmetric angle ply laminates.

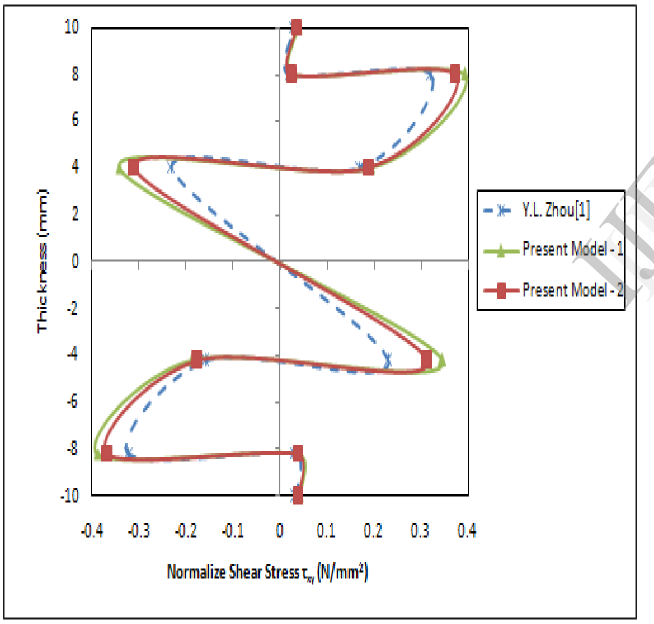

Fig. 5. Variation of transverse shear stress (τ_{xy}) against thickness of piezoelectric composite laminate for antisymmetric angle ply laminates.

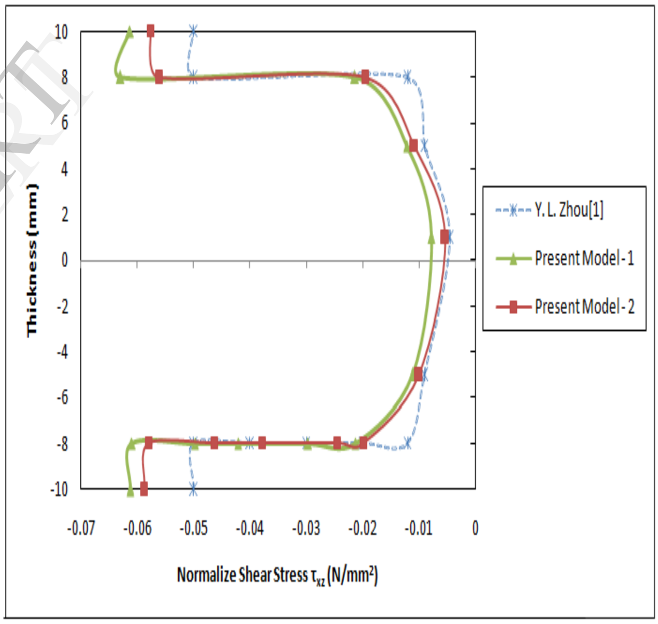

Fig. 7. Variation of transverse shear stress (τ_{xz}) against thickness of piezoelectric composite laminate for antisymmetric angle ply laminates.

IV. CONCLUSIONS

Analytical procedure has been developed for composite laminate plates attached with piezoelectric layer under electromechanical loading is discussed in this paper. Higher order shear deformation theory is used to model elastic substrate response to voltages. Comparative numerical results for across the thickness variations of stresses are presented. It can be concluded from results that the actuating effects are more in case of thick than thin laminates. From the results it is found that the obtained values are in close form with available literature.

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