Analysis of Ohmic Loss in Oversized Smooth Walled Circular Waveguide using FEM

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Abstract—In this paper we discuss the attenuation occur due to ohmic losses in oversized smooth walled circular waveguide. Ohmic losses depend on the radius of the waveguide and material used for the wall of the waveguide. It also depends on the mode propagating in the waveguide. We calculate the ohmic loss for W-band frequency that ranges from 70-110 GHz and have verified the results using COMSOL v2.a software.

Keywords—oversized waveguide; smooth walled waveguide; ohmic loss; attenuation constant

I. INTRODUCTION

Delivery of electromagnetic waves in dielectric and conducting media is difficult because of radiation at high frequencies. Circular waveguides are one type of transmission line in which air is used as dielectric because it has low loss than other insulation material. Circular waveguides offer implementation advantage over rectangular waveguide as its installation is much simpler. The circular waveguide supports Transverse Electric (TE) & Transverse Magnetic (TM) modes. The cut-off frequency of circular waveguide depends on the waveguide dimensions and shape. The electromagnetic wave at frequency greater than the cut-off frequency can propagate in the waveguide. The TE\textsubscript{0n} modes of circular waveguide have very low attenuation in the case of oversized waveguide. More than one mode propagates in the oversized waveguide so it is called overmoded. The reason it is used is that the resistive losses are reduced as the waveguide becomes oversized. \cite{1, 4}

II. MODES IN SMOOTH WALL CIRCULAR WAVEGUIDE

We consider the parameters of smooth wall circular waveguide for mode calculations as shown in figure 1.

![Smooth walled cylindrical waveguide with radius 'a'][2]

Fig. 1 smooth walled cylindrical waveguide with radius ‘a’[2]

A. TM Mode

We know that the wave made up of Transverse Electric (TE) and Transverse Magnetic (TM) components. First we see the TM modes. In these modes \( Z \) component of magnetic field must be zero (\( H_z = 0 \)). \cite{2} Therefore

\[
E_z = E_0 J_m(k, r) \left\{ \frac{\sin(m\phi)}{\cos(m\phi)} \right\} e^{-jk_zz} \] (1)

Where, \( E_0 \) is arbitrary amplitude of the mode.

Rest of the field equations can be derived using Maxwell equations and dispersion relation for cylindrical waveguide.

\[
E_r = \frac{j k_r k_z E_0}{k^2 - k_z^2} J'_m(k, r) \left\{ \frac{\sin(m\phi)}{\cos(m\phi)} \right\} e^{-jk_zz} \] (2)

\[
E_\phi = \frac{j k_z E_0 m}{k^2 - k_z^2} J_m(k, r) \left\{ \frac{\cos(m\phi)}{\cos(m\phi)} \right\} e^{-jk_zz} \] (3)

\[
H_r = -\frac{j \omega \varepsilon E_0 m}{k^2 - k_z^2} J_m(k, r) \left\{ \frac{\cos(m\phi)}{\cos(m\phi)} \right\} e^{-jk_zz} \] (4)

\[
H_\phi = \frac{j \omega \kappa E_0}{k^2 - k_z^2} J'_m(k, r) \left\{ \frac{\sin(m\phi)}{\cos(m\phi)} \right\} e^{-jk_zz} \] (5)

Where,

\[
k_r = \frac{X_{mn}}{a} \] (6)

And \( X_{mn} \) is the nth root of the mth Bessel function of the first kind, so that \( J_m(X_{mn}) = 0 \).

\[
k_z = \sqrt{\frac{\omega^2 \varepsilon \mu - k_r^2}{\varepsilon}} \] (7)
Fig. 2 TM₀₁ & TM₁₁ mode pattern respectively

As a result of mode analysis with radius of 36 mm we get the mode patterns of TM mode as shown in figure 2.

B. TE Mode

We can perform the same for the TE modes. In these modes Z component of electric field must be zero (E_z = 0).[2] Therefore

\[ H_z = J_m(k_r) \left[ \sin(m \phi) \cos(m \phi) \right] e^{-jk_z z} \] (8)

Using Maxwell equations solution for other transvers electric (TE) field is given by,

\[ H_r = \frac{j k_z J'_m(k_r)}{k^2 - k_z^2} \left[ \sin(m \phi) \cos(m \phi) \right] e^{-jk_z z} \] (9)

\[ H_\phi = \frac{j k_z m}{k^2 - k_z^2} \frac{J_m(k_r)}{r} \left[ \cos(m \phi) \sin(m \phi) \right] e^{-jk_z z} \] (10)

\[ E_r = \frac{j \omega \mu m}{k^2 - k_z^2} \frac{J_m(k_r)}{r} \left[ \cos(m \phi) \sin(m \phi) \right] e^{-jk_z z} \] (11)

\[ E_\phi = -\frac{j \omega \mu k_z J'_m(k_r)}{k^2 - k_z^2} \left[ \sin(m \phi) \cos(m \phi) \right] e^{-jk_z z} \] (12)

As shown in figure 3 we can observe that the field intensity is low in the TE₀₁ mode at the boundary compare to the TE₁₁ mode. So the ohmic attenuation of the TE₀₁ is low for the oversized waveguide.

III. OHMIC LOSS

The loss occurs in waveguide known as ohmic loss as a result of finite conductivity of waveguide wall. Ohmic losses arise due to small field penetration into the conductor walls of the waveguide. The attenuation coefficient for the TEₘₙ modes inside circular waveguide are given by,[3]

\[ (\alpha_c)_{TE} = \frac{R_f}{a} \frac{R_f}{\pi} \sqrt{1 - \left( \frac{f}{f_c} \right)^2} \left[ \left( \frac{f}{f_c} \right)^2 + \left( \frac{m}{n} \right)^2 \right] Np / m \] (13)

And for the TM modes (c) is given by,

\[ (\alpha_c)_{TM} = \frac{R_f}{a} Np / m \] (14)

Where a, f, fc, χ", η, n, and m are the radius of waveguide, propagating frequency, cut-off frequency, nth zero of the derivative of the Bessel function, impedance of the material, no of circumferential variations, no of radial variations respectively and surface resistance given by

\[ R_s = \frac{\sigma \mu}{2\pi} \] (15)

Where σ = electrical conductivity of material. The ohmic loss in circular waveguide depends on the mode propagate in it and radius of the waveguide.

![Comparison of ohmic attenuation of TE₁₁ mode](image)

We calculate the ohmic attenuation of the TE₁₁ mode into entire W-band for the diameter 72mm and 80 mm for the same conducting material aluminum. And we observe that for higher diameter ohmic loss is low.
As shown in Fig. 5 we calculate the ohmic loss for the TE\(_{11}\) mode using two different material copper and aluminum for the 72 mm diameter of the waveguide. Continuous and dotted lines represent the theoretical and simulated values respectively. Result shows that there is a good agreement between theoretical and simulation values.

By observing the above values, we can say that the ohmic attenuation of the TE\(_{01}\) mode is much lesser than the TE\(_{11}\).

**CONCLUSION**

We have seen that the ohmic attenuation of the waveguide mainly depends on the radius and conductivity of material used by waveguide wall. Ohmic loss can be decreases by increasing the diameter of the waveguide and using highly conductive material. By observing the characteristics of TE\(_{11}\) and TE\(_{01}\) mode in smooth wall circular waveguide we can conclude that the TE\(_{01}\) is the lowest attenuated mode in oversized smooth walled circular waveguide.

**REFERENCES**


**TABLE 1. Attenuation constants of TE\(_{11}\) & TE\(_{01}\) mode**

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>(\alpha_c) of TE(_{11}) (dB/m)</th>
<th>(\alpha_c) of TE(_{01}) (dB/m)</th>
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