Analysis of Multi-Objective Programming Using Goal Programming Methodology

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***Abstract*— This research proposes a goal programming (GP) approach for solving linear multiple objective programming problem with priority level. In the proposed approach, priority is assigned for achievement of goals to their aspired levels on the basis of their relative importance is considered in an uncertain environment. In the model formulation of the problem, the membership functions for each of the priority goals are defined first. Then, the membership functions are transformed into membership goals by assigning the highest membership value (priority) and introducing under-and over-deviational variables to each of them. In the solution process, the priority goals are solved by simplex method for the optimality purpose which helps in the achievement of a function for minimizing them to reach the aspired goal levels of the problem. To illustrate the proposed approach, a numerical case study is presented.**

Keywords*— *Multi-objective programming, Goal programming, simplex method.

I. INTRODUCTION

Multi objective programming is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. Multi-objective programming [1] and planning represents a very useful generalization if more traditional, single-objective approaches to planning problems. The consideration of many objectives in the planning process accomplishes three major improvements in problem solving. First, multi-objective programming and planning promotes more appropriate role for the participants in the planning and decision making processes. Second, a wider range of alternatives is usually identified when a multi-objective methodology is employed. Third, models (if they are used) or the analyst perception of a problem will be more realistic if many objectives are considered.

Multi objective programming can be handled by many methods such as Sum of objective(SO) [2] , Fuzzy logic[3] , TOPSIS , Goal Programming and many But Goal Programming is most commonly used approach to address the situation which have multiple objectives.

The disadvantage of Sum of objective method is that it is feasible only when it contains all the objective functions either minimization or maximization. GP technique can be used for the mixtures of objective functions such as both minimization and maximization. Goal programming is helpful over other techniques is with dealing with real-world decision problems is that it reflects the way manages actually make decisions and GP [4] is a powerful technique since it can handle multiple objectives. Unlike linear programming, the GP model does not optimize (maximize/minimize) the objectives directly. GP is started as an extension of LP in order to solve unsolvable LP problems, e.g. infeasible LP problems.

The goal programming necessitates the establishment of a weighting system for the goals such that lower-ranked (or weighted) goals are considered only after higher-ranked goals have been satisfied or have reached the point beyond which no further improvement is desirable. Goal programming can be solved by Graphical method, modified simplex method and soon.

The simplex method is faster than the algebraic method because the simplex method contains less iteration than the algebraic method and the goodness of simplex algorithm is it contains less memory as compare to other methods. The simplex method contains two key elements such as leaving variable and entering variable. The leaving variable ensures that feasibility is maintained and entering variable ensures that optimality is maintained.

II. ANALYSIS

In this proposed work resort scenario has been considered as follows.

A resort owner has been estimated Rs.6, 00,000 lakhs to expand its different facilities recreational works. Four facilities areas have been identified, by construction department such as play area, restaurant, parking area, guest house. The total demand for identified facilities are seven play area, ten restaurant, eight parking area and twelve guest house. Each facility costs a certain amount, requires a certain number of hectares and has an expected usage as shown in the following table.

Facility	Cost	Required Acres	Excepted Usage (People/Week)
Play area	80000	4	1500

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Restaurant	24000	8	3000
Parking area	15000	3	500
Guest House	40000	5	1000

The resort owner has located a total of 50 acres of land for construction (although more land could be located if necessary). The owner has established the following list of prioritised goals:

1. The construction department must spend the total grant (otherwise the amount not spent will be returned to the owner).
2. The facilities should be used by 20,000 or more people weekly.
3. If more land is required, the additional amount should be limited to ten hectares.
4. The construction department would like to meet the demands of the other department members of the resort. However, this priority should be weighted according to the number of people expected to use each facility.
5. The construction department wants to avoid having to acquire land beyond the 50 acres presently available.

III. METHODOLOGY

In the proposed work Goal Programming methodology is used to handle the multi objective programming. In which further solution can be solved using modified simplex method. The process flow can be stated below.

a) Transforming a baseline model into a GP model
b) Converting Inequalities - Standard Approach
c) Assigning Deviation Variables - "Distance to target"
d) Two Deviation Variables instead of One
e) Values of Deviation Variables
f) "Desired" Values of Deviation Variables
g) Minimizing deviations
h) Prioritizing Goals using Rank order approach.
i) Simplex method is used for solving mathematical formulation.

ADVANTAGES OF GOAL PROGRAMMING

1. Allows for an ordinal ranking of goal, where the low priority goals are considered only after higher priority goals have been satisfied to the fullest extent possible.
2. Useful in situations where the multiple goals are conflicting and cannot all be fully achieved.
3. Used to "Satisfies" rather than to "optimize" the problem. In linear programming, what one wants is to optimize the solution. But in using the goal programming, the goal may be incorporated into the model at a value that is judged to be satisfactory, not necessarily optimal.
4. Appropriate to find a satisfactory solution where many objectives or goals are to be considered.

ALGORITHM FOR GOAL PROGRAMMING

a) Transforming a baseline model into a GP model

Step1: Transform all objectives into goals by establishing associated aspiration levels based on the belief that a real world decision maker can usually cite (initial) estimates of his or her aspiration levels. Hence,

maximize $A_r(\underline{X})$ becomes $A_r(\underline{X}) \geq T_r$ for all r
minimize $A_s(\underline{X})$ becomes $A_s(\underline{X}) \leq T_s$ for all s.
where and T_r and T_s are the respective aspiration levels (targets).

Step2: Rank-order each goal according to its perceived importance. Hence, the set of hard goals (i.e., constraints in traditional math programming) is always assigned the top priority or rank.

Step3: All the goals must be converted into equations through the addition of deviation variables

b) Converting Inequalities - Standard Approach

In general, converting equalities to inequalities is achieved by adding variables

c) Assigning Deviation Variables - "Distance to target"

- In Goal Programming and other approaches (like compromise Decision Support Problem) "deviation" variables are used to convert inequalities to equalities.
- The deviation variable d is (then) defined as:

$$d = T_i - A_i(X)$$

- Note: Mathematically, the deviation variable d can be negative, positive, or zero.
- From a reality point of view, a deviation variable represents the distance (deviation) between the aspiration level (target) and the actual attainment of the goal.

d) Two Deviation Variables instead of One

- The deviation variable d can be replaced by two variables:

$$d = d_i^- - d_i^+$$

$$\text{where } d_i^- \bullet d_i^+ = 0 \text{ and } d_i^-, d_i^+ \geq 0$$

- The goal formulation (now) becomes:

$$A_i(X) + d_i^- - d_i^+ = T_i;\ i = 1,2, \ldots, m$$

$$\text{subject to } d_i^- \bullet d_i^+ = 0 \text{ and } d_i^-, d_i^+ \geq 0$$

e) Values of Deviation Variables

Note that a goal is always expressed as equality:

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$A_i(\underline{X}) + d_i^- - d_i^+ = T_i$; $i = 1, 2, \ldots, m$

And when considering this equality, the following will be true:

if $A_i(\underline{X}) < T_i$ is true, then ($d_i^- > 0$ AND $d_i^+ = 0$) must be true;

if $A_i(\underline{X}) > T_i$ is true, then ($d_i^- = 0$ AND $d_i^+ > 0$) must be true;

if $A_i(\underline{X}) = T_i$ is true, then ($d_i^- = 0$ AND $d_i^+ = 0$) must be true.

f) "Desired" Values of Deviation Variables

Again, note that a goal is always expressed as equality.

$A_i(X) + d_i^- - d_i^+ = T_i$; $i = 1,2, \ldots, m$

To achieve a goal (i.e., reach the target), 3 situations are possible:

1. To satisfy $A_i(\underline{X}) \leq T_i$, we must ensure that the deviation variable d_i^+ is zero.
- The deviation variable d_i^- is a measure of how far the performance of the actual design is from the goal.
2. To satisfy $A_i(\underline{X}) \geq T_i$, the deviation variable d_i^- must be made equal to zero.
- In this case, the degree of "overachievement" is indicated by the positive deviation variable d_i^+.
3. To satisfy $A_i(\underline{X}) = T_i$, both deviation variables, d_i^- and d_i^+ must be zero.

Thus, to achieve a target, we must minimize the unwanted deviation(s).

g) Minimizing deviations

Consider the preceding three situations again.

To achieve a goal (i.e., reach the target), 3 situations are possible:

1. To achieve $A_i(\underline{X}) \leq T_i$, we must *minimize* (d_i^+)
2. To achieve $A_i(\underline{X}) \geq T_i$, we must *minimize* (d_i^-)
3. To achieve $A_i(\underline{X}) = T_i$, we must *minimize* ($d_i^- + d_i^+$).

h) Prioritizing Goals

1. Rank order goal deviations in priority levels, often referred to as a preemptive formulation. The preemptive formulation does not exclude the assignment of weights.
2. In Rank Ordering, you prioritize one goal/objective above each other without giving explicit mathematical weights.
3. Basically, in words, Goal A has to be achieved before Goal B. I should not even think about Goal B yet if Goal A has not been achieved yet.

IV. NUMERICAL EXAMPLE

Scenario:

A resort owner has been estimated Rs.6, 00,000 lakhs to expand its different facilities recreational works. Four facilities areas have been identified, by construction department such as play area, restaurant, parking area, guest house. The total demand for identified facilities are seven play area, ten restaurant, eight parking area and twelve guest house. Each facility costs a certain amount, requires a certain number of hectares and has an expected usage as shown in the following table.

Facility	Cost	Required Acres	Excepted Usage (People/Week)
Play area	80000	4	1500
Restaurant	24000	8	3000
Parking area	15000	3	500
Guest House	40000	5	1000

Solution:

Let X_1 = number of Play Area.
X_2 = number of Restaurant.
X_3 = number of Parking Area.
X_4 = number of Guest House.

Goal 1 Funding
Upper limit = 600m

$80X_1 + 24X_2 + 15X_3 + 40X_4 + d_1^- - d_1^+ = 6, 00,000$
Min $P_1 d_1^-$

Goal 2 Usage
Usage by 20000 or more per week

$15000X_1 + 3000X_2 + 500X_3 + 1000X_4 + d_2^- - d_2^+ = 20000$
Min $P_2 d_2^-$

Goal 3 Land Requirement
50 acres available

$4X_1 + 8X_2 + 3X_3 + 5X_4 + d_3^- - d_3^+ = 50$
$d_3^+ + d_4^- - d_4^+ = 10$
Min $P_3 d_4^+$

Note alternative formulation possible:
$4X_1 + 8X_2 + 3X_3 + 5X_4 + d_4^- - d_4^+ = 60$
Min $P_3 d_4^+$
But first formulation will also accommodate Goal 5.

Goal 4 Demand

$X_1 + d_5^- - d_5^+ = 7$
$X_2 + d_6^- - d_6^+ = 10$
$X_3 + d_7^- - d_7^+ = 8$
$X_4 + d_8^- - d_8^+ = 12$

The resort owner would like to meet demand i.e. try not to under-supply i.e. Minimize the d_i^- variables, weighted according to usage.

$$\text{Min } P_4 \,(\, 1.5d_5^- + 3d_6^- + 0.5d_7^- + d_8^- \,)$$
$$\text{i.e. Min } P_4 \,(\, 3d_5^- + 6d_6^- + d_7^- + 2d_8^- \,)$$

Goal 5 Land requirement
Avoid exceeding the 50 acres
Min $P_5 d_3^+$

Complete Problem:

Min $P_1d_1^- + P_2d_2^- + P_3d_4^+ + P_4\,(\, 3d_5^- + 6d_6^- + d_7^- + 2d_8^- \,) + P_5d_3^+$

s.t.c

$80X_1 + 24X_2 + 15X_3 + 40X_4 + d_1^- - d_1^+ = 6,00,000$

$15000X_1 + 3000X_2 + 500X_3 + 1000X_4 + d_2^- - d_2^+ = 20000$
$4X_1 + 8X_2 + 3X_3 + 5X_4 + d_3^- - d_3^+ = 50$
$d_3^+ + d_4^- - d_4^+ = 10$
$X_1 + d_5^- - d_5^+ = 7$
$X_2 + d_6^- - d_6^+ = 10$
$X_3 + d_7^- - d_7^+ = 8$
$X_4 + d_8^- - d_8^+ = 12$
$X_j \geq 0$, j= 1…..4, $d_i^-, d_i^+ \geq 0$, i=1……8

2. Carry out two iterations of the modified simplex method on the following goal programming problem:

Min $P_1d_1^- + P_2d_2^+ + 5P_3d_2^- + 3P_3d_3^- + P_4d_1^+$
s.t $X_1 + X_2 + d_1^- - d_1^+ = 80{,}000$
$X_1 + d_2^- = 70{,}000$
$X_2 + d_3^- = 45{,}000$
$X_1 + X_2 + d_4^- - d_4^+ = 90{,}000$
$X_j \geq 0$, j= 1….2, $d_i^-, d_i^+ \geq 0$, i=1……4

	X_1	X_2	d_1^+	d_4^+	d_1^-	d_2^-	d_3^-	d_4^-	B
P_1					-1				0
P_2				-1		-5	-3		0
P_3		2							0
P_4			-1						0
d_1^-	1	1	-1		1				80,000
X_1	1					1			70,000
d_3^-		1					1		45,000
d_2^-	1	1						1	90,000

Not in canonical form. For d_i^- variables to be basic, non-zero entries in their columns need to be eliminated for P_1 and P_3 rows.

P_1 row = P_1 row + d_1^- row
P_3 row = P_3 row + $5d_2^-$ row + $3d_3^-$ row

	X_1	X_2	d_1^+	d_4^+	d_1^-	d_2^-	d_3^-	d_4^-	B
P_1	1	1	-1						80,000
P_2				-1					0
P_3	5	3							4,85,000
P_4			-1						0
d_1^-	1	1	-1		1				80,000
d_2^-	1					1			70,000
d_3^-		1					1		45,000
d_4^-	1	1						1	90,000

X_1 enters and d_2^- leaves

	X_1	X_2	d_1^+	d_4^+	d_1^-	d_2^-	d_3^-	d_4^-	B
P_1		1	-1						10,000
P_2				-1					0
P_3		3				-5			1,35,000
P_4			-1						0
d_1^-		1	-1		1	-1			10,000
X_1	1					1			70,000
d_3^-		1					1		45,000
d_4^-		1		-1		-1		1	20,000

X_2 enters and d_1^- leaves

	X_1	X_2	d_1^+	d_4^+	d_1^-	d_2^-	d_3^-	d_4^-	B
P_1					-1				0
P_2				-1					0
P_3			3		-3	-2			1,05,000
P_4			-1						0
X_2		1	-1		1	-1			10,000
X_1	1					1			70,000
d_3^-			1				1		35,000
d_4^-			1	1	-1	-1		1	10,000

Solution after 2nd iteration $X_1 = 70{,}000$ $X2 = 10{,}000$
Goals 1, 2 and 4 are achieved, while Goal 3 is out by 1, 05,000

After one more iteration

	X_1	X_2	d_1^+	d_4^+	d_1^-	d_2^-	d_3^-	d_4^-	B
P_1					-1				0
P_2				-1					0
P_3			3			-2		-3	75,000
P_4				-1	-1			1	10,000
X_2		1		-1		-1		1	20,000
X_1	1					1			70,000
d_3^-				1		1	1		25,000
d_1^+				1	-1	1		1	10,000

Optimal solution $X^*_1 = 70{,}000$ $X^*_2 = 20{,}000$
Goal 1 achieved, Goal 2 achieved, Goal 3 out by 75,000, Goal 4 out by 10,000

V. CONCLUSION

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This study is more flexible approach for handling multiple objectives because in linear programming only one objective function is considered that is either maximum or minimum. By using goal programming we can consider one or more objective functions and can provide priorities for the objectives, based on priority the optimal result can be obtained. This helps the decision maker to take optimal decisions based on multiple criteria's.

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